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Chapter 2: Geometric Concepts of Data Manipulation

2.1 The euclidean distances can be computed using equation 2.1:

(a) $D_{AB} = \sqrt{(4-2)^2 + (-1-2)^2 + (0-2)^2} = 4.123$

(b) $D_{BC} = \sqrt{(2-(-5))^2 + (2-3)^2 + (2-(-2))^2} = 8.124$

(c) $D_{CO} = \sqrt{(-5-0)^2 + (3-0)^2 + (-2-0)^2} = 6.164$

2.2

(a) $\mathbf{a} + \mathbf{c} = (4\mathbf{e_1} - \mathbf{e_2}) + (-5\mathbf{e_1} + 3\mathbf{e}_2 - \mathbf{2e}_3) = -\mathbf{e}_1 + 2\mathbf{e}_2 - 2\mathbf{e}_3 = (-1,2,-2)$

(b) $3\mathbf{a} - 2\mathbf{b} + 5\mathbf{c} = 3(4\mathbf{e_1} - \mathbf{e_2}) - 2(2\mathbf{e}_1 + 2\mathbf{e}_2 + 2\mathbf{e}_3) + 5(-5\mathbf{e_1} + 3\mathbf{e}_2 - \mathbf{2e}_3)$
$= -17\mathbf{e}_1 + 8\mathbf{e}_2 - 14\mathbf{e}_3 = (-17,8,-14)$

(c) $\mathbf{ac} = \{(4)(-5) + (-1)(3) + (0)(-2)\} = -23$ (using equation 2.9)

2.3 The lengths of the vectors can be computed using equation 2.3 :

(a) $||\mathbf{a}|| = \sqrt{(3)^2 + (2)^2} = 3.606$
$||\mathbf{b}|| = \sqrt{(-5)^2 + (0)^2} = 5.000$
$||\mathbf{c}|| = \sqrt{(3)^2 + (-2)^2} = 3.606$

The angle between two vectors can be computed using equation 2.13 :

(b) $\alpha_{ab} = \cos^{-1}\left[\frac{\mathbf{ab}}{||\mathbf{a}||.||\mathbf{b}||}\right] = \cos^{-1}\left[\frac{(3)(-5)+(2)(0)}{(3.606)(5.000)}\right] = 146.299^0$
$\alpha_{ac} = \cos^{-1}\left[\frac{\mathbf{ac}}{||\mathbf{a}||.||\mathbf{c}||}\right] = \cos^{-1}\left[\frac{(3)(3)+(2)(-2)}{(3.606)(3.606)}\right] = 67.386^0$

The distance between two vectors can be computed by two methods. Using equation 2.1 :

(c) $D_{ab} = \sqrt{(3-(-5))^2 + (2-0)^2} = 8.246$
$D_{ac} = \sqrt{(3-3)^2 + (2-(-2))^2} = 4.000$

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and using equation 2.12 :

$$D_{ab} = \sqrt{3.606^2 + 5.000^2 - 2(3.606)(5.000)(\cos 146.299^0)} = 8.246$$
$$D_{ac} = \sqrt{3.606^2 + 3.606^2 - 2(3.606)(3.606)(\cos 67.386^0)} = 4.000$$

(d) Projection of **a** on **b** can be determined using equation 2.17 :

$$\mathbf{a}_p = \frac{(3.606\cos 146.299^0)(-5\mathbf{e}_1)}{5.000} = \frac{-3(-5\mathbf{e}_1)}{5} = 3\mathbf{e}_1 = (3,0)$$

Similarly projection of **a** on **c** is given by :

$$\mathbf{a}_p = \frac{(3.606\cos 67.386^0)(3\mathbf{e}_1 - 2\mathbf{e}_2)}{3.606} = \frac{1.387(3\mathbf{e}_1 - 2\mathbf{e}_2)}{3.606}$$
$$= 1.153\mathbf{e}_1 - 0.769\mathbf{e}_2 = (1.153, -0.769)$$

(e) Scalar product **ad** is given by :

$$\mathbf{ad} = (3)(2) + (2)(-3) = 0$$

Thus **a** and **d** are orthogonal.

