Contents

Chapter 1

Functions

1.1 Review of Functions

1.1.1 A function is a rule that assigns each to each value of the independent variable in the domain a unique value of the dependent variable in the range.

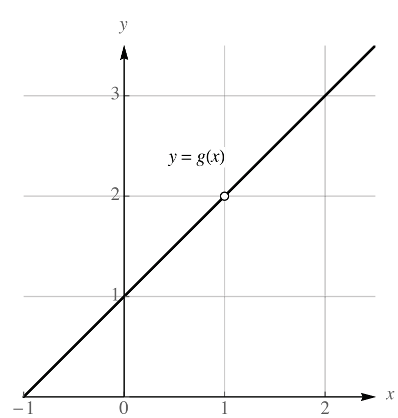
1.1.2 The independent variable belongs to the domain, while the dependent variable belongs to the range.

1.1.3 Graph A does not represent a function, while graph B does. Note that graph A fails the vertical line test, while graph B passes it.

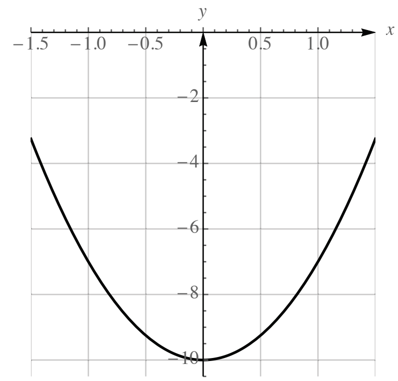
1.1.4 The domain of f is $[1, 4)$, while the range of f is $(1, 5]$. Note that the domain is the "shadow" of the graph on the x-axis, while the range is the "shadow" of the graph on the y-axis.

1.1.5 Item i. is true while item ii. isn't necessarily true. In the definition of function, item i. is stipulated. However, item ii. need not be true – for example, the function $f(x) = x^2$ has two different domain values associated with the one range value 4, because $f(2) = f(-2) = 4$.

1.1.6 $g(x) = \frac{x^2+1}{x-1} = \frac{(x+1)(x-1)}{x-1} = x + 1,\ x \neq 1$. The domain is $\{x : x \neq 1\}$ and the range is $\{x : x \neq 2\}$.



1.1.7 The domain of this function is the set of a real numbers. The range is $[-10, \infty)$.



1.1.8 The independent variable t is elapsed time and the dependent variable d is distance above the ground. The domain in context is $[0, 8]$

1.1.9 The independent variable h is the height of the water in the tank and the dependent variable V is the volume of water in the tank. The domain in context is $[0, 50]$

1.1.10 $f(2) = \dfrac{1}{2^3+1} = \dfrac{1}{9}$. $f(y^2) = \frac{1}{(y^2)^3+1} = \frac{1}{y^6+1}$.

1.1.11 $f(g(1/2)) = f(-2) = -3$; $g(f(4)) = g(9) = \frac{1}{8}$; $g(f(x)) = g(2x+1) = \dfrac{1}{(2x+1)-1} = \dfrac{1}{2x}$.

1.1.12 One possible answer is $g(x) = x^2 + 1$ and $f(x) = x^5$, because then $f(g(x)) = f(x^2+1) = (x^2+1)^5$. Another possible answer is $g(x) = x^2$ and $f(x) = (x+1)^5$, because then $f(g(x)) = f(x^2) = (x^2+1)^5$.

1.1.13 The domain of $f \circ g$ consists of all x in the domain of g such that $g(x)$ is in the domain of f.

1.1.14 $(f \circ g)(3) = f(g(3)) = f(25) = \sqrt{25} = 5$.
$(f \circ f)(64) = f(\sqrt{64}) = f(8) = \sqrt{8} = 2\sqrt{2}$.
$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = x^{3/2} - 2$.
$(f \circ g)(x) = f(g(x)) = f(x^3 - 2) = \sqrt{x^3 - 2}$

1.1.15

a. $(f \circ g)(2) = f(g(2)) = f(2) = 4$.

b. $g(f(2)) = g(4) = 1$.

c. $f(g(4)) = f(1) = 3$.

d. $g(f(5)) = g(6) = 3$.

e. $f(f(8)) = f(8) = 8$.

f. $g(f(g(5))) = g(f(2)) = g(4) = 1$.

1.1.16

a. $h(g(0)) = h(0) = -1$.

b. $g(f(4)) = g(-1) = -1$.

c. $h(h(0)) = h(-1) = 0$.

d. $g(h(f(4))) = g(h(-1)) = g(0) = 0$.

e. $f(f(f(1))) = f(f(0)) = f(1) = 0$.

f. $h(h(h(0))) = h(h(-1)) = h(0) = -1$.

g. $f(h(g(2))) = f(h(3)) = f(0) = 1$.

h. $g(f(h(4))) = g(f(4)) = g(-1) = -1$.

i. $g(g(g(1))) = g(g(2)) = g(3) = 4$.

j. $f(f(h(3))) = f(f(0)) = f(1) = 0$.

1.1.17 $\dfrac{f(5) - f(0)}{5 - 0} = \dfrac{83 - 6}{5} = 15.4$; the radiosonde rises at an average rate of 15.4 ft/s during the first 5 seconds after it is released.

1.1.18 $f(0) = 0$. $f(34) = 127852.4 - 109731 = 18121.4$. $f(64) = 127852.4 - 75330.4 = 52522$.

$$\frac{f(64) - f(34)}{64 - 34} = \frac{52522 - 18121.4}{30} \approx 1146.69 \text{ ft/s}.$$

1.1.19 $f(-2) = f(2) = 2$; $g(-2) = -g(2) = -(-2) = 2$; $f(g(2)) = f(-2) = f(2) = 2$; $g(f(-2)) = g(f(2)) = g(2) = -2$.

1.1.20 The graph would be the result of leaving the portion of the graph in the first quadrant, and then also obtaining a portion in the third quadrant which would be the result of reflecting the portion in the first quadrant around the y-axis and then the x-axis.

1.1.21 Function A is symmetric about the y-axis, so is even. Function B is symmetric about the origin so is odd. Function C is symmetric about the y-axis, so is even.

1.1.22 Function A is symmetric about the y-axis, so is even. Function B is symmetric about the origin, so is odd. Function C is also symmetric about the origin, so is odd.

1.1.23 $f(x) = \dfrac{x^2 - 5x + 6}{x - 2} = \dfrac{(x-2)(x-3)}{x-2} = x - 3$, $x \neq 2$. The domain of f is $\{x : x \neq 2\}$. The range is $\{y : y \neq -1\}$.

1.1.24 $f(x) = \frac{x-2}{2-x} = \frac{x-2}{-(x-2)} = -1$, $x \neq 2$. The domain is $\{x : x \neq 2\}$. The range is $\{-1\}$.

1.1.25 The domain of the function is the set of numbers x which satisfy $7 - x^2 \geq 0$. This is the interval $[-\sqrt{7}, \sqrt{7}]$. Note that $f(\sqrt{7}) = 0$ and $f(0) = \sqrt{7}$. The range is $[0, \sqrt{7}]$.

1.1.26 The domain of the function is the set of numbers x which satisfy $25 - x^2 \geq 0$. This is the interval $[-5, 5]$. Note that $f(0) = -5$ and $f(5) = 0$. The range is $[-5, 0]$.

1.1.27 Because the cube root function is defined for all real numbrs, the domain is $\mathbb{R}$, the set of all real numbers.

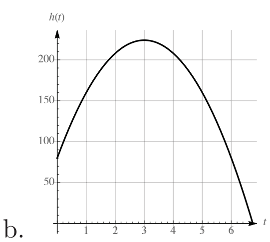
1.1.28 The domain consists of the set of numbers w for which $2 - w \geq 0$, so the interval $(-\infty, 2]$.

1.1.29 The domain consists of the set of numbers x for which $9 - x^2 \geq 0$, so the interval $[-3, 3]$.

1.1.30 Because $1 + t^2$ is never zero for any real numbered value of t, the domain of this function is $\mathbb{R}$, the set of all real numbers.

1.1.31

a. The formula for the height of the rocket is valid from $t = 0$ until the rocket hits the ground, which is the positive solution to $-16t^2 + 96t + 80 = 0$, which the quadratic formula reveals is $t = 3 + \sqrt{14}$. Thus, the domain is $[0, 3 + \sqrt{14}]$.

b. 

The maximum appears to occur at $t = 3$. The height at that time would be 224.

1.1.32

a. $d(0) = (10 - (2.2) \cdot 0)^2 = 100$.

b. The tank is first empty when $d(t) = 0$, which is when $10 - (2.2)t = 0$, or $t = 50/11$.

c. An appropriate domain would $[0, 50/11]$.

1.1.33 $g(1/z) = (1/z)^3 = \frac{1}{z^3}$

1.1.34 $F(y^4) = \frac{1}{y^4-3}$

1.1.35 $F(g(y)) = F(y^3) = \frac{1}{y^3-3}$

1.1.36 $f(g(w)) = f(w^3) = (w^3)^2 - 4 = w^6 - 4$

1.1.37 $g(f(u)) = g(u^2 - 4) = (u^2 - 4)^3$

1.1.38 $\dfrac{f(2+h) - f(2)}{h} = \dfrac{(2+h)^2 - 4 - 0}{h} = \dfrac{4 + 4h + h^2 - 4}{h} = \dfrac{4h + h^2}{h} = 4 + h$

1.1.39 $F(F(x)) = F\left(\dfrac{1}{x-3}\right) = \dfrac{1}{\frac{1}{x-3} - 3} = \dfrac{1}{\frac{1}{x-3} - \frac{3(x-3)}{x-3}} = \dfrac{1}{\frac{10-3x}{x-3}} = \dfrac{x-3}{10-3x}$

1.1.40 $g(F(f(x))) = g(F(x^2-4)) = g\left(\dfrac{1}{x^2-4-3}\right) = \left(\dfrac{1}{x^2-7}\right)^3$

1.1.41 $f(\sqrt{x+4}) = (\sqrt{x+4})^2 - 4 = x + 4 - 4 = x.$

1.1.42 $F((3x+1)/x) = \dfrac{1}{\frac{3x+1}{x} - 3} = \dfrac{1}{\frac{3x+1-3x}{x}} = \dfrac{x}{3x+1-3x} = x.$

1.1.43 $g(x) = x^3 - 5$ and $f(x) = x^{10}$.

