CHAPTER 2

Section 2-1

Provide a reasonable description of the sample space for each of the random experiments in Exercises 2-1 to 2-17. There can be more than one acceptable interpretation of each experiment. Describe any assumptions you make.

2-1. Each of three machined parts is classified as either above or below the target specification for the part. Let *a* and *b* denote a part above and below the specification, respectively.

$$S = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$$

2-2. Each of four transmitted bits is classified as either in error or not in error.

Let *e* and *o* denote a bit in error and not in error (*o* denotes okay), respectively.

$$S = \left\{\begin{matrix} eeee, eoee, oeee, ooee, \\ eeeo, eoeo, oeeo, ooeo, \\ eeoe, eooe, oeoe, oooe, \\ eeoo, eooo, oeoo, oooo \end{matrix}\right\}$$

2-3. In the final inspection of electronic power supplies, either units pass, or three types of nonconformities might occur: functional, minor, or cosmetic. Three units are inspected.

Let *a* denote an acceptable power supply.
Let *f*, *m*, and *c* denote a power supply that has a functional, minor, or cosmetic error, respectively.

$$S = \{a, f, m, c\}$$

2-4. The number of hits (views) is recorded at a high-volume Web site in a day.

$S = \{0,1,2,\ldots\}$ = set of nonnegative integers

2-5. Each of 24 Web sites is classified as containing or not containing banner ads.

Let *y* and *n* denote a web site that contains and does not contain banner ads.
The sample space is the set of all possible sequences of *y* and *n* of length 24. An example outcome in the sample space is $S = \{yynnynyyymynynnnnnyynnyy\}$

2-6. An ammeter that displays three digits is used to measure current in milliamperes.

A vector with three components can describe the three digits of the ammeter. Each digit can be 0,1,2,...,9.
The sample space *S* is 1000 possible three digit integers, $S = \{000, 001, \ldots, 999\}$

2-7. A scale that displays two decimal places is used to measure material feeds in a chemical plant in tons.

S is the sample space of 100 possible two digit integers.

2-8. The following two questions appear on an employee survey questionnaire. Each answer is chosen from the five point scale 1 (never), 2, 3, 4, 5 (always).

Is the corporation willing to listen to and fairly evaluate new ideas?
How often are my coworkers important in my overall job performance?

Let an ordered pair of numbers, such as 43 denote the response on the first and second question. Then, S consists of the 25 ordered pairs $\{11, 12, \ldots, 55\}$

2-9. The concentration of ozone to the nearest part per billion.

$S = \{0,1,2,\ldots,1E09\}$ in ppb.

2-10. The time until a service transaction is requested of a computer to the nearest millisecond.

$S = \{0,1,2,\ldots,\}$ in milliseconds

2-11. The pH reading of a water sample to the nearest tenth of a unit.

$S = \{1.0,1.1,1.2,\ldots 14.0\}$

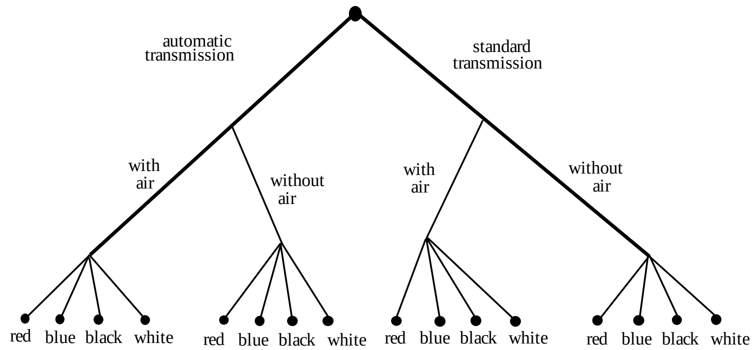
2-12. The voids in a ferrite slab are classified as small, medium, or large. The number of voids in each category is measured by an optical inspection of a sample.

Let s, m, and l denote small, medium, and large, respectively. Then $S = \{s, m, l, ss, sm, sl, \ldots.\}$

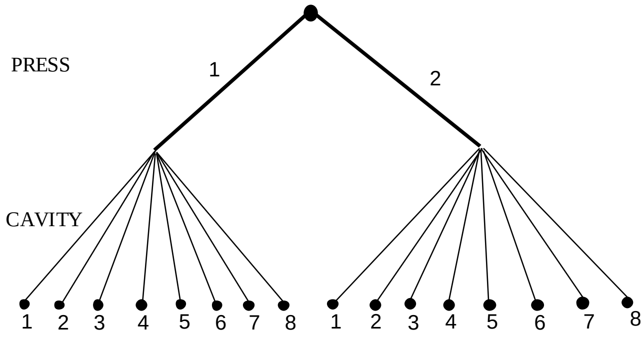
2-13 The time of a chemical reaction is recorded to the nearest millisecond.

$S = \{0,1,2,\ldots,\}$ in milliseconds.

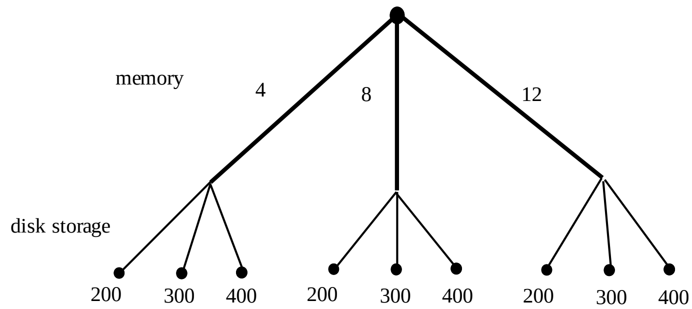
2-14. An order for an automobile can specify either an automatic or a standard transmission, either with or without air conditioning, and with any one of the four colors red, blue, black, or white. Describe the set of possible orders for this experiment.



2-15. A sampled injection-molded part could have been produced in either one of two presses and in any one of the eight cavities in each press.



2-16. An order for a computer system can specify memory of 4, 8, or 12 gigabytes and disk storage of 200, 300, or 400 gigabytes. Describe the set of possible orders.



2-17. Calls are repeatedly placed to a busy phone line until a connection is achieved.

Let *c* and *b* denote connect and busy, respectively. Then $S = \{c, bc, bbc, bbbc, bbbbc, \ldots\}$

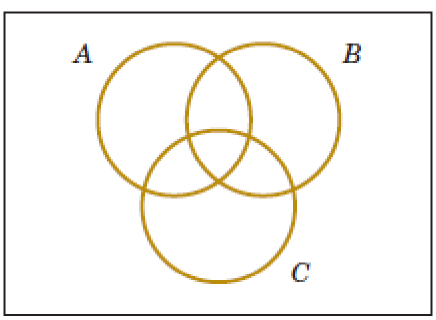
2-18. Three attempts are made to read data in a magnetic storage device before an error recovery procedure that repositions the magnetic head is used. The error recovery procedure attempts three repositionings before an "abort" message is sent to the operator. Let

s denote the success of a read operation
f denote the failure of a read operation
S denote the success of an error recovery procedure
F denote the failure of an error recovery procedure
A denote an abort message sent to the operator

Describe the sample space of this experiment with a tree diagram.

$$S = \{s, fs, ffs, fffS, fffFS, fffFFS, fffFFFA\}$$

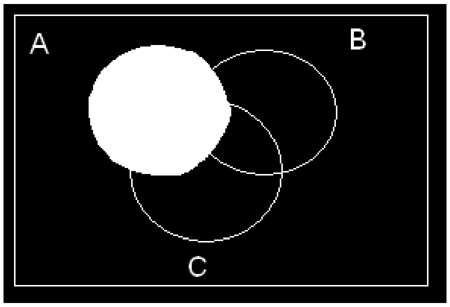
2-19. Three events are shown on the Venn diagram in the following figure:



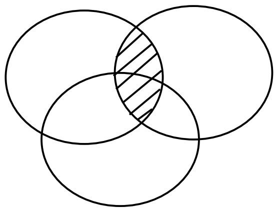
Reproduce the figure and shade the region that corresponds to each of the following events.