(f) Since **a** and **d** are orthogonal, the length of the projection vector is zero and the projection vector itself is given by :

$$\mathbf{a}_p = (0,0)$$

2.4 The euclidean distances can be computed as follows :

$$D_{AB} = \sqrt{(3-(-5))^2 + (-2-3)^2} = 9.434$$
$$D_{BC} = \sqrt{(-5-0)^2 + (3-1)^2} = 5.385$$
$$D_{AC} = \sqrt{(3-0)^2 + (-2-1)^2} = 4.243$$

The new coordinates of A,B and C with respect to O^* are (see section 2.1.1) :

$$A_{new} = \{(3-2),(-2-(-5))\} = (1,3)$$
$$B_{new} = \{(-5-2),(3-(-5))\} = (-7,8)$$
$$C_{new} = \{(0-2),(1-(-5))\} = (-2,6)$$

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The distances between A,B and C based on the new coordinates are given by:

$$D_{AB_{new}} = \sqrt{(1-(-7))^2 + (3-8)^2} = 9.434$$
$$D_{BC_{new}} = \sqrt{(-7-(-2))^2 + (8-6)^2} = 5.385$$
$$D_{AC_{new}} = \sqrt{(1-(-2))^2 + (3-6)^2} = 4.243$$

It can be seen that the distances based on the new coordinates are identical to those based on the old coordinates. Thus the shift of origin did not change the orientation of the points.

2.5

(a) The new coordinates can be computed using equations 2.24 and 2.25 :

$$a_1^* = 3\cos 20^0 - 2\sin 20^0 = 2.135$$
$$a_2^* = -3\sin 20^0 - 2\cos 20^0 = -2.905$$
$$\therefore A^* = (2.135, -2.905)$$

$$b_1^* = 5\cos 20^0 + \sin 20^0 = 5.040$$
$$b_2^* = -5\sin 20^0 + \cos 20^0 = -0.770$$
$$\therefore B^* = (5.040, -0.770)$$

(b) Using equations 2.24 and 2.25, we have :

$$3.69 = 5\cos\theta + 2\sin\theta \qquad (1)$$
$$3.93 = 2\cos\theta - 5\sin\theta \qquad (2)$$

Multiplying equation (1) by -2 and equation (2) by 5 and adding the resulting equations gives:

$$12.27 = -29\sin\theta$$
$$\therefore \theta = \sin^{-1}\left(\frac{12.27}{-29}\right) = -25.03^0$$

Thus the axes are rotated clockwise by an angle of 25.03^o.

Note : If you solve for cos θ, θ works out to be 24.87^o. This value of θ does not satisfy equations (1) and (2) above. Also, since the rotation is in the clockwise direction, θ must be negative so 24.87^o is not correct.

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2.6 To represent **a** with respect to $\mathbf{f}_1$ and $\mathbf{f}_2$ we need to represent $\mathbf{e}_1$ and $\mathbf{e}_2$ with respect to $\mathbf{f}_1$ and $\mathbf{f}_2$. This can be done as follows :

$$\mathbf{e}_1 = \frac{\mathbf{f}_1 - .600\mathbf{e}_2}{.800} \qquad \text{using the first given relation} \qquad - \quad (3)$$

Substituting $\mathbf{e}_1$ in the second given relation gives :

$$\mathbf{f_2} = .707\left(\frac{\mathbf{f}_1 - .600\mathbf{e}_2}{.800}\right) + .707\mathbf{e_2}$$

Solving for $\mathbf{e}_2$ gives :

$$\mathbf{e}_2 = -5\mathbf{f}_1 + 5.658\mathbf{f}_2$$

Substituting the above in (3) gives :

$$\mathbf{e}_1 = 5\mathbf{f}_1 - 4.243\mathbf{f}_2$$

Thus,

$$\begin{aligned}\mathbf{a} &= .500(5\mathbf{f}_1 - 4.243\mathbf{f}_2) + .866(-5\mathbf{f}_1 + 5.658\mathbf{f_2}) \\ &= -1.83\mathbf{f}_1 + 2.778\mathbf{f}_2\end{aligned}$$

The representation of **b** with respect to $\mathbf{e}_1$ and $\mathbf{e}_2$ can be obtained as follows :

$$\begin{aligned}\mathbf{b} &= .700(.800\mathbf{e}_1 + .600\mathbf{e}_2) + .500(.707\mathbf{e}_1 + .707\mathbf{e}_2) \\ &= .9135\mathbf{e}_1 + .7735\mathbf{e}_2\end{aligned}$$