1.1.44 $g(x) = x^6 + x^2 + 1$ and $f(x) = \frac{2}{x^2}$.

1.1.45 $g(x) = x^4 + 2$ and $f(x) = \sqrt{x}$.

1.1.46 $g(x) = x^3 - 1$ and $f(x) = \frac{1}{\sqrt{x}}$.

1.1.47 $(f \circ g)(x) = f(g(x)) = f(x^2-4) = |x^2-4|$. The domain of this function is the set of all real numbers.

1.1.48 $(g \circ f)(x) = g(f(x)) = g(|x|) = |x|^2 - 4 = x^2 - 4$. The domain of this function is the set of all real numbers.

1.1.49 $(f \circ G)(x) = f(G(x)) = f\left(\frac{1}{x-2}\right) = \left|\frac{1}{x-2}\right| = \frac{1}{|x-2|}$. The domain of this function is the set of all real numbers except for the number 2.

1.1.50 $(f \circ g \circ G)(x) = f(g(G(x))) = f\left(g\left(\frac{1}{x-2}\right)\right) = f\left(\left(\frac{1}{x-2}\right)^2 - 4\right) = \left|\left(\frac{1}{x-2}\right)^2 - 4\right|$. The domain of this function is the set of all real numbers except for the number 2.

1.1.51 $(G \circ g \circ f)(x) = G(g(f(x))) = G(g(|x|)) = G(x^2-4) = \frac{1}{x^2-4-2} = \frac{1}{x^2-6}$. The domain of this function is the set of all real numbers except for the numbers $\pm\sqrt{6}$.

1.1.52 $(g \circ F \circ F)(x) = g(F(F(x))) = g(F(\sqrt{x})) = g(\sqrt{\sqrt{x}}) = \sqrt{x} - 4$. The domain is $[0, \infty)$.

1.1.53 $(g \circ g)(x) = g(g(x)) = g(x^2-4) = (x^2-4)^2 - 4 = x^4 - 8x^2 + 16 - 4 = x^4 - 8x^2 + 12$. The domain is the set of all real numbers.

1.1.54 $(G \circ G)(x) = G(G(x)) = G(1/(x-2)) = \dfrac{1}{\frac{1}{x-2} - 2} = \dfrac{1}{\frac{1-2(x-2)}{x-2}} = \dfrac{x-2}{1-2x+4} = \dfrac{x-2}{5-2x}$. Then $G \circ G$ is defined except where the denominator vanishes, so its domain is the set of all real numbers except for $x = \frac{5}{2}$.

1.1.55 Because $(x^2+3) - 3 = x^2$, we may choose $f(x) = x - 3$.

1.1.56 Because the reciprocal of $x^2 + 3$ is $\frac{1}{x^2+3}$, we may choose $f(x) = \frac{1}{x}$.

1.1.57 Because $(x^2+3)^2 = x^4 + 6x^2 + 9$, we may choose $f(x) = x^2$.

1.1.58 Because $(x^2+3)^2 = x^4+6x^2+9$, and the given expression is 11 more than this, we may choose $f(x) = x^2+11$.

1.1.59 Because $(x^2)^2+3 = x^4+3$, this expression results from squaring x^2 and adding 3 to it. Thus we may choose $f(x) = x^2$.

1.1.60 Because $x^{2/3}+3 = (\sqrt[3]{x})^2+3$, we may choose $f(x) = \sqrt[3]{x}$.

1.1.61

a. True. A real number z corresponds to the domain element $z/2+19$, because $f(z/2+19) = 2(z/2+19)-38 = z+38-38 = z$.

b. False. The definition of function does not require that each range element comes from a unique domain element, rather that each domain element is paired with a unique range element.

c. True. $f(1/x) = \frac{1}{1/x} = x$, and $\frac{1}{f(x)} = \frac{1}{1/x} = x$.

d. False. For example, suppose that f is the straight line through the origin with slope 1, so that $f(x) = x$. Then $f(f(x)) = f(x) = x$, while $(f(x))^2 = x^2$.

e. False. For example, let $f(x) = x+2$ and $g(x) = 2x-1$. Then $f(g(x)) = f(2x-1) = 2x-1+2 = 2x+1$, while $g(f(x)) = g(x+2) = 2(x+2)-1 = 2x+3$.

f. True. This is the definition of $f\circ g$.

g. True. If f is even, then $f(-z) = f(z)$ for all z, so this is true in particular for $z = ax$. So if $g(x) = cf(ax)$, then $g(-x) = cf(-ax) = cf(ax) = g(x)$, so g is even.

h. False. For example, $f(x) = x$ is an odd function, but $h(x) = x+1$ isn't, because $h(2) = 3$, while $h(-2) = -1$ which isn't $-h(2)$.

i. True. If $f(-x) = -f(x) = f(x)$, then in particular $-f(x) = f(x)$, so $0 = 2f(x)$, so $f(x) = 0$ for all x.

1.1.62 $\frac{f(x+h)-f(x)}{h} = \frac{10-10}{h} = \frac{0}{h} = 0.$

1.1.63 $\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)-3x}{h} = \frac{3x+3h-3x}{h} = \frac{3h}{h} = 3.$

1.1.64 $\frac{f(x+h)-f(x)}{h} = \frac{4(x+h)-3-(4x-3)}{h} = \frac{4x+4h-3-4x+3}{h} = \frac{4h}{h} = 4.$

1.1.65 $\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-x^2}{h} = \frac{(x^2+2hx+h^2)-x^2}{h} = \frac{h(2x+h)}{h} = 2x+h.$

1.1.66 $\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-3(x+h)+1-(2x^2-3x+1)}{h} =$
$\frac{2x^2+4xh+2h^2-3x-3h+1-2x^2+3x-1}{h} = \frac{4xh+2h^2-3h}{h} = \frac{h(4x+2h-3)}{h} = 4x+2h-3.$

1.1.67 $\frac{f(x+h)-f(x)}{h} = \frac{\frac{2}{x+h}-\frac{2}{x}}{h} = \frac{\frac{2x-2(x+h)}{x(x+h)}}{h} = \frac{2x-2x-2h}{hx(x+h)} = -\frac{2h}{hx(x+h)} = -\frac{2}{x(x+h)}.$

1.1.68 $\frac{f(x+h)-f(x)}{h} = \frac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h} = \frac{\frac{(x+h)(x+1)-x(x+h+1)}{(x+1)(x+h+1)}}{h} = \frac{x^2+x+hx+h-x^2-xh-x}{h(x+1)(x+h+1)} =$
$\frac{h}{h(x+1)(x+h+1)} = \frac{1}{(x+1)(x+h+1)}$

1.1.69 $\dfrac{f(x)-f(a)}{x-a} = \dfrac{x^2+x-(a^2+a)}{x-a} = \dfrac{(x^2-a^2)+(x-a)}{x-a} = \dfrac{(x-a)(x+a)+(x-a)}{x-a} = \dfrac{(x-a)(x+a+1)}{x-a} = x+a+1.$

1.1.70

$$\frac{f(x)-f(a)}{x-a} = \frac{4-4x-x^2-(4-4a-a^2)}{x-a} = \frac{-4(x-a)-(x^2-a^2)}{x-a} = \frac{-4(x-a)-(x-a)(x+a)}{x-a} = \frac{(x-a)(-4-(x+a))}{x-a} = -4-x-a.$$

1.1.71 $\dfrac{f(x)-f(a)}{x-a} = \dfrac{x^3-2x-(a^3-2a)}{x-a} = \dfrac{(x^3-a^3)-2(x-a)}{x-a} = \dfrac{(x-a)(x^2+ax+a^2)-2(x-a)}{x-a} = \dfrac{(x-a)(x^2+ax+a^2-2)}{x-a} = x^2+ax+a^2-2.$

1.1.72 $\dfrac{f(x)-f(a)}{x-a} = \dfrac{x^4-a^4}{x-a} = \dfrac{(x^2-a^2)(x^2+a^2)}{x-a} = \dfrac{(x-a)(x+a)(x^2+a^2)}{x-a} = (x+a)(x^2+a^2).$

1.1.73 $\dfrac{f(x)-f(a)}{x-a} = \dfrac{\frac{-4}{x^2}-\frac{-4}{a^2}}{x-a} = \dfrac{\frac{-4a^2+4x^2}{a^2x^2}}{x-a} = \dfrac{4(x^2-a^2)}{(x-a)a^2x^2} = \dfrac{4(x-a)(x+a)}{(x-a)a^2x^2} = \dfrac{4(x+a)}{a^2x^2}.$

1.1.74 $\dfrac{f(x)-f(a)}{x-a} = \dfrac{\frac{1}{x}-x^2-(\frac{1}{a}-a^2)}{x-a} = \dfrac{\frac{1}{x}-\frac{1}{a}}{x-a} - \dfrac{x^2-a^2}{x-a} = \dfrac{\frac{a-x}{ax}}{x-a} - \dfrac{(x-a)(x+a)}{x-a} = -\dfrac{1}{ax}-(x+a).$

1.1.75

a. The slope is $\frac{12227-10499}{3-1} = 864$ ft/h. The hiker's elevation increases at an average rate of 874 feet per hour.

b. The slope is $\frac{12144-12631}{5-4} = -487$ ft/h. The hiker's elevation decreases at an average rate of 487 feet per hour.

c. The hiker might have stopped to rest during this interval of time or the trail is level on this portion of the hike.