(a) A' (b) $A \cap B$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $(A \cap B)' \cup C$

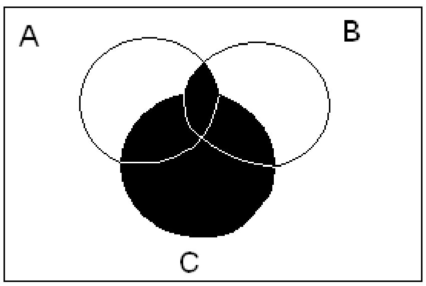
(a)



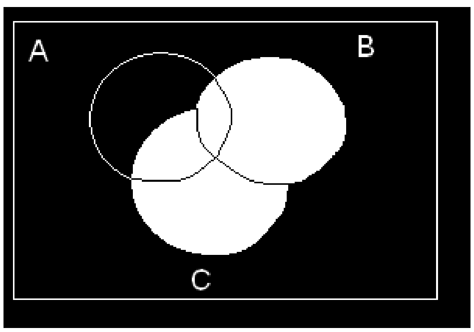
(b)



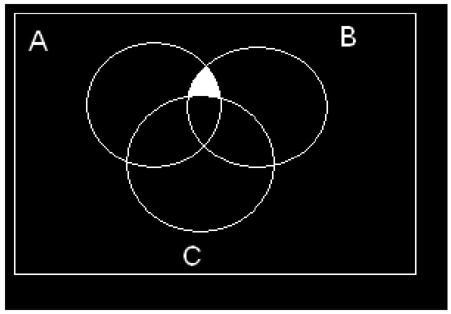
(c)



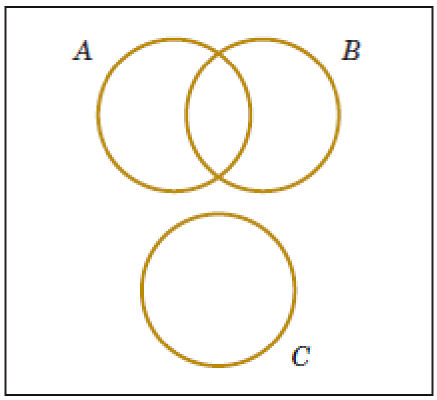
(d)



(e)



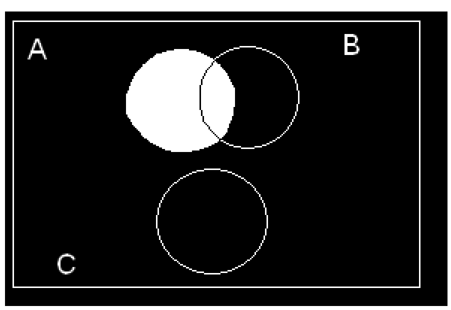
2-20. Three events are shown on the Venn diagram in the following figure:



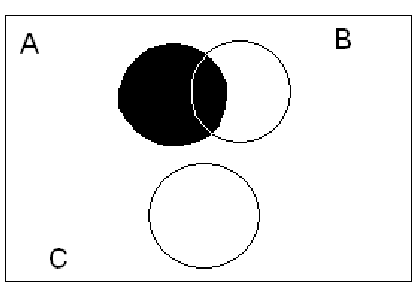
Reproduce the figure and shade the region that corresponds to each of the following events.

(a) A' (b) $(A \cap B) \cup (A \cap B')$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $(A \cap B)' \cup C$

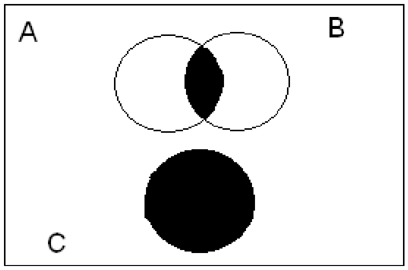
(a)



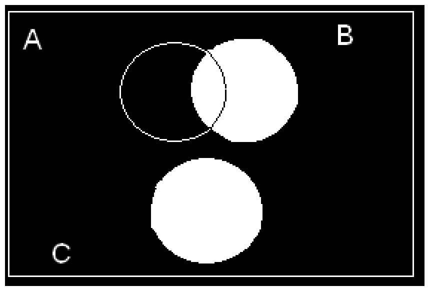
(b)



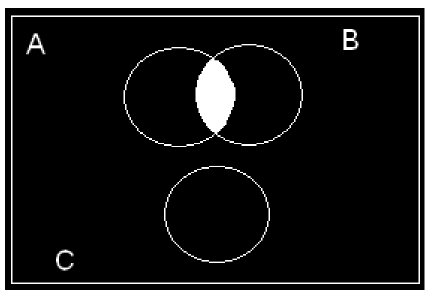
(c)



(d)



(e)



2-21. A digital scale that provides weights to the nearest gram is used.

(a) What is the sample space for this experiment?
Let A denote the event that a weight exceeds 11 grams, let B denote the event that a weight is less than or equal to 15 grams, and let C denote the event that a weight is greater than or equal to 8 grams and less than 12 grams.

Describe the following events.

(b) $A \cup B$ (c) $A \cap B$ (d) A' (e) $A \cup B \cup C$ (f) $(A \cup C)'$ (g) $A \cap B \cap C$ (h) $B' \cap C$ (i) $A \cup (B \cap C)$

(a) Let S = the nonnegative integers from 0 to the largest integer that can be displayed by the scale.
Let X denote the weight.

A is the event that $X > 11$ B is the event that $X \le 15$ C is the event that $8 \le X < 12$
$S = \{0, 1, 2, 3, \ldots\}$
(b) S
(c) $11 < X \le 15$ or $\{12, 13, 14, 15\}$
(d) $X \le 11$ or $\{0, 1, 2, \ldots, 11\}$
(e) S
(f) $A \cup C$ contains the values of X such that: $X \ge 8$
Thus $(A \cup C)'$ contains the values of X such that: $X < 8$ or $\{0, 1, 2, \ldots, 7\}$
(g) $\varnothing$

(h) B' contains the values of X such that $X > 15$. Therefore, $B' \cap C$ is the empty set. They have no outcomes in common or $\varnothing$.
(i) $B \cap C$ is the event $8 \le X < 12$. Therefore, $A \cup (B \cap C)$ is the event $X \ge 8$ or $\{8, 9, 10, \ldots\}$

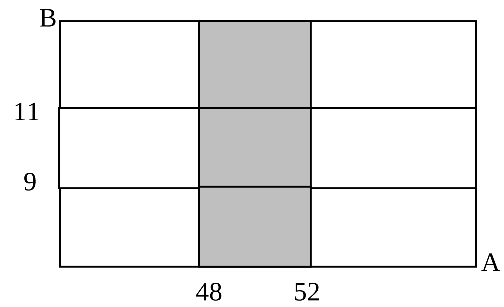
2-22. In an injection-molding operation, the length and width, denoted as X and Y, respectively, of each molded part are evaluated. Let

A denote the event of $48 < X < 52$ centimeters
B denote the event of $9 < Y < 11$ centimeters

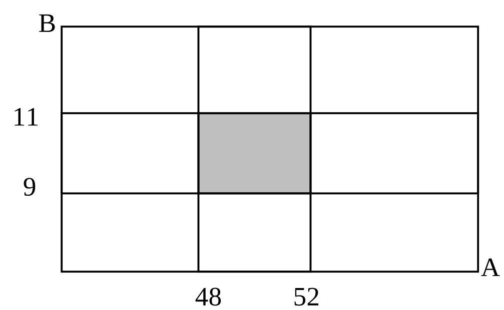
Construct a Venn diagram that includes these events. Shade the areas that represent the following:
(a) A (b) $A \cap B$ (c) $A' \cup B$ (d) $A \cap B$

(e) If these events were mutually exclusive, how successful would this production operation be? Would the process produce parts with $X = 50$ centimeters and $Y = 10$ centimeters?