The angle between **a** and **b** is given by :

$$\alpha = \cos^{-1}\left[\frac{(.5)(.9135) + (.866)(.7735)}{\sqrt{.5^2 + .866^2}\,.\sqrt{.9135^2 + .7735^2}}\right] = 19.743^0$$

2.7 Distance traveled by car A = 50 × 1.75 = 87.5 miles

Distance traveled by car B = 55 × 1.75 = 96.25 miles

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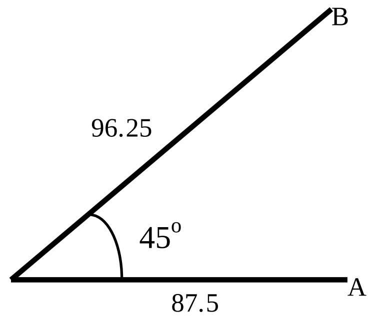
The relative positions of the 2 cars can be shown as under :

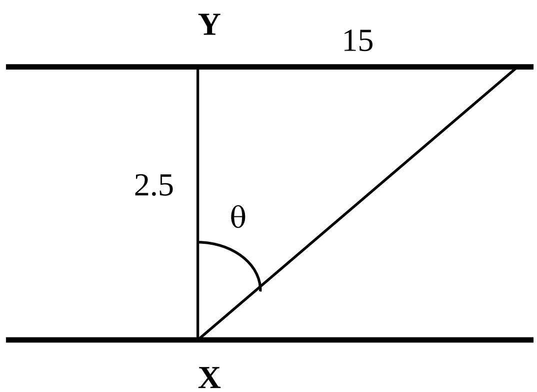


The distance between the two cars can be computed using equation 2.12 :

$$D_{AB} = \sqrt{96.25^2 + 87.5^2 - 2(96.25)(87.5)\cos 45^0} = 70.781 \text{ miles}$$

2.8 A schematic representation of the problem is given below :

City B



City A

The angle can then be computed as follows :

$$\therefore \theta = \tan^{-1}\left(\frac{15}{2.5}\right) = 80.54^0$$

2.9 *Enterprise*'s position is given by :

$$Ent. = 0.5\mathbf{e}_1 + 2\mathbf{e_2}$$

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where $\mathbf{e}_1$ and $\mathbf{e}_2$ are orthogonal basis vectors with Sun as the origin. Thus *Bakh-ra*'s position with respect to the same basis vectors is also :

$$Bak. = 0.5\mathbf{e}_1 + 2\mathbf{e}_2$$

In order to express *Bakh-ra*'s position with respect to $\mathbf{k}_1$ and $\mathbf{k}_2$, the oblique basis vectors with Sun as the origin, we need to express $\mathbf{e}_1$ and $\mathbf{e}_2$ with respect to $\mathbf{k}_1$ and $\mathbf{k}_2$. This can be done as follows :

$$\mathbf{e}_1 = \frac{\mathbf{k}_1 - .586\mathbf{e}_2}{.810} \quad \text{using the first given relation} \quad - \quad (4)$$

Substituting $\mathbf{e}_1$ in the second given relation gives :

$$\mathbf{k}_2 = .732\left(\frac{\mathbf{k}_1 - .586\mathbf{e}_2}{.810}\right) + .681\mathbf{e}_2$$

Solving for $\mathbf{e}_2$ gives :

$$\mathbf{e}_2 = -5.951\mathbf{k}_1 + 6.585\mathbf{k}_2$$

Substituting the above in (4) gives :

$$\mathbf{e}_1 = 5.537\mathbf{k}_1 - 4.764\mathbf{k}_2$$

Thus,

$$\begin{aligned} Bak. &= .500(5.537\mathbf{k}_1 - 4.764\mathbf{k}_2) + 2(-5.951\mathbf{k}_1 + 6.585\mathbf{k}_2) \\ &= -9.134\mathbf{k}_1 + 10.788\mathbf{k}_2 \end{aligned}$$

Thus *Bakh-ra*'s position is (-9.134,10.788) using the Kling-on system of axes.

The distance between Earth and Kling-on is given by :

$$D_{EK} = \sqrt{(5.2 - 2.5)^2 + (-1.5 - 3.2)^2} = 5.42 \text{ light years}$$