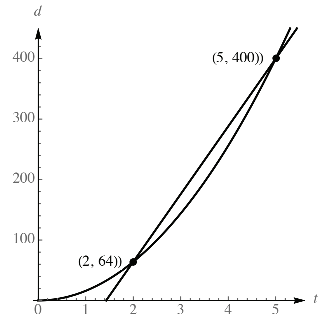
1.1.76

a. The slope is $\frac{11302-9954}{3-1} = 674$ ft/m. The elevation of the trail increases by an average of 674 feet per mile for $1 \le d \le 3$.

b. The slope is $\frac{12237-12357}{6-5} = -120$ ft/m. The elevation of the trail decreases by an average of 120 feet per mile for $5 \le d \le 6$.

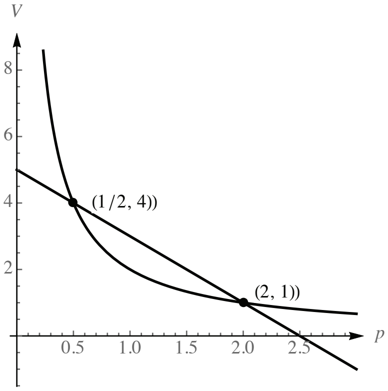
c. The elevation of the trail doesn't change much for $4.5 \le d \le 5$.

1.1.77

a. 

b. The slope of the secant line is given by $\frac{400-64}{5-2} = \frac{336}{3} = 112$ feet per second. The object falls at an average rate of 112 feet per second over the interval $2 \le t \le 5$.

1.1.78

a. 

b. The slope of the secant line is given by $\frac{4-1}{.5-2} = \frac{3}{-1.5} = -2$ cubic centimeters per atmosphere. The volume decreases by an average of 2 cubic centimeters per atmosphere over the interval $0.5 \le p \le 2$.

1.1.79 This function is symmetric about the y-axis, because $f(-x) = (-x)^4 + 5(-x)^2 - 12 = x^4 + 5x^2 - 12 = f(x)$.

1.1.80 This function is symmetric about the origin, because $f(-x) = 3(-x)^5 + 2(-x)^3 - (-x) = -3x^5 - 2x^3 + x = -(3x^5 + 2x^3 - x) = f(x)$.

1.1.81 This function has none of the indicated symmetries. For example, note that $f(-2) = -26$, while $f(2) = 22$, so f is not symmetric about either the origin or about the y-axis, and is not symmetric about the x-axis because it is a function.

1.1.82 This function is symmetric about the y-axis. Note that $f(-x) = 2|-x| = 2|x| = f(x)$.

1.1.83 This curve (which is not a function) is symmetric about the x-axis, the y-axis, and the origin. Note that replacing either x by $-x$ or y by $-y$ (or both) yields the same equation. This is due to the fact that $(-x)^{2/3} = ((-x)^2)^{1/3} = (x^2)^{1/3} = x^{2/3}$, and a similar fact holds for the term involving y.

1.1.84 This function is symmetric about the origin. Writing the function as $y = f(x) = x^{3/5}$, we see that $f(-x) = (-x)^{3/5} = -(x)^{3/5} = -f(x)$.

1.1.85 This function is symmetric about the origin. Note that $f(-x) = (-x)|(-x)| = -x|x| = -f(x)$.

1.1.86 This curve (which is not a function) is symmetric about the x-axis, the y-axis, and the origin. Note that replacing either x by $-x$ or y by $-y$ (or both) yields the same equation. This is due to the fact that $|-x| = |x|$ and $|-y| = |y|$.

1.1.87

a. $f(g(-2)) = f(-g(2)) = f(-2) = 4$

b. $g(f(-2)) = g(f(2)) = g(4) = 1$

c. $f(g(-4)) = f(-g(4)) = f(-1) = 3$

d. $g(f(5) - 8) = g(-2) = -g(2) = -2$

e. $g(g(-7)) = g(-g(7)) = g(-4) = -1$

f. $f(1 - f(8)) = f(-7) = 7$

1.1.88

a. $f(g(-1)) = f(-g(1)) = f(3) = 3$

b. $g(f(-4)) = g(f(4)) = g(-4) = -g(4) = 2$

c. $f(g(-3)) = f(-g(3)) = f(4) = -4$

d. $f(g(-2)) = f(-g(2)) = f(1) = 2$

e. $g(g(-1)) = g(-g(1)) = g(3) = -4$

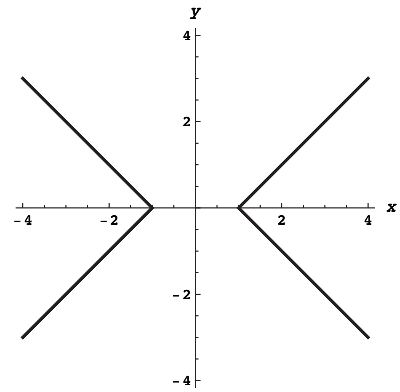
f. $f(g(0) - 1) = f(-1) = f(1) = 2$

g. $f(g(g(-2))) = f(g(-g(2))) = f(g(1)) = f(-3) = 3$

h. $g(f(f(-4))) = g(f(-4)) = g(-4) = 2$

i. $g(g(g(-1))) = g(g(-g(1))) = g(g(3)) = g(-4) = 2$

1.1.89 We will make heavy use of the fact that $|x|$ is x if $x > 0$, and is $-x$ if $x < 0$. In the first quadrant where x and y are both positive, this equation becomes $x - y = 1$ which is a straight line with slope 1 and y-intercept -1. In the second quadrant where x is negative and y is positive, this equation becomes $-x - y = 1$, which is a straight line with slope -1 and y-intercept -1. In the third quadrant where both x and y are negative, we obtain the equation $-x - (-y) = 1$, or $y = x + 1$, and in the fourth quadrant, we obtain $x + y = 1$. Graphing these lines and restricting them to the appropriate quadrants yields the following curve:



1.1.90 We have $y = 10 + \sqrt{-x^2 + 10x - 9}$, so by subtracting 10 from both sides and squaring we have $(y - 10)^2 = -x^2 + 10x - 9$, which can be written as

$$x^2 - 10x + (y - 10)^2 = -9.$$

To complete the square in x, we add 25 to both sides, yielding

$$x^2 - 10x + 25 + (y - 10)^2 = -9 + 25,$$

or

$$(x - 5)^2 + (y - 10)^2 = 16.$$

This is the equation of a circle of radius 4 centered at $(5, 10)$. Because $y \geq 10$, we see that the graph of f is the upper half of this circle. The domain of the function is $[1, 9]$ and the range is $[10, 14]$.

1.1.91 We have $y = 2 - \sqrt{-x^2 + 6x + 16}$, so by subtracting 2 from both sides and squaring we have $(y - 2)^2 = -x^2 + 6x + 16$, which can be written as

$$x^2 - 6x + (y - 2)^2 = 16.$$

To complete the square in x, we add 9 to both sides, yielding

$$x^2 - 6x + 9 + (y - 2)^2 = 16 + 9,$$

or

$$(x - 3)^2 + (y - 2)^2 = 25.$$

This is the equation of a circle of radius 5 centered at $(3, 2)$. Because $y \leq 2$, we see that the graph of f is the lower half of this circle. The domain of the function is $[-2, 8]$ and the range is $[-3, 2]$.

1.1.92

a. No. For example $f(x) = x^2 + 3$ is an even function, but $f(0)$ is not 0.

b. Yes. because $f(-x) = -f(x)$, and because $-0 = 0$, we must have $f(-0) = f(0) = -f(0)$, so $f(0) = -f(0)$, and the only number which is its own additive inverse is 0, so $f(0) = 0$.

1.1.93 Because the composition of f with itself has first degree, f has first degree as well, so let $f(x) = ax+b$. Then $(f \circ f)(x) = f(ax + b) = a(ax + b) + b = a^2x + (ab + b)$. Equating coefficients, we see that $a^2 = 9$ and $ab + b = -8$. If $a = 3$, we get that $b = -2$, while if $a = -3$ we have $b = 4$. So the two possible answers are $f(x) = 3x - 2$ and $f(x) = -3x + 4$.

1.1.94 Since the square of a linear function is a quadratic, we let $f(x) = ax+b$. Then $f(x)^2 = a^2x^2+2abx+b^2$. Equating coefficients yields that $a = \pm 3$ and $b = \pm 2$. However, a quick check shows that the middle term is correct only when one of these is positive and one is negative. So the two possible such functions f are $f(x) = 3x - 2$ and $f(x) = -3x + 2$.

1.1.95 Let $f(x) = ax^2 + bx + c$. Then $(f \circ f)(x) = f(ax^2 + bx + c) = a(ax^2 + bx + c)^2 + b(ax^2 + bx + c) + c$. Expanding this expression yields $a^3x^4 + 2a^2bx^3 + 2a^2cx^2 + ab^2x^2 + 2abcx + ac^2 + abx^2 + b^2x + bc + c$, which simplifies to $a^3x^4 + 2a^2bx^3 + (2a^2c + ab^2 + ab)x^2 + (2abc + b^2)x + (ac^2 + bc + c)$. Equating coefficients yields $a^3 = 1$, so $a = 1$. Then $2a^2b = 0$, so $b = 0$. It then follows that $c = -6$, so the original function was $f(x) = x^2 - 6$.