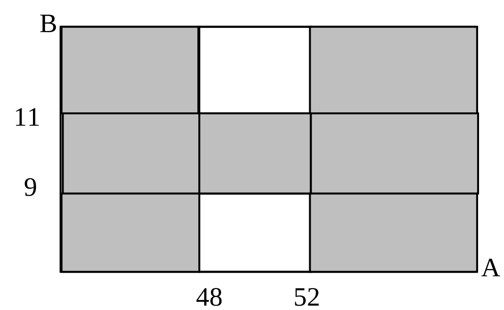
(a)



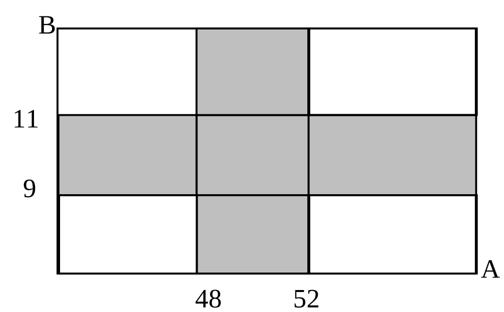
(b)



(c)



(d)



(e) If the events are mutually exclusive, then A∩B is the null set. Therefore, the process does not produce product parts with X = 50 cm and Y = 10 cm. The process would not be successful.

2-23. Four bits are transmitted over a digital communications channel. Each bit is either distorted or received without distortion. Let Ai denote the event that the ith bit is distorted, i = 1,....... ,4.

(a) Describe the sample space for this experiment.
(b) Are the A_i's mutually exclusive?

Describe the outcomes in each of the following events:

(c) A_1 (d) A_1' (e) $A_1 \cap A_2 \cap A_3 \cap A_4$ (f) $(A_1 \cap A_2) \cup (A_3 \cap A_4)$

Let d and o denote a distorted bit and one that is not distorted (o denotes okay), respectively.

(a) $S = \left\{ \begin{matrix} dddd, dodd, oddd, oodd, \\ dddo, dodo, oddo, oodo, \\ ddod, dood, odod, oood, \\ ddoo, dooo, odoo, oooo \end{matrix} \right\}$

(b) No, for example $A_1 \cap A_2 = \{dddd, dddo, ddod, ddoo\}$

(c) $A_1 = \left\{ \begin{matrix} dddd, dodd, \\ dddo, dodo \\ ddod, dood \\ ddoo, dooo \end{matrix} \right\}$

(d) $A_1' = \left\{ \begin{matrix} oddd, oodd, \\ oddo, oodo, \\ odod, oood, \\ odoo, oooo \end{matrix} \right\}$

(e) $A_1 \cap A_2 \cap A_3 \cap A_4 = \{dddd\}$

(f) $(A_1 \cap A_2) \cup (A_3 \cap A_4) = \{dddd, dodd, dddo, oddd, ddod, oodd, ddoo\}$

2-24. In light-dependent photosynthesis, light quality refers to the wavelengths of light that are important. The wavelength of a sample of photosynthetically active radiations (PAR) is measured to the nearest nanometer. The red range is 675–700 nm and the blue range is 450–500 nm. Let A denote the event that PAR occurs in the red range, and let B denote the event that PAR occurs in the blue range. Describe the sample space and indicate each of the following events:

(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

Let w denote the wavelength. The sample space is $\{w \mid w = 0, 1, 2, \ldots\}$
(a) $A=\{w \mid w = 675, 676, \ldots, 700 \text{ nm}\}$
(b) $B=\{ w \mid w = 450, 451, \ldots, 500 \text{ nm}\}$
(c) $A \cap B = \Phi$
(d) $A \cup B = \{w \mid w = 450, 451, \ldots, 500, 675, 676, \ldots, 700 \text{ nm}\}$

2-25. In control replication, cells are replicated over a period of two days. Not until mitosis is completed can freshly synthesized DNA be replicated again. Two control mechanisms have been identified—one positive and one negative. Suppose that a replication is observed in three cells. Let A denote the event that all cells are identified as positive, and let B denote the event that all cells are negative. Describe the sample space graphically and display each of the following events:

(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

Let P and N denote positive and negative, respectively.
The sample space is {PPP, PPN, PNP, NPP, PNN, NPN, NNP, NNN}.

(a) A={ PPP }
(b) B={ NNN }
(c) $A \cap B = \Phi$
(d) $A \cup B = \{ PPP, NNN \}$

2-26. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized here:

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

Let A denote the event that a disk has high shock resistance, and let B denote the event that a disk has high scratch resistance. Determine the number of disks in $A \cap B$, A', and $A \cup B$.

$A \cap B = 70$, $A' = 14$, $A \cup B = 95$

2-27. Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarized as follows:

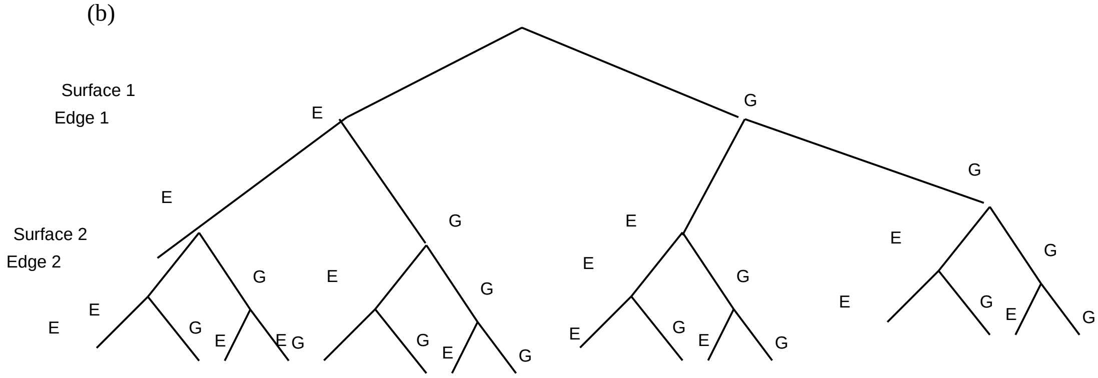
		Edge Finish	
		Excellent	Good
Surface	Excellent	80	2
Finish	Good	10	8

(a) Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent edge finish. Determine the number of samples in $A' \cap B$, B' and in $A \cup B$.

(b) Assume that each of two samples is to be classified on the basis of surface finish, either excellent or good, and on the basis of edge finish, either excellent or good. Use a tree diagram to represent the possible outcomes of this experiment.

(a) $A' \cap B = 10$, $B' = 10$, $A \cup B = 92$

(b)



2-28. Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

		Conforms	
		Yes	No
	1	22	8
Supplier	2	25	5
	3	30	10

Let A denote the event that a sample is from supplier 1, and let B denote the event that a sample conforms to specifications. Determine the number of samples in $A' \cap B$, B' and in $A \cup B$.

$A' \cap B = 55$, $B' = 23$, $A \cup B = 85$

2-29. The rise time of a reactor is measured in minutes (and fractions of minutes). Let the sample space be positive, real numbers. Define the events A and B as follows:
$A = \{x \mid x < 72.5\}$ and $B = \{x \mid x > 52.5\}$.

Describe each of the following events:

(a) A' (b) B' (c) $A \cap B$ (d) $A \cup B$

(a) $A' = \{x \mid x \geq 72.5\}$
(b) $B' = \{x \mid x \leq 52.5\}$
(c) $A \cap B = \{x \mid 52.5 < x < 72.5\}$
(d) $A \cup B = \{x \mid x > 0\}$

2-30. A sample of two items is selected without replacement from a batch. Describe the (ordered) sample space for each of the following batches:
(a) The batch contains the items $\{a, b, c, d\}$.
(b) The batch contains the items $\{a, b, c, d, e, f, g\}$.
(c) The batch contains 4 defective items and 20 good items.
(d) The batch contains 1 defective item and 20 good items.

(a) $\{ab, ac, ad, bc, bd, cd, ba, ca, da, cb, db, dc\}$
(b) $\{ab, ac, ad, ae, af, ag, ba, bc, bd, be, bf, bg, ca, cb, cd, ce, cf, cg, da, db, dc, de, df, dg, ea, eb, ec, ed, ef, eg, fa, fb, fc, fg, fd, fe, ga, gb, gc, gd, ge, gf\}$, contains 42 elements
(c) Let d and g denote defective and good, respectively. Then $S = \{gg, gd, dg, dd\}$
(d) $S = \{gd, dg, gg\}$

2-31. A sample of two printed circuit boards is selected without replacement from a batch. Describe the (ordered) sample space for each of the following batches:

(a) The batch contains 90 boards that are not defective, 8 boards with minor defects, and 2 boards with major defects.

(b) The batch contains 90 boards that are not defective, 8 boards with minor defects, and 1 board with major defects.

Let *g* denote a good board, *m* a board with minor defects, and *j* a board with major defects.

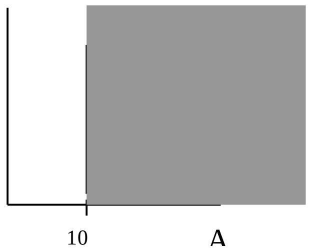
(a) $S = \{gg, gm, gj, mg, mm, mj, jg, jm, jj\}$
(b) $S = \{gg,gm,gj,mg,mm,mj,jg,jm\}$

2-32. Counts of the Web pages provided by each of two computer servers in a selected hour of the day are recorded. Let *A* denote the event that at least 10 pages are provided by server 1, and let *B* denote the event that at least 20 pages are provided by server 2. Describe the sample space for the numbers of pages for the two servers graphically in an *x* □ *y* plot. Show each of the following events on the sample space graph:

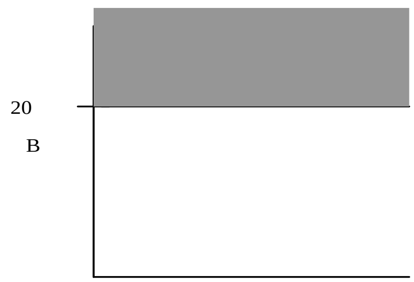
(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

(a) The sample space contains all points in the nonnegative *X*-*Y* plane.

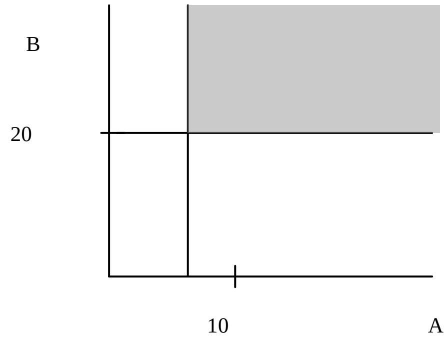
(b)



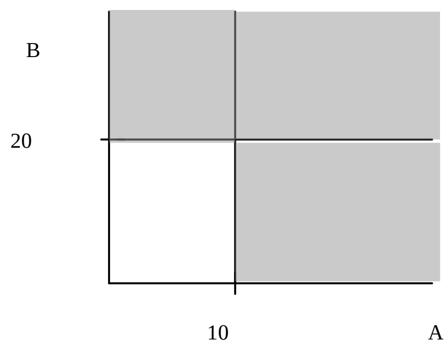
(c)



(d)



(e)

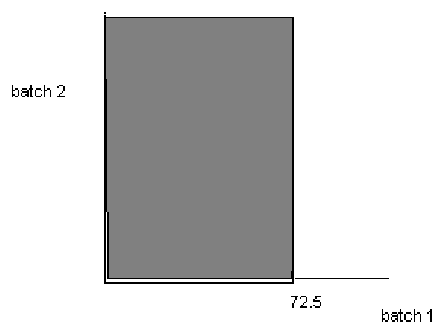


2-33. A reactor's rise time is measured in minutes (and fractions of minutes). Let the sample space for the rise time of each batch be positive, real numbers. Consider the rise times of *two* batches. Let *A* denote the event that the rise time of batch 1 is less than 72.5 minutes, and let *B* denote the event that the rise time of batch 2 is greater than 52.5 minutes.

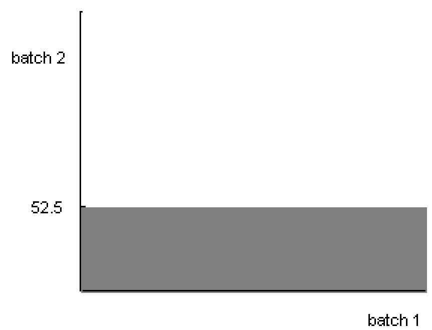
Describe the sample space for the rise time of two batches graphically and show each of the following events on a two dimensional plot:

(a) A (b) B' (c) $A \cap B$ (d) $A \cup B$

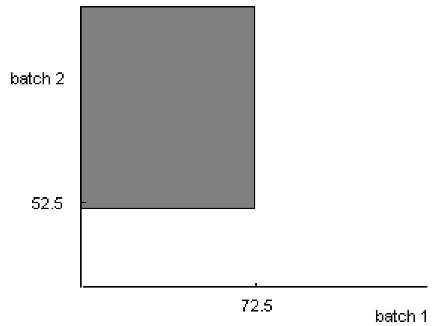
(a)



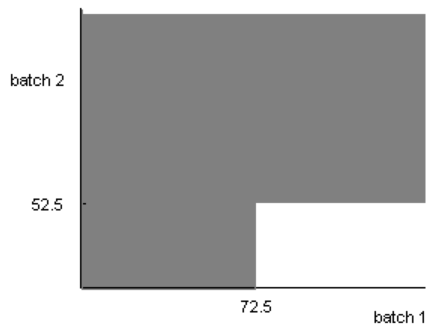
(b)



(c)



(d)



2-34. A wireless garage door opener has a code determined by the up or down setting of 12 switches. How many outcomes are in the sample space of possible codes?

$2^{12} = 4096$

2-35. An order for a computer can specify any one of five memory sizes, any one of three types of displays, and any one of four sizes of a hard disk, and can either include or not include a pen tablet. How many different systems can be ordered?

From the multiplication rule, the answer is $5 \times 3 \times 4 \times 2 = 120$

2-36. In a manufacturing operation, a part is produced by machining, polishing, and painting. If there are three machine tools, four polishing tools, and three painting tools, how many different routings (consisting of machining, followed by polishing, and followed by painting) for a part are possible?

From the multiplication rule, $3 \times 4 \times 3 = 36$

2-37. New designs for a wastewater treatment tank have proposed three possible shapes, four possible sizes, three locations for input valves, and four locations for output valves. How many different product designs are possible?