1.1.96 Because the square of a quadratic is a quartic, we let $f(x) = ax^2 + bx + c$. Then the square of f is $c^2 + 2bcx + b^2x^2 + 2acx^2 + 2abx^3 + a^2x^4$. By equating coefficients, we see that $a^2 = 1$ and so $a = \pm 1$. Because the coefficient on x^3 must be 0, we have that $b = 0$. And the constant term reveals that $c = \pm 6$. A quick check shows that the only possible solutions are thus $f(x) = x^2 - 6$ and $f(x) = -x^2 + 6$.

1.1.97 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{x+h}-\sqrt{x}}{h} = \dfrac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \dfrac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} =$
$\dfrac{1}{\sqrt{x+h}+\sqrt{x}}$.

$$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{x}-\sqrt{a}}{x-a} = \frac{\sqrt{x}-\sqrt{a}}{x-a} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}} = \frac{x-a}{(x-a)(\sqrt{x}+\sqrt{a})} = \frac{1}{\sqrt{x}+\sqrt{a}}.$$

1.1.98 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h} =$
$\dfrac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h} \cdot \dfrac{\sqrt{1-2(x+h)}+\sqrt{1-2x}}{\sqrt{1-2(x+h)}+\sqrt{1-2x}} =$
$\dfrac{1-2(x+h)-(1-2x)}{h(\sqrt{1-2(x+h)}+\sqrt{1-2x})} =$
$-\dfrac{2}{\sqrt{1-2(x+h)}+\sqrt{1-2x}}$.

$$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{1-2x}-\sqrt{1-2a}}{x-a} = \frac{\sqrt{1-2x}-\sqrt{1-2a}}{x-a} \cdot \frac{\sqrt{1-2x}+\sqrt{1-2a}}{\sqrt{1-2x}+\sqrt{1-2a}} =$$
$$\frac{(1-2x)-(1-2a)}{(x-a)(\sqrt{1-2x}+\sqrt{1-2a})} = \frac{(-2)(x-a)}{(x-a)(\sqrt{1-2x}+\sqrt{1-2a})} = -\frac{2}{(\sqrt{1-2x}+\sqrt{1-2a})}.$$

1.1.99 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{-3}{\sqrt{x+h}} - \frac{-3}{\sqrt{x}}}{h} = \dfrac{-3(\sqrt{x}-\sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}} = \dfrac{-3(\sqrt{x}-\sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}} \cdot \dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}} =$
$\dfrac{-3(x-(x+h))}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \dfrac{3}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$.

$$\frac{f(x)-f(a)}{x-a} = \frac{\frac{-3}{\sqrt{x}} - \frac{-3}{\sqrt{a}}}{x-a} = \frac{-3\left(\frac{\sqrt{a}-\sqrt{x}}{\sqrt{a}\sqrt{x}}\right)}{x-a} = \frac{(-3)(\sqrt{a}-\sqrt{x})}{(x-a)\sqrt{a}\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} =$$
$$\frac{(3)(x-a)}{(x-a)(\sqrt{a}\sqrt{x})(\sqrt{a}+\sqrt{x})} = \frac{3}{\sqrt{ax}(\sqrt{a}+\sqrt{x})}.$$

1.1.100 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h} =$
$\dfrac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h} \cdot \dfrac{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}} =$
$\dfrac{(x+h)^2+1-(x^2+1)}{h(\sqrt{(x+h)^2+1}+\sqrt{x^2+1})} = \dfrac{x^2+2hx+h^2-x^2}{h(\sqrt{(x+h)^2+1}+\sqrt{x^2+1})} = \dfrac{2x+h}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}$.

$$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{x^2+1}-\sqrt{a^2+1}}{x-a} = \frac{\sqrt{x^2+1}-\sqrt{a^2+1}}{x-a} \cdot \frac{\sqrt{x^2+1}+\sqrt{a^2+1}}{\sqrt{x^2+1}+\sqrt{a^2+1}} =$$
$$\frac{x^2+1-(a^2+1)}{(x-a)(\sqrt{x^2+1}+\sqrt{a^2+1})} = \frac{(x-a)(x+a)}{(x-a)(\sqrt{x^2+1}+\sqrt{a^2+1})} = \frac{x+a}{\sqrt{x^2+1}+\sqrt{a^2+1}}.$$

1.1.101 This would not necessarily have either kind of symmetry. For example, $f(x) = x^2$ is an even function and $g(x) = x^3$ is odd, but the sum of these two is neither even nor odd.

1.1.102 This would be an odd function, so it would be symmetric about the origin. Suppose f is even and g is odd. Then $(f \cdot g)(-x) = f(-x)g(-x) = f(x) \cdot (-g(x)) = -(f \cdot g)(x)$.

1.1.103 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is odd. Then $g(f(-x)) = g(f(x))$, because $f(-x) = f(x)$.

1.1.104 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is odd. Then $f(g(-x)) = f(-g(x)) = f(g(x))$.

1.2 Representing Functions

1.2.1 Functions can be defined and represented by a formula, through a graph, via a table, and by using words.

1.2.2 The domain of every polynomial is the set of all real numbers.

1.2.3 The slope of the line shown is $m = \frac{-3-(-1)}{3-0} = -2/3$. The y-intercept is $b = -1$. Thus the function is given by $f(x) = -\frac{2}{3}x - 1$.

1.2.4 Because it is to be parallel to a line with slope 2, it must also have slope 2. Using the point-slope form of the equation of the line, we have $y - 0 = 2(x-5)$, or $y = 2x - 10$.

1.2.5 The domain of a rational function $\frac{p(x)}{q(x)}$ is the set of all real numbers for which $q(x) \neq 0$.

1.2.6 A piecewise linear function is one which is linear over intervals in the domain.

1.2.7 For $x < 0$, the graph is a line with slope 1 and y- intercept 3, while for $x > 0$, it is a line with slope $-1/2$ and y-intercept 3. Note that both of these lines contain the point $(0,3)$. The function shown can thus be written

$$f(x) = \begin{cases} x+3 & \text{if } x < 0; \\ -\frac{1}{2}x + 3 & \text{if } x \geq 0. \end{cases}$$

1.2.8 The transformed graph would have equation $y = \sqrt{x-2} + 3$.

1.2.9 Compared to the graph of $f(x)$, the graph of $f(x+2)$ will be shifted 2 units to the left.

1.2.10 Compared to the graph of $f(x)$, the graph of $-3f(x)$ will be scaled vertically by a factor of 3 and flipped about the x axis.

1.2.11 Compared to the graph of $f(x)$, the graph of $f(3x)$ will be compressed horizontally by a factor of $\frac{1}{3}$.

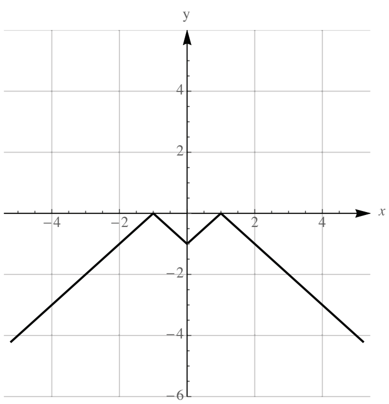
1.2.12 To produce the graph of $y = 4(x+3)^2 + 6$ from the graph of x^2, one must

1. shift the graph horizontally by 3 units to left
2. scale the graph vertically by a factor of 4
3. shift the graph vertically up 6 units.

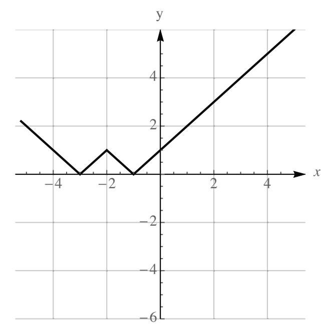
1.2.13 $f(x) = |x-2| + 3$, because the graph of f is obtained from that of $|x|$ by shifting 2 units to the right and 3 units up.

$g(x) = -|x+2| - 1$, because the graph of g is obtained from the graph of $|x|$ by shifting 2 units to the left, then reflecting about the x-axis, and then shifting 1 unit down.

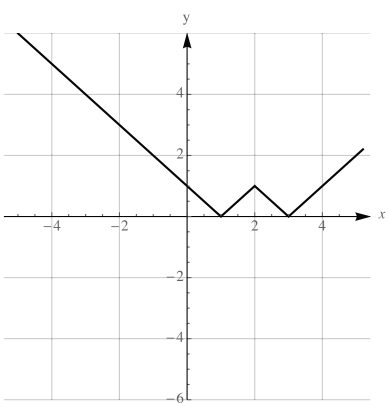
1.2.14



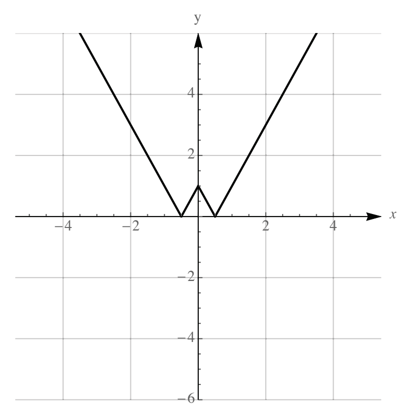
a.



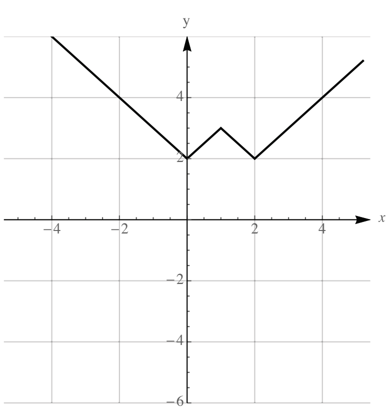
b.