From the multiplication rule, $3 \times 4 \times 3 \times 4 = 144$

2-38. A manufacturing process consists of 10 operations that can be completed in any order. How many different production sequences are possible?

From equation 2-1, the answer is 10! = 3,628,800

2-39. A manufacturing operation consists of 10 operations. However, five machining operations must be completed before any of the remaining five assembly operations can begin. Within each set of five, operations can be completed in any order. How many different production sequences are possible?

From the multiplication rule and equation 2-1, the answer is 5!5! = 14,400

2-40. In a sheet metal operation, three notches and four bends are required. If the operations can be done in any order, how many different ways of completing the manufacturing are possible?

From equation 2-3, $\frac{7!}{3!4!} = 35$ sequences are possible

2-41. A batch of 140 semiconductor chips is inspected by choosing a sample of 5 chips. Assume 10 of the chips do not conform to customer requirements.

(a) How many different samples are possible?
(b) How many samples of five contain exactly one nonconforming chip?
(c) How many samples of five contain at least one nonconforming chip?

(a) From equation 2-4, the number of samples of size five is $\binom{140}{5} = \frac{140!}{5!135!} = 416{,}965{,}528$

(b) There are 10 ways of selecting one nonconforming chip and there are $\binom{130}{4} = \frac{130!}{4!126!} = 11{,}358{,}880$ ways of selecting four conforming chips. Therefore, the number of samples that contain exactly one nonconforming chip is $10 \times \binom{130}{4} = 113{,}588{,}800$

(c) The number of samples that contain at least one nonconforming chip is the total number of samples $\binom{140}{5}$ minus the number of samples that contain no nonconforming chips $\binom{130}{5}$. That is

$$\binom{140}{5} - \binom{130}{5} = \frac{140!}{5!135!} - \frac{130!}{5!125!} = 130{,}721{,}752$$

2-42. In the layout of a printed circuit board for an electronic product, 12 different locations can accommodate chips.

(a) If five different types of chips are to be placed on the board, how many different layouts are possible?
(b) If the five chips that are placed on the board are of the same type, how many different layouts are possible?

(a) If the chips are of different types, then every arrangement of 5 locations selected from the 12 results in a different layout. Therefore, $P_5^{12} = \frac{12!}{7!} = 95{,}040$ layouts are possible.

(b) If the chips are of the same type, then every subset of 5 locations chosen from the 12 results in a different layout. Therefore, $\binom{12}{5} = \frac{12!}{5!7!} = 792$ layouts are possible.

2-43. In the laboratory analysis of samples from a chemical process, five samples from the process are analyzed daily. In addition, a control sample is analyzed twice each day to check the calibration of the laboratory instruments.

(a) How many different sequences of process and control samples are possible each day? Assume that the five process samples are considered identical and that the two control samples are considered identical.
(b) How many different sequences of process and control samples are possible if we consider the five process samples to be different and the two control samples to be identical?
(c) For the same situation as part (b), how many sequences are possible if the first test of each day must be a control sample?

(a) $\frac{7!}{2!5!} = 21$ sequences are possible.

(b) $\frac{7!}{1!1!1!1!1!2!} = 2520$ sequences are possible.

(c) 6! = 720 sequences are possible.

2-44. In the design of an electromechanical product, 12 components are to be stacked into a cylindrical casing in a manner that minimizes the impact of shocks. One end of the casing is designated as the bottom and the other end is the top.

(a) If all components are different, how many different designs are possible?
(b) If seven components are identical to one another, but the others are different, how many different designs are possible?
(c) If three components are of one type and identical to one another, and four components are of another type and identical to one another, but the others are different, how many different designs are possible?

(a) Every arrangement selected from the 12 different components comprises a different design.
Therefore, $12! = 479{,}001{,}600$ designs are possible.

(b) 7 components are the same, others are different, $\frac{12!}{7!1!1!1!1!1!} = 95040$ designs are possible.

(c) $\frac{12!}{3!4!} = 3326400$ designs are possible.

2-45. Consider the design of a communication system.

(a) How many three-digit phone prefixes that are used to represent a particular geographic area (such as an area code) can be created from the digits 0 through 9?
(b) As in part (a), how many three-digit phone prefixes are possible that do not start with 0 or 1, but contain 0 or 1 as the middle digit?
(c) How many three-digit phone prefixes are possible in which no digit appears more than once in each prefix?

(a) From the multiplication rule, $10^3 = 1000$ prefixes are possible
(b) From the multiplication rule, $8 \times 2 \times 10 = 160$ are possible
(c) Every arrangement of three digits selected from the 10 digits results in a possible prefix.
$P_3^{10} = \frac{10!}{7!} = 720$ prefixes are possible.

2-46. A *byte* is a sequence of eight bits and each bit is either 0 or 1.

(a) How many different bytes are possible?
(b) If the first bit of a byte is a parity check, that is, the first byte is determined from the other seven bits, how many different bytes are possible?

(a) From the multiplication rule, $2^8 = 256$ bytes are possible
(b) From the multiplication rule, $2^7 = 128$ bytes are possible

2-47. In a chemical plant, 24 holding tanks are used for final product storage. Four tanks are selected at random and without replacement. Suppose that six of the tanks contain material in which the viscosity exceeds the customer requirements.

(a) What is the probability that exactly one tank in the sample contains high-viscosity material?
(b) What is the probability that at least one tank in the sample contains high-viscosity material?
(c) In addition to the six tanks with high viscosity levels, four different tanks contain material with high impurities. What is the probability that exactly one tank in the sample contains high-viscosity material and exactly one tank in the sample contains material with high impurities?

(a) The total number of samples possible is $\binom{24}{4} = \frac{24!}{4!20!} = 10{,}626.$ The number of samples in which exactly one tank has high viscosity is $\binom{6}{1}\binom{18}{3} = \frac{6!}{1!5!} \times \frac{18!}{3!15!} = 4896$. Therefore, the probability is

$$\frac{4896}{10626} = 0.461$$

(b) The number of samples that contain no tank with high viscosity is $\binom{18}{4} = \frac{18!}{4!14!} = 3060$. Therefore, the requested probability is $1 - \frac{3060}{10626} = 0.712$.

(c) The number of samples that meet the requirements is $\binom{6}{1}\binom{4}{1}\binom{14}{2} = \frac{6!}{1!5!} \times \frac{4!}{1!3!} \times \frac{14!}{2!12!} = 2184$.

Therefore, the probability is $\frac{2184}{10626} = 0.206$

2-48. Plastic parts produced by an injection-molding operation are checked for conformance to specifications. Each tool contains 12 cavities in which parts are produced, and these parts fall into a conveyor when the press opens. An inspector chooses 3 parts from among the 12 at random. Two cavities are affected by a temperature malfunction that results in parts that do not conform to specifications.

(a) How many samples contain exactly 1 nonconforming part?
(b) How many samples contain at least 1 nonconforming part?

(a) The total number of samples is $\binom{12}{3} = \frac{12!}{3!9!} = 220$. The number of samples that result in one nonconforming part is $\binom{2}{1}\binom{10}{2} = \frac{2!}{1!1!} \times \frac{10!}{2!8!} = 90$. Therefore, the requested probability is 90/220 = 0.409.

(b) The number of samples with no nonconforming part is $\binom{10}{3} = \frac{10!}{3!7!} = 120$. The probability of at least one nonconforming part is $1 - \frac{120}{220} = 0.455$.

2-49. A bin of 50 parts contains 5 that are defective. A sample of 10 parts is selected at random, without replacement. How many samples contain at least four defective parts?