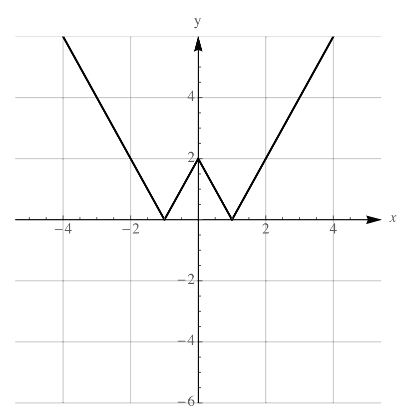
c.



d.



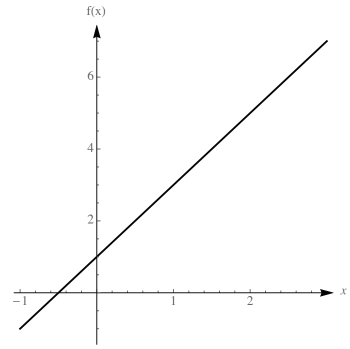
e.



f.

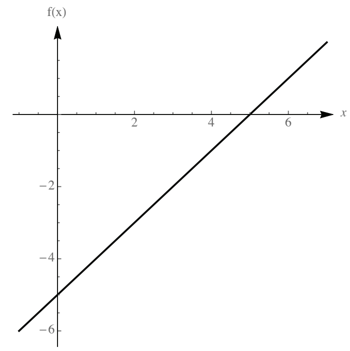
1.2.15

The slope is given by $\frac{5-3}{2-1} = 2$, so the equation of the line is $y - 3 = 2(x - 1)$, which can be written as $f(x) = 2x - 2 + 3$, or $f(x) = 2x + 1$.



1.2.16

The slope is given by $\frac{0-(-3)}{5-2} = 1$, so the equation of the line is $y - 0 = 1(x - 5)$, or $f(x) = x - 5$.



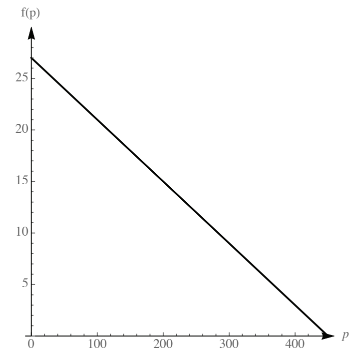
1.2.17 We are looking for the line with slope 3 that goes through the point $(3, 2)$. Using the point-slope form of the equation of a line, we have $y - 2 = 3(x - 3)$, which can be written as $y = 2 + 3x - 9$, or $y = 3x - 7$.

1.2.18 We are looking for the line with slope -4 which goes through the point $(-1, 4)$. Using the point-slope form of the equation of a line, we have $y - 4 = -4(x - (-1))$, which can be written as $y = 4 - 4x - 4$, or $y = -4x$.

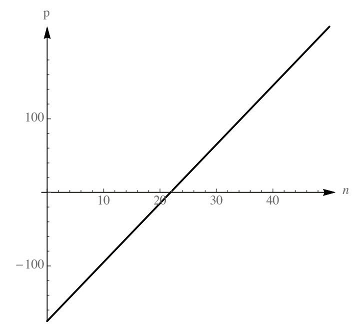
1.2.19 We have $571 = C_s(100)$, so $C_s = 5.71$. Therefore $N(150) = 5.71(150) = 856.5$ million.

1.2.20 We have $226 = C_s(100)$, so $C_s = 2.26$. Therefore $N(150) = 2.26(150) = 339$ million.

1.2.21 Using price as the independent variable p and the average number of units sold per day as the dependent variable d, we have the ordered pairs $(250, 12)$ and $(200, 15)$. The slope of the line determined by these points is $m = \frac{15-12}{200-250} = \frac{3}{-50}$. Thus the demand function has the form $d(p) = (-3/50)p + b$ for some constant b. Using the point $(200, 15)$, we find that $15 = (-3/50) \cdot 200 + b$, so $b = 27$. Thus the demand function is $d = (-3p/50) + 27$. While the domain of this linear function is the set of all real numbers, the formula is only likely to be valid for some subset of the interval $(0, 450)$, because outside of that interval either $p \leq 0$ or $d \leq 0$.



1.2.22 The profit is given by $p = f(n) = 8n - 175$. The break-even point is when $p = 0$, which occurs when $n = 175/8 = 21.875$, so they need to sell at least 22 tickets to not have a negative profit.

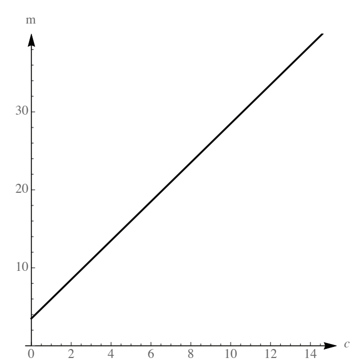


1.2.23

a. Using the points $(1986, 1875)$ and $(2000, 6471)$ we see that the slope is about 328.3. At $t = 0$, the value of p is 1875. Therefore a line which reasonably approximates the data is $p(t) = 328.3t + 1875$.

b. Using this line, we have that $p(9) = 4830$ breeding pairs.

1.2.24 The cost per mile is the slope of the desired line, and the intercept is the fixed cost of 3.5. Thus, the cost per mile is given by $c(m) = 2.5m + 3.5$. When $m = 9$, we have $c(9) = (2.5)(9) + 3.5 = 22.5 + 3.5 = 26$ dollars.



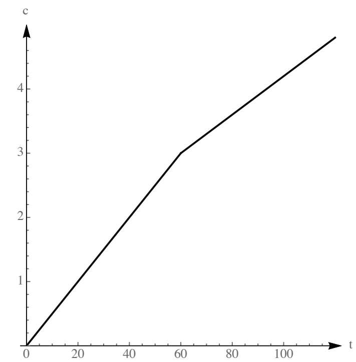
1.2.25 For $x \le 3$, we have the constant function 3. For $x \ge 3$, we have a straight line with slope 2 that contains the point $(3, 3)$. So its equation is $y - 3 = 2(x - 3)$, or $y = 2x - 3$. So the function can be written as $f(x) = \begin{cases} 3 & \text{if } x \le 3; \\ 2x - 3 & \text{if } x > 3 \end{cases}$

1.2.26 For $x < 3$ we have straight line with slope 1 and y-intercept 1, so the equation is $y = x+1$. For $x \geq 3$, we have a straight line with slope $-\frac{1}{3}$ which contains the point $(3, 2)$, so its equation is $y - 2 = -\frac{1}{3}(x-3)$, or $y = -\frac{1}{3}x + 3$. Thus the function can be written as $f(x) = \begin{cases} x+1 & \text{if } x < 3; \\ -\frac{1}{3}x + 3 & \text{if } x \geq 3 \end{cases}$

1.2.27

The cost is given by

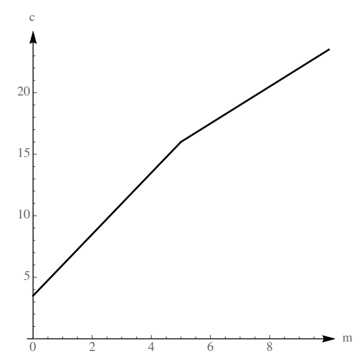
$$c(t) = \begin{cases} 0.05t & \text{for } 0 \leq t \leq 60 \\ 1.2 + 0.03t & \text{for } 60 < t \leq 120 \end{cases}.$$



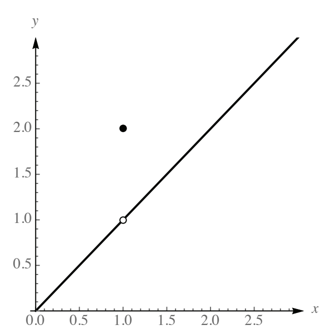
1.2.28

The cost is given by

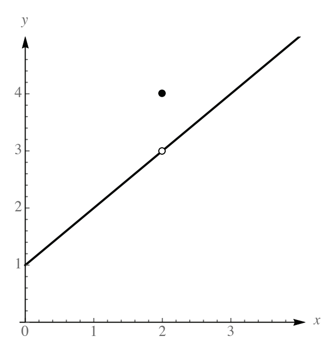
$$c(m) = \begin{cases} 3.5 + 2.5m & \text{for } 0 \leq m \leq 5 \\ 8.5 + 1.5m & \text{for } m > 5 \end{cases}.$$



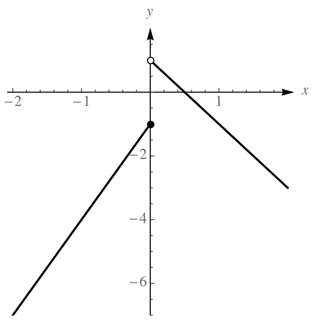
1.2.29



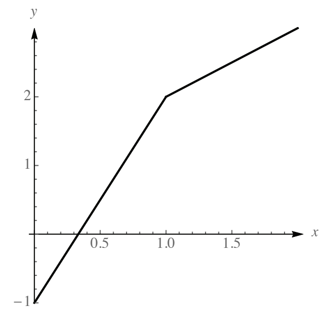
1.2.30



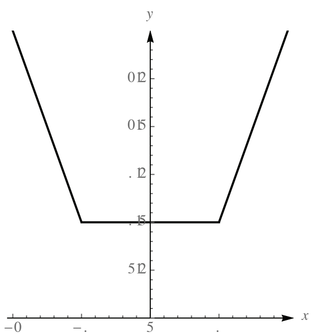
1.2.31



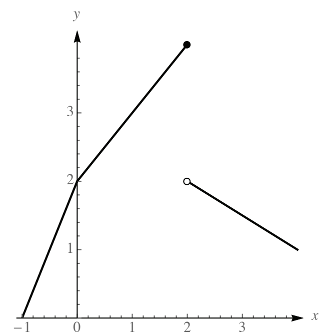
1.2.32



1.2.33

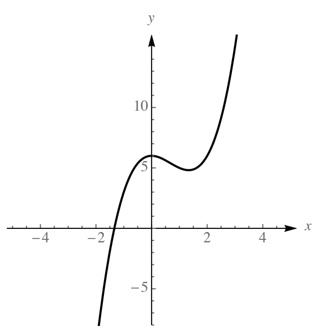


1.2.34



1.2.35

a.

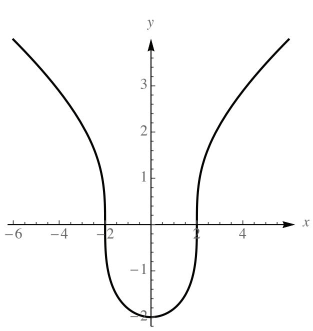


b. The function is a polynomial, so its domain is the set of all real numbers.

c. It has one peak near its y-intercept of $(0, 6)$ and one valley between $x = 1$ and $x = 2$. Its x-intercept is near $x = -4/3$.

1.2.36

a.

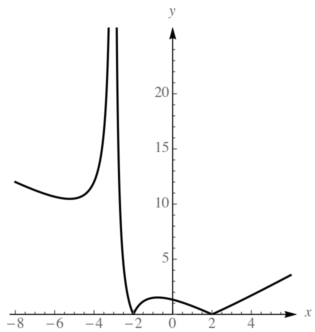


b. The function's domain is the set of all real numbers.

c. It has a valley at the y-intercept of $(0, -2)$, and is very steep at $x = -2$ and $x = 2$ which are the x-intercepts. It is symmetric about the y-axis.

1.2.37

a.

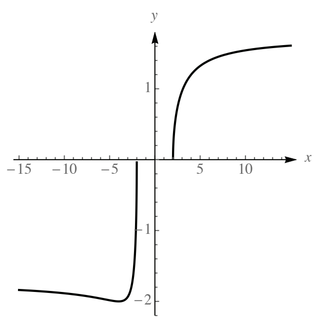


b. The domain of the function is the set of all real numbers except −3.

c. There is a valley near $x = -5.2$ and a peak near $x = -0.8$. The x-intercepts are at −2 and 2, where the curve does not appear to be smooth. There is a vertical asymptote at $x = -3$. The function is never below the x-axis. The y-intercept is $(0, 4/3)$.

1.2.38

a.

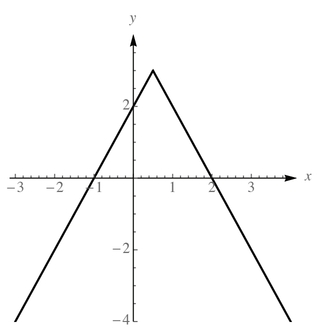


b. The domain of the function is $(-\infty, -2] \cup [2, \infty)$

c. x-intercepts are at −2 and 2. Because 0 isn't in the domain, there is no y-intercept. The function has a valley at $x = -4$.

1.2.39

a.

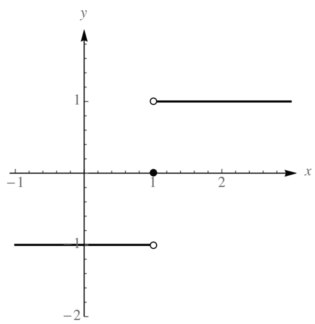


b. The domain of the function is $(-\infty, \infty)$

c. The function has a maximum of 3 at $x = 1/2$, and a y-intercept of 2.

1.2.40

a.



b. The domain of the function is $(-\infty, \infty)$

c. The function contains a jump at $x = 1$. The maximum value of the function is 1 and the minimum value is -1.

1.2.41

a. The zeros of f are the points where the graph crosses the x-axis, so these are points A, D, F, and I.

b. The only high point, or peak, of f occurs at point E, because it appears that the graph has larger and larger y values as x increases past point I and decreases past point A.

c. The only low points, or valleys, of f are at points B and H, again assuming that the graph of f continues its apparent behavior for larger values of x.

d. Past point H, the graph is rising, and is rising faster and faster as x increases. It is also rising between points B and E, but not as quickly as it is past point H. So the marked point at which it is rising most rapidly is I.

e. Before point B, the graph is falling, and falls more and more rapidly as x becomes more and more negative. It is also falling between points E and H, but not as rapidly as it is before point B. So the marked point at which it is falling most rapidly is A.

1.2.42

a. The zeros of g appear to be at $x = 0$, $x = 1$, $x = 1.6$, and $x \approx 3.15$.

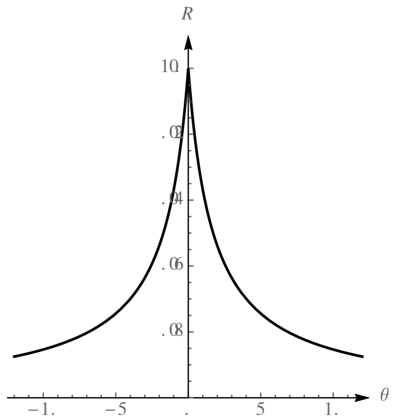
b. The two peaks of g appear to be at $x \approx 0.5$ and $x \approx 2.6$, with corresponding points $\approx (0.5, 0.4)$ and $\approx (2.6, 3.4)$.

c. The only valley of g is at $\approx (1.3, -0.2)$.

d. Moving right from $x \approx 1.3$, the graph is rising more and more rapidly until about $x = 2$, at which point it starts rising less rapidly (because, by $x \approx 2.6$, it is not rising at all). So the coordinates of the point at which it is rising most rapidly are approximately $(2.1, g(2)) \approx (2.1, 2)$. Note that while the curve is also rising between $x = 0$ and $x \approx 0.5$, it is not rising as rapidly as it is near $x = 2$.

e. To the right of $x \approx 2.6$, the curve is falling, and falling more and more rapidly as x increases. So the point at which it is falling most rapidly in the interval $[0, 3]$ is at $x = 3$, which has the approximate coordinates $(3, 1.4)$. Note that while the curve is also falling between $x \approx 0.5$ and $x \approx 1.3$, it is not falling as rapidly as it is near $x = 3$.

1.2.43

a. 

b. This appears to have a maximum when $\theta = 0$. Our vision is sharpest when we look straight ahead.

c. For $|\theta| \le .19°$. We have an extremely narrow range where our eyesight is sharp.

1.2.44 Because the line is horizontal, the slope is constantly 0. So $S(x) = 0$.

1.2.45 The slope of this line is constantly 2, so the slope function is $S(x) = 2$.

1.2.46 The function can be written as $|x| = \begin{cases} -x & \text{if } x \le 0 \\ x & \text{if } x > 0 \end{cases}$.

The slope function is $S(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$.

1.2.47 The slope function is given by $S(x) = \begin{cases} 1 & \text{if } x < 0; \\ -1/2 & \text{if } x > 0. \end{cases}$

1.2.48 The slope function is given by $s(x) = \begin{cases} 1 & \text{if } x < 3; \\ -1/3 & \text{if } x > 3. \end{cases}$

1.2.49

a. Because the area under consideration is that of a rectangle with base 2 and height 6, $A(2) = 12$.

b. Because the area under consideration is that of a rectangle with base 6 and height 6, $A(6) = 36$.

c. Because the area under consideration is that of a rectangle with base x and height 6, $A(x) = 6x$.

1.2.50

a. Because the area under consideration is that of a triangle with base 2 and height 1, $A(2) = 1$.

b. Because the area under consideration is that of a triangle with base 6 and height 3, the $A(6) = 9$.

c. Because $A(x)$ represents the area of a triangle with base x and height $(1/2)x$, the formula for $A(x)$ is $\frac{1}{2} \cdot x \cdot \frac{x}{2} = \frac{x^2}{4}$.