From the 5 defective parts, select 4, and the number of ways to complete this step is 5!/(4!1!) = 5
From the 45 non-defective parts, select 6, and the number of ways to complete this step is 45!/(6!39!) = 8,145,060
Therefore, the number of samples that contain exactly 4 defective parts is 5(8,145,060) = 40,725,300
Similarly, from the 5 defective parts, the number of ways to select 5 is 5!(5!1!) = 1
From the 45 non-defective parts, select 5, and the number of ways to complete this step is 45!/(5!40!) = 1,221,759
Therefore, the number of samples that contain exactly 5 defective parts is
1(1,221,759) = 1,221,759

Therefore, the number of samples that contain at least 4 defective parts is
40,725,300 + 1,221,759 = 41,947,059

2-50. The following table summarizes 204 endothermic reactions involving sodium bicarbonate.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is below target. Determine the number of reactions in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

(a) $A \cap B = 56$
(b) $A' = 36 + 56 = 92$
(c) $A \cup B = 40 + 12 + 16 + 44 + 56 = 168$
(d) $A \cup B' = 40+12+16+44+36=148$
(e) $A' \cap B' = 36$

2-51. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. How many different designs are possible?

Total number of possible designs = $4\times3\times5\times3\times5=900$

2-52. Consider the hospital emergency department data given below. Let *A* denote the event that a visit is to hospital 1, and let *B* denote the event that a visit results in admittance to any hospital.

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the number of persons in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

(a) $A \cap B = 1277$
(b) $A' = 22252 - 5292 = 16960$
(c) $A \cup B = 1685 + 3733 + 1403 + 2 + 14 + 29 + 46 + 3 = 6915$
(d) $A \cup B' = 195 + 270 + 246 + 242 + 3820 + 5163 + 4728 + 3103 + 1277 = 19044$
(e) $A' \cap B' = 270 + 246 + 242 + 5163 + 4728 + 3103 = 13752$

2-53. An article in *The Journal of Data Science* ["A Statistical Analysis of Well Failures in Baltimore County" (2009, Vol. 7, pp. 111–127)] provided the following table of well failures for different geological formation groups in Baltimore County.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed. Determine the number of wells in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cap B'$ (e) $A' \cap B'$

(a) $A \cap B = 170 + 443 + 60 = 673$
(b) $A' = 28 + 363 + 309 + 933 + 39 = 1672$
(c) $A \cup B = 1685 + 3733 + 1403 + 2 + 14 + 29 + 46 + 3 = 6915$
(d) $A \cup B' = 1685 + (28 - 2) + 3733 + (363 - 14) + (309 - 29) + 1403 + (933 - 46) + (39 - 3) = 8399$
(e) $A' \cap B' = 28 - 2 + 363 - 14 + 306 - 29 + 933 - 46 + 39 - 3 = 1578$

2-54. A hospital operating room needs to schedule three knee surgeries and two hip surgeries in a day. Suppose that an operating room needs to handle three knee, four hip, and five shoulder surgeries.

(a) How many different sequences are possible?
(b) How many different sequences have all hip, knee, and shoulder surgeries scheduled consecutively?
(c) How many different schedules begin and end with a knee surgery?

(a) From the formula for the number of sequences $\frac{12!}{3!4!5!}$ = 27,720 sequences are possible.

(b) Combining all hip surgeries into one single unit, all knee surgeries into one single unit and all shoulder surgeries into one unit, the possible number of sequences of these units = 3! = 6

(c)With two surgeries specified, 10 remain and there are $\frac{10!}{4!5!1!}$ = 1,260 different sequences.

2-55. Consider the bar code code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). One code is still held back as a delimiter. For each of the following cases, how many characters can be encoded?

(a) The constraint of exactly two wide bars is replaced with one that requires exactly one wide bar.
(b) The constraint of exactly two wide bars is replaced with one that allows either one or two wide bars.
(c) The constraint of exactly two wide bars is dropped.
(d) The constraints of exactly two wide bars and one wide space are dropped.

(a) The constraint of exactly two wide bars is replaced with one that requires exactly one wide bar.
Focus first on the bars. There are 5!/(4!1!) = 5 permutations of the bars with one wide bar and four narrow bars. As in the example, the number of permutations of the spaces = 4. Therefore, the possible number of codes = 5(4) = 20, and if one is held back as a delimiter, 19 characters can be coded.

(b) The constraint of exactly two wide bars is replaced with one that allows either one or two wide bars.
As in the example, the number of codes with exactly two wide bars = 40. From part (a), the number of codes with exactly one wide bar = 20. Therefore, is the possible codes are 40 + 20 = 60, and if one code is held back as a delimiter, 59 characters can be coded.

(c) The constraint of exactly two wide bars is dropped.
There are 2 choices for each bar (wide or narrow) and 5 bars are used in total. Therefore, the number of possibilities for the bars = 2^5 = 32. As in the example, there are 4 possibilities for the spaces. Therefore, the number of codes is 32(3) = 128, and if one is held back as a delimiter, 127 characters can be coded.
(d) The constraints of exactly 2 wide bars and 1 wide space is dropped.
As in part (c), there are 32 possibilities for the bars, and there are also 2^4 = 16 possibilities for the spaces. Therefore, 32(16) = 512 codes are possible, and if one is held back as a delimiter, 511 characters can be coded.

2-56. A computer system uses passwords that contain exactly eight characters, and each character is 1 of the 26 lowercase letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let Ω denote the set of all possible passwords, and let *A* and *B* denote the events that consist of passwords with only letters or only integers, respectively. Determine the number of passwords in each of the following events.

(a) Ω (b) *A* (c) $A' \cap B'$
(d) Passwords that contain at least 1 integer
(e) Passwords that contain exactly 1 integer

Let |A| denote the number of elements in the set A.
(a) The number of passwords in Ω is $|\Omega| = 62^8$ (from multiplication rule).
(b) The number of passwords in A is $|A| = 52^8$ (from multiplication rule)
(c) $A' \cap B' = (A \cup B)'$. Also, $|A| = 52^8$ and $|B| = 10^8$ and $A \cap B$ = null. Therefore,
$(A \cup B)' = |\Omega| - |A| - |B| = 62^8 - 52^8 - 10^8 \approx 1.65 \times 10^{14}$
(d) Passwords that contain at least 1 integer = $|\Omega| - |A| = 62^8 - 52^8 \approx 1.65 \times 10^{14}$
(e) Passwords that contain exactly 1 integer. The number of passwords with 7 letters is 52^7. Also, 1 integer is selected

in 10 ways, and can be inserted into 8 positions in the password. Therefore, the solution is $8(10)(52^7) \approx 8.22 \times 10^{13}$

2-57. The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let A denote the event that the patient was treated with ribavirin plus interferon alfa, and let B denote the event that the response was complete. Determine the number of patients in each of the following events.

(a) A (b) $A \cap B$ (c) $A \cup B$ (d) $A' \cap B'$

Let |A| denote the number of elements in the set A.
(a) |A| = 21
(b) |A∩B| = 16
(c) |A∪B| = A+B - (A∩B) = 21+22 – 16 = 27
(d) |A'∩B'| = 60 - |A∪B| = 60 – 27 = 33

Section 2-2

2-58. Each of the possible five outcomes of a random experiment is equally likely. The sample space is $\{a, b, c, d, e\}$. Let A denote the event $\{a, b\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∪B) (e) P(A∩B)

All outcomes are equally likely
(a) P(A) = 2/5
(b) P(B) = 3/5
(c) P(A') = 3/5
(d) P(A∪B) = 1
(e) P(A∩B) = P(∅)= 0

2-59. The sample space of a random experiment is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∪B) (e) P(A∩B)

(a) P(A) = 0.4
(b) P(B) = 0.8
(c) P(A') = 0.6
(d) P(A∪B) = 1
(e) P(A∩B) = 0.2

2-60. Orders for a computer are summarized by the optional features that are requested as follows:

	Proportion of Orders
No optional features	0.3
One optional feature	0.5
More than one optional feature	0.2

(a) What is the probability that an order requests at least one optional feature?
(b) What is the probability that an order does not request more than one optional feature?

(a) 0.5 + 0.2 = 0.7

(b) 0.3 + 0.5 = 0.8

2-61. If the last digit of a weight measurement is equally likely to be any of the digits 0 through 9,

(a) What is the probability that the last digit is 0?
(b) What is the probability that the last digit is greater than or equal to 5?

(a) 1/10
(b) 5/10

2-62. A part selected for testing is equally likely to have been produced on any one of six cutting tools.

(a) What is the sample space?
(b) What is the probability that the part is from tool 1?
(c) What is the probability that the part is from tool 3 or tool 5?
(d) What is the probability that the part is not from tool 4?

(a) S = {1, 2, 3, 4, 5, 6}
(b) 1/6
(c) 2/6
(d) 5/6

2-63. An injection-molded part is equally likely to be obtained from any one of the eight cavities on a mold.

(a) What is the sample space?
(b) What is the probability that a part is from cavity 1 or 2?
(c) What is the probability that a part is from neither cavity 3 nor 4?

(a) S = {1,2,3,4,5,6,7,8}
(b) 2/8
(c) 6/8

2-64. In an acid-base titration, a base or acid is gradually added to the other until they have completely neutralized each other. Because acids and bases are usually colorless (as are the water and salt produced in the neutralization reaction), pH is measured to monitor the reaction. Suppose that the equivalence point is reached after approximately 100 mL of an NaOH solution has been added (enough to react with all the acetic acid present) but that replicates are equally likely to indicate from 95 to 104 mL to the nearest mL. Assume that volumes are measured to the nearest mL and describe the sample space.

(a) What is the probability that equivalence is indicated at 100 mL?
(b) What is the probability that equivalence is indicated at less than 100 mL?
(c) What is the probability that equivalence is indicated between 98 and 102 mL (inclusive)?

The sample space is {95, 96, 97,…, 103, and 104}.

(a) Because the replicates are equally likely to indicate from 95 to 104 mL, the probability that equivalence is indicated at 100 mL is 0.1.
(b) The event that equivalence is indicated at less than 100 mL is {95, 96, 97, 98, 99}. The probability that the event occurs is 0.5.
(c) The event that equivalence is indicated between 98 and 102 mL is {98, 99, 100, 101, 102}. The probability that the event occurs is 0.5.

2-65. In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states and that is usually found in the following states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

(a) What is the probability that a cell has at least one of the positive nickel-charged options?
(b) What is the probability that a cell is *not* composed of a positive nickel charge greater than +3?

The sample space is {0, +2, +3, and +4}.

(a) The event that a cell has at least one of the positive nickel charged options is {+2, +3, and +4}. The probability is 0.35 + 0.33 + 0.15 = 0.83.

(b) The event that a cell is not composed of a positive nickel charge greater than +3 is {0, +2, and +3}. The probability is 0.17 + 0.35 + 0.33 = 0.85.

2-66. A credit card contains 16 digits between 0 and 9. However, only 100 million numbers are valid. If a number is entered randomly, what is the probability that it is a valid number?

Total possible: 10^{16}, but only 10^8 are valid. Therefore, P(valid) = $10^8/10^{16} = 1/10^8$

2-67. Suppose your vehicle is licensed in a state that issues license plates that consist of three digits (between 0 and 9) followed by three letters (between *A* and *Z*). If a license number is selected randomly, what is the probability that yours is the one selected?

3 digits between 0 and 9, so the probability of any three numbers is 1/(10*10*10).
3 letters A to Z, so the probability of any three numbers is 1/(26*26*26). The probability your license plate is chosen is then $(1/10^3)*(1/26^3) = 5.7 \times 10^{-8}$

2-68. A message can follow different paths through servers on a network. The sender's message can go to one of five servers for the first step; each of them can send to five servers at the second step; each of those can send to four servers at the third step; and then the message goes to the recipient's server.

(a) How many paths are possible?
(b) If all paths are equally likely, what is the probability that a message passes through the first of four servers at the third step?

(a) 5*5*4 = 100
(b) (5*5)/100 = 25/100=1/4

2-69. Magnesium alkyls are used as homogenous catalysts in the production of linear low-density polyethylene (LLDPE), which requires a finer magnesium powder to sustain a reaction. Redox reaction experiments using four different amounts of magnesium powder are performed. Each result may or may not be further reduced in a second step using three different magnesium powder amounts. Each of these results may or may not be further reduced in a third step using three different amounts of magnesium powder.

(a) How many experiments are possible?
(b) If all outcomes are equally likely, what is the probability that the best result is obtained from an experiment that uses all three steps?
(c) Does the result in part (b) change if five or six or seven different amounts are used in the first step? Explain.

(a) The number of possible experiments is $4 + 4 \times 3 + 4 \times 3 \times 3 = 52$
(b) There are 36 experiments that use all three steps. The probability the best result uses all three steps is 36/52 = 0.6923.
(c) No, it will not change. With *k* amounts in the first step the number of experiments is $k + 3k + 9k = 13k$.
The number of experiments that complete all three steps is $9k$ out of $13k$. The probability is 9/13 = 0.6923.

2-70. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

Let *A* denote the event that a disk has high shock resistance, and let *B* denote the event that a disk has high scratch resistance. If a disk is selected at random, determine the following probabilities:

(a) P(A) (b) P(B) (c) P(A') (d) $P(A \cap B)$ (e) $P(A \cup B)$ (f) $P(A' \cup B)$

(a) P(A) = 86/100 = 0.86
(b) P(B) = 79/100 = 0.79
(c) P(A') = 14/100 = 0.14
(d) P(A∩B) = 70/100 = 0.70
(e) P(A∪B) = (70+9+16)/100 = 0.95
(f) P(A'∪B) = (70+9+5)/100 = 0.84

2-71. Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

		Conforms	
		Yes	No
	1	22	8
Supplier	2	25	5
	3	30	10

Let *A* denote the event that a sample is from supplier 1, and let *B* denote the event that a sample conforms to specifications. If a sample is selected at random, determine the following probabilities:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∩B) (e) P(A∪B) (f) P(A'∪B)

(a) P(A) = 30/100 = 0.30
(b) P(B) = 77/100 = 0.77
(c) P(A') = 1 – 0.30 = 0.70
(d) P(A∩B) = 22/100 = 0.22
(e) P(A∪B) = 85/100 = 0.85
(f) P(A'∪B) =92/100 = 0.92

2-72. An article in the *Journal of Database Management* ["Experimental Study of a Self-Tuning Algorithm for DBMS Buffer Pools" (2005, Vol. 16, pp. 1–20)] provided the workload used in the TPC-C OLTP (Transaction Processing Performance Council's Version C On-Line Transaction Processing) benchmark, which simulates a typical order entry application.