1.2.51

a. Because the area under consideration is that of a trapezoid with base 2 and heights 8 and 4, we have $A(2) = 2 \cdot \frac{8+4}{2} = 12$.

b. Note that $A(3)$ represents the area of a trapezoid with base 3 and heights 8 and 2, so $A(3) = 3\cdot\frac{8+2}{2} = 15$. So $A(6) = 15+(A(6)-A(3))$, and $A(6)-A(3)$ represents the area of a triangle with base 3 and height 2. Thus $A(6) = 15+6 = 21$.

c. For x between 0 and 3, $A(x)$ represents the area of a trapezoid with base x, and heights 8 and $8-2x$. Thus the area is $x\cdot\frac{8+8-2x}{2} = 8x-x^2$. For $x > 3$, $A(x) = A(3)+A(x)-A(3) = 15+2(x-3) = 2x+9$. Thus

$$A(x) = \begin{cases} 8x - x^2 & \text{if } 0 \le x \le 3; \\ 2x+9 & \text{if } x > 3. \end{cases}$$

1.2.52

a. Because the area under consideration is that of trapezoid with base 2 and heights 3 and 1, we have $A(2) = 2\cdot\frac{3+1}{2} = 4$.

b. Note that $A(6) = A(2)+(A(6)-A(2)$, and that $A(6)-A(2)$ represents a trapezoid with base $6-2=4$ and heights 1 and 5. The area is thus $4+\left(4\cdot\frac{1+5}{2}\right) = 4+12 = 16$.

c. For x between 0 and 2, $A(x)$ represents the area of a trapezoid with base x, and heights 3 and $3-x$. Thus the area is $x\cdot\frac{3+3-x}{2} = 3x-\frac{x^2}{2}$. For $x > 2$, $A(x) = A(2)+A(x)-A(2) = 4+(A(x)-A(2))$. Note that $A(x)-A(2)$ represents the area of a trapezoid with base $x-2$ and heights 1 and $x-1$. Thus $A(x) = 4+(x-2)\cdot\frac{1+x-1}{2} = 4+(x-2)\left(\frac{x}{2}\right) = \frac{x^2}{2}-x+4$. Thus

$$A(x) = \begin{cases} 3x - \frac{x^2}{2} & \text{if } 0 \le x \le 2; \\ \frac{x^2}{2} - x + 4 & \text{if } x > 2. \end{cases}$$

1.2.53

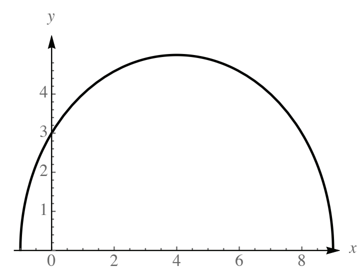
a. True. A polynomial $p(x)$ can be written as the ratio of polynomials $\frac{p(x)}{1}$, so it is a rational function. However, a rational function like $\frac{1}{x}$ is not a polynomial.

b. False. For example, if $f(x) = 2x$, then $(f\circ f)(x) = f(f(x)) = f(2x) = 4x$ is linear, not quadratic.

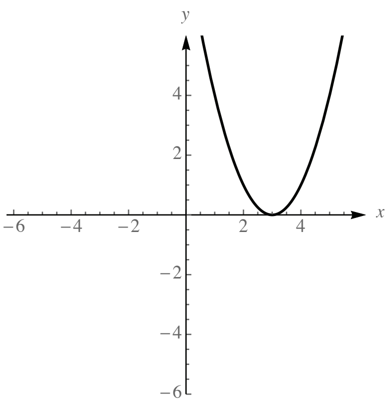
c. True. In fact, if f is degree m and g is degree n, then the degree of the composition of f and g is $m\cdot n$, regardless of the order they are composed.

d. False. The graph would be shifted two units to the left.

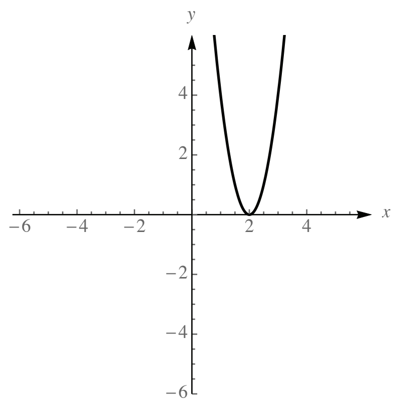
1.2.54 We complete the square for $-x^2+8x+9$. Call this quantity z. Then $z = -(x^2-8x-9)$, so $z = -(x^2-8x+16+(-16-9)) = -((x-4)^2-25 = 25-(x-4)^2$. Thus $f(x)$ is obtained from the graph of $g(x) = \sqrt{25-x^2}$ by shifting 4 units to the right. Thus the graph of f is the upper half of a circle of radius 5 centered at $(4,0)$.



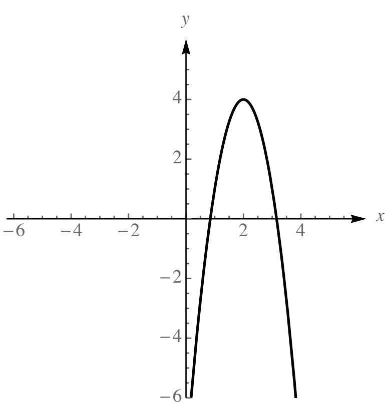
1.2.55

a.


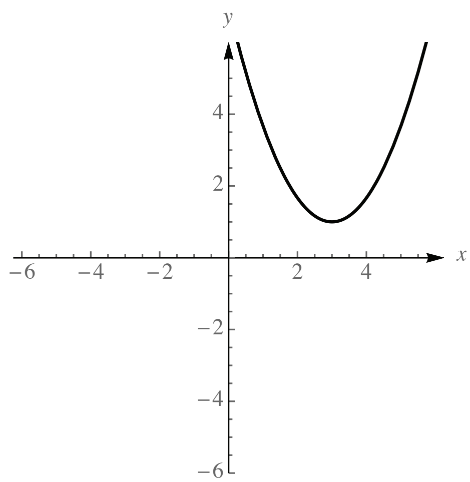
Shift 3 units to the right.

b.


Horizontal compression by a factor of $\frac{1}{2}$, then shift 2 units to the right.

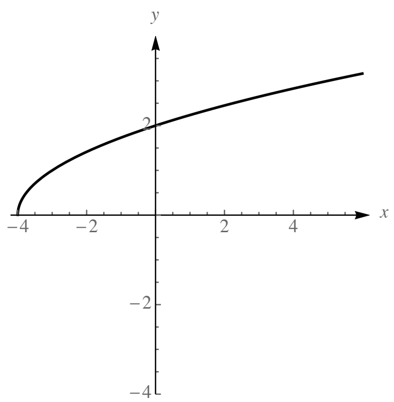
c.


Shift to the right 2 units, vertically stretch by a factor of 3, reflect across the x-axis, and shift up 4 units.

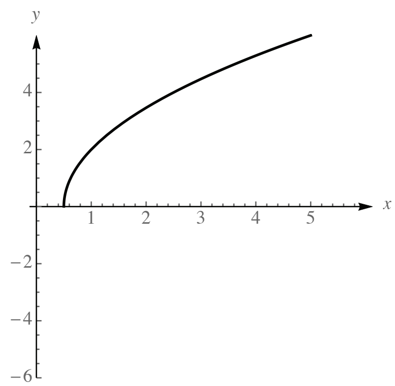
d.


Horizontal stretch by a factor of 3, horizontal shift right 2 units, vertical stretch by a factor of 6, and vertical shift up 1 unit.

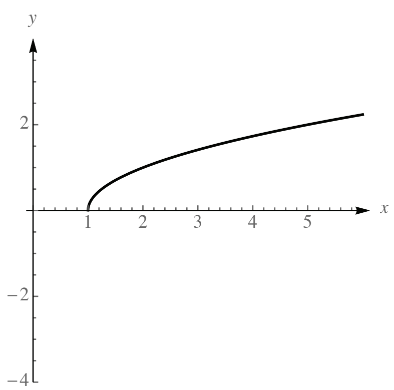
1.2.56



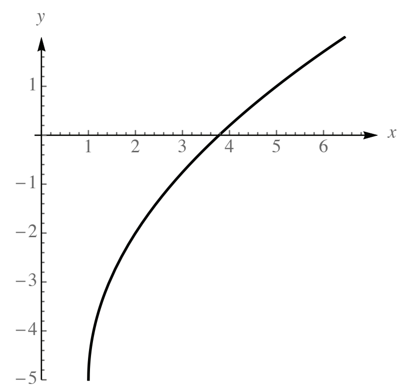
a. Shift 4 units to the left.



b. Horizontal compression by a factor of $\frac{1}{2}$, then shift $\frac{1}{2}$ units to the right. Then stretch vertically by a factor of 2.

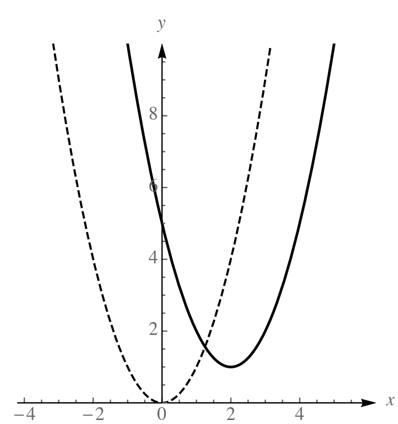


c. Shift 1 unit to the right.

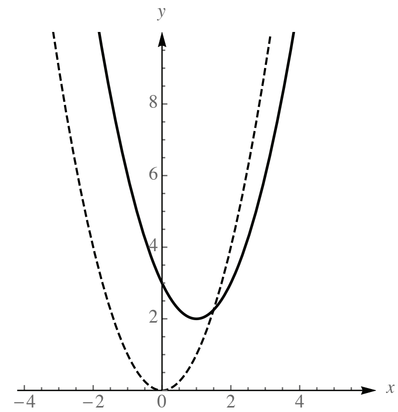


d. Shift 1 unit to the right, then stretch vertically by a factor of 3, then shift down 5 units.

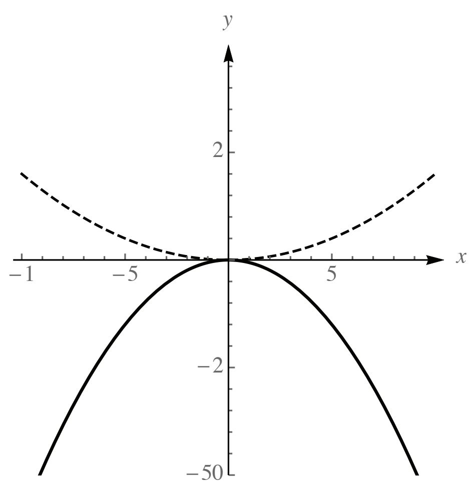
1.2.57 The graph is obtained by shifting the graph of x^2 two units to the right and one unit up.