TABLE • 2E-1 Average Frequencies and Operations in TPC-C

Transaction	Frequency	Selects	Updates	Inserts	Deletes	Nonunique Selects	Joins
New order	43	23	11	12	0	0	0
Payment	44	4.2	3	1	0	0.6	0
Order status	4	11.4	0	0	0	0.6	0
Delivery	5	130	120	0	10	0	0
Stock level	4	0	0	0	0	0	1

The frequency of each type of transaction (in the second column) can be used as the percentage of each type of transaction. The average number of *selects* operations required for each type of transaction is shown. Let *A* denote the event of transactions with an average number of *selects* operations of 12 or fewer. Let *B* denote the event of transactions with an average number of *updates* operations of 12 or fewer. Calculate the following probabilities:

(a) P(A) (b) P(B) (c) P(A∩B) (d) P(A∩B') (f) P(A∪B)

(a) The total number of transactions is 43+44+4+5+4=100

$$P(A) = \frac{44+4+4}{100} = 0.52$$

(b) $P(B) = \dfrac{100-5}{100} = 0.95$

(c) $P(A \cap B) = \dfrac{44+4+4}{100} = 0.52$

(d) $P(A \cap B') = 0$

(e) $P(A \cup B) = \dfrac{100-5}{100} = 0.95$

2-73. Use the axioms of probability to show the following: $A \cup B$(d) $A \cap B'$

(a) For any event E, $P(E') = 1 - P(E)$. (b) $P(\varnothing) = 0$ (c) If A is contained in B, then $P(A) \le P(B)$.

(a) Because E and E' are mutually exclusive events and $E \cup E' = S$
$1 = P(S) = P(E \cup E') = P(E) + P(E')$. Therefore, $P(E') = 1 - P(E)$

(b) Because S and $\varnothing$ are mutually exclusive events with $S = S \cup \varnothing$
$P(S) = P(S) + P(\varnothing)$. Therefore, $P(\varnothing) = 0$

(c) Now, $B = A \cup (A' \cap B)$ and the events A and $A' \cap B$ are mutually exclusive. Therefore,
$P(B) = P(A) + P(A' \cap B)$. Because $P(A' \cap B) \ge 0$, $P(B) \ge P(A)$.

2.74. Consider the endothermic reaction's table given below. Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A \cup B')$ (e) $P(A' \cap B')$

(a) $P(A \cap B) = (40 + 16)/204 = 0.2745$
(b) $P(A') = (36 + 56)/204 = 0.4510$
(c) $P(A \cup B) = (40 + 12 + 16 + 44 + 36)/204 = 0.7255$
(d) $P(A \cup B') = (40 + 12 + 16 + 44 + 56)/204 = 0.8235$
(e) $P(A' \cap B') = 56/204 = 0.2745$

2-75. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. If you visit the site five times, what is the probability that you will not see the same design?

A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. If you visit the site five times, what is the probability that you will not see the same design?

Total number of possible designs is 900. The sample space of all possible designs that may be seen on five visits. This space contains 900^5 outcomes.

The number of outcomes in which all five visits are different can be obtained as follows. On the first visit any one of 900 designs may be seen. On the second visit there are 899 remaining designs. On the third visit there are 898 remaining designs. On the fourth and fifth visits there are 897 and 896 remaining designs, respectively. From the multiplication rule, the number of outcomes where all designs are different is 900*899*898*897*896. Therefore, the probability that a design is not seen again is

$$(900*899*898*897*896)/ 900^5 = 0.9889$$

2-76. Consider the hospital emergency room data is given below. Let A denote the event that a visit is to hospital 4, and let B denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A \cup B')$ (e) $P(A' \cap B')$

(a) $P(A \cap B)$ = 242/22252 = 0.0109
(b) $P(A')$ = (5292+6991+5640)/22252 = 0.8055
(c) $P(A \cup B)$ = (195 + 270 + 246 + 242 + 984 + 3103)/22252 = 0.2265
(d) $P(A \cup B')$ = (4329 + (5292 – 195) + (6991 – 270) + 5640 – 246))/22252 = 0.9680
(e) $P(A' \cap B')$ = (1277 + 1558 + 666 + 3820 + 5163 + 4728)/22252 = 0.7735

2-77. Consider the well failure data is given below. Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A \cup B')$ (e) $P(A' \cap B')$

(a) P(A ∩ B) = (170 + 443 + 60)/8493 = 0.0792
(b) P(A′) = (28 + 363 + 309 + 933 + 39)/8493 = 1672/8493 = 0.1969
(c) P(A ∪ B) = (1685+3733+1403+2+14+29+46+3)/8493 = 6915/8493 = 0.8142
(d) P(A ∪ B′) = (1685 + (28 – 2) + 3733 + (363 – 14) + (309 – 29) + 1403 + (933 – 46) + (39 – 3))/8493 = 8399/8493 = 0.9889
(e) P(A′ ∩ B′) = (28 – 2 + 363 – 14 + 306 – 29 + 933 – 46 + 39 – 3)/8493 = 1578/8493 = 0.1858

2-78. Consider the bar code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). Suppose that all 40 codes are equally likely (none is held back as a delimiter).

Determine the probability for each of the following:
(a) A wide space occurs before a narrow space.
(b) Two wide bars occur consecutively.
(c) Two consecutive wide bars are at the start or end.
(d) The middle bar is wide.

(a) There are 4 spaces and exactly one is wide.
Number of permutations of the spaces where the wide space appears first is 1.
Number of permutations of the bars is 5!/(2!3!) = 10.
Total number of permutations where a wide space occurs before a narrow space 1(10) = 10.

P(wide space occurs before a narrow space) =10/40 = 1/4

(b) There are 5 bars and 2 are wide.
The spaces are handled as in part (a).
Number of permutations of the bars where 2 wide bars are consecutive is 4.
Therefore, the probability is 16/40 = 0.4

(c) The spaces are handled as in part (a).
Number of permutations of the bars where the 2 consecutive wide bars are at the start or end is 2. Therefore, the probability is 8/40 = 0.2

(d) The spaces are handled as in part (a).
Number of permutations of the bars where a wide bar is at the center is 4 because there are 4 remaining positions for the second wide bar. Therefore, the probability is 16/40 = 0.4.

2-79. A hospital operating room needs to schedule three knee surgeries and two hip surgeries in a day. Suppose that an operating room needs to schedule three knee, four hip, and five shoulder surgeries. Assume that all schedules are equally likely.

Determine the probability for each of the following:
(a) All hip surgeries are completed before another type of surgery.
(b) The schedule begins with a hip surgery.
(c) The fi rst and last surgeries are hip surgeries.
(d) The fi rst two surgeries are hip surgeries.

(a) P(all hip surgeries before another type)= $\frac{\frac{8!}{3!5!}}{\frac{12!}{3!4!5!}} = \frac{8!4!}{12!} = \frac{1}{495} = 0.00202$

(b) P(begins with hip surgery)= $\frac{\frac{11!}{3!3!5!}}{\frac{12!}{3!4!5!}} = \frac{11!4!}{12!3!} = \frac{1}{3}$

(c) P(first and last are hip surgeries)= $\frac{\frac{10!}{2!3!5!}}{\frac{12!}{3!4!5!}} = \frac{1}{11}$

(d) P(first two are hip surgeries)= $\frac{\frac{10!}{2!3!5!}}{\frac{12!}{3!4!5!}} = \frac{1}{11}$

2-80. Suppose that a patient is selected randomly from the those described ,The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let *A* denote the event that the patient is in the group treated with interferon alfa, and let *B* denote the event that the patient has a complete response.
Determine the following probabilities.

(a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$ (e) $P(A' \cup B)$

(a) P(A)= 19/60 = 0.3167
(b) P(B)= 22/60 = 0.3667
(c) P(A∩B) = 6/60 = 0.1
(d) P(AUB) = P(A)+P(B)-P(A∩B) = (19+22-6)/60 =0.5833
(e) P(A'UB) = P(A')+P(B)-P(A'∩B) = $\frac{21+20}{60} + \frac{22}{60} - \frac{16}{60} = 0.7833$