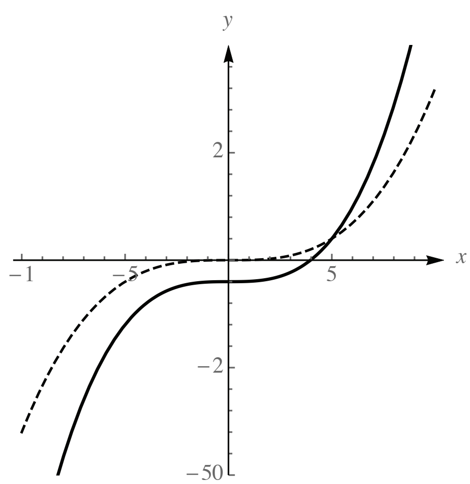
1.2.58 Write x^2-2x+3 as $(x^2-2x+1)+2=(x-1)^2+2$. The graph is obtained by shifting the graph of x^2 one unit to the right and two units up.



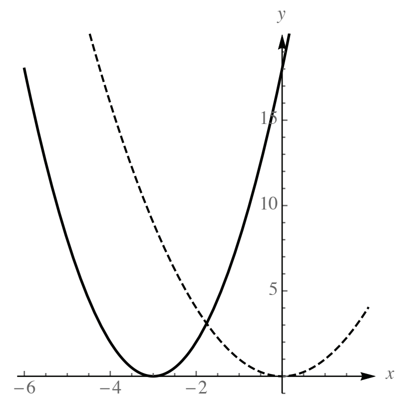
1.2.59 Stretch the graph of $y=x^2$ vertically by a factor of 3 and then reflect across the x-axis.



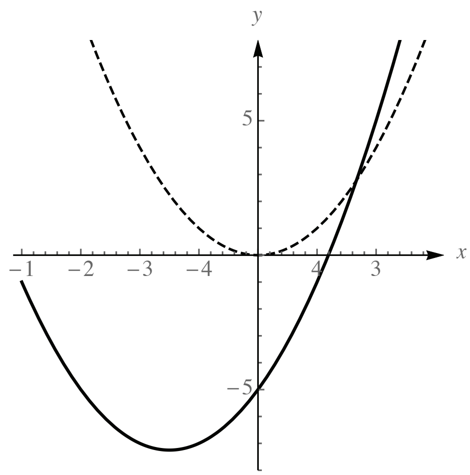
1.2.60 Scale the graph of $y=x^3$ vertically by a factor of 2, and then shift down 1 unit.



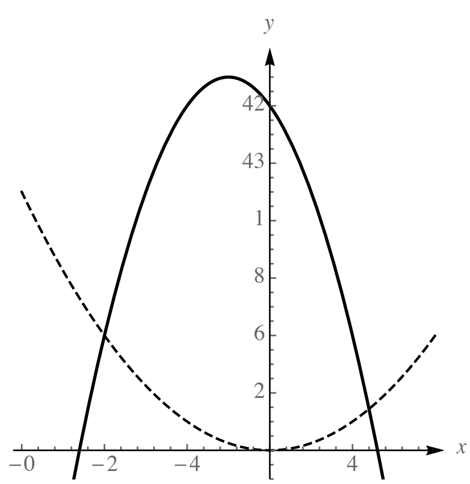
1.2.61 Shift the graph of $y = x^2$ left 3 units and stretch vertically by a factor of 2.



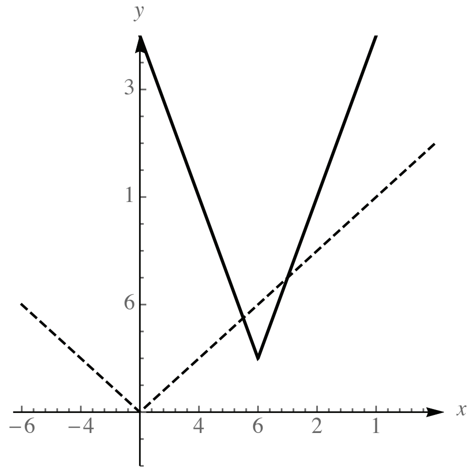
1.2.62 By completing the square, we have that $p(x) = x^2 + 3x - 5 = x^2 + 3x + \frac{9}{4} - 5 - \frac{9}{4} = (x + \frac{3}{2})^2 - \frac{29}{4}$. So it is $f(x + \frac{3}{2}) - (\frac{29}{4})$ where $f(x) = x^2$. The graph is shifted $\frac{3}{2}$ units to the left and then down $\frac{29}{4}$ units.



1.2.63 By completing the square, we have that $h(x) = -4(x^2 + x - 3) = -4\left(x^2 + x + \frac{1}{4} - \frac{1}{4} - 3\right) = -4(x + \frac{1}{2})^2 + 13$. So it is $-4f(x + (\frac{1}{2})) + 13$ where $f(x) = x^2$. The graph is shifted $\frac{1}{2}$ unit to the left, stretched vertically by a factor of 4, then reflected about the x-axis, then shifted up 13 units.



1.2.64 Because $|3x-6|+1 = 3|x-2|+1$, this is $3f(x-2)+1$ where $f(x) = |x|$. The graph is shifted 2 units to the right, then stretched vertically by a factor of 3, and then shifted up 1 unit.

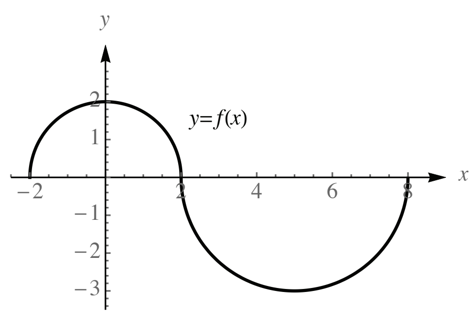


1.2.65 The curves intersect where $4\sqrt{2x} = 2x^2$. If we square both sides, we have $32x = 4x^4$, which can be written as $4x(8-x^3) = 0$, which has solutions at $x = 0$ and $x = 2$. So the points of intersection are $(0,0)$ and $(2,8)$.

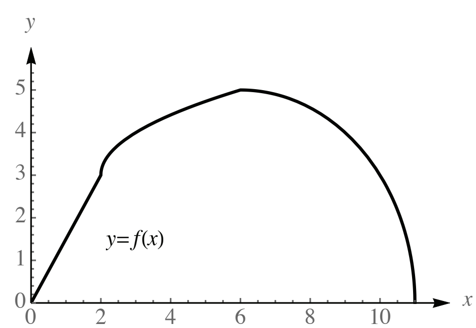
1.2.66 The points of intersection are found by solving $x^2 + 2 = x + 4$. This yields the quadratic equation $x^2 - x - 2 = 0$ or $(x-2)(x+1) = 0$. So the x-values of the points of intersection are 2 and -1. The actual points of intersection are $(2,6)$ and $(-1,3)$.

1.2.67 The points of intersection are found by solving $x^2 = -x^2 + 8x$. This yields the quadratic equation $2x^2 - 8x = 0$ or $(2x)(x-4) = 0$. So the x-values of the points of intersection are 0 and 4. The actual points of intersection are $(0,0)$ and $(4,16)$.

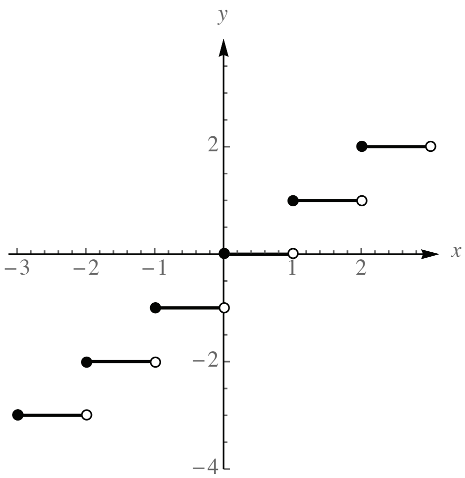
1.2.68 $f(x) = \begin{cases} \sqrt{4-x^2} & \text{if } -2 \le x \le 2 \\ -\sqrt{9-(x-5)^2} & \text{if } 2 < x \le 6. \end{cases}$.



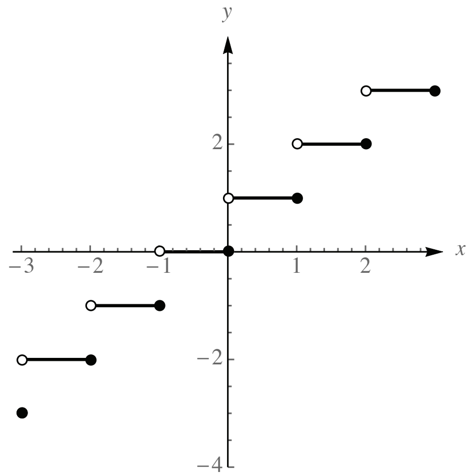
1.2.69



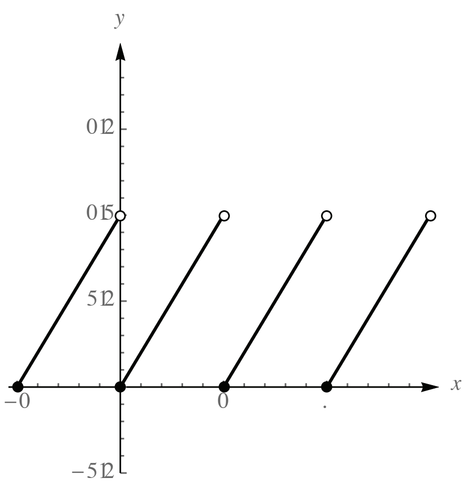
1.2.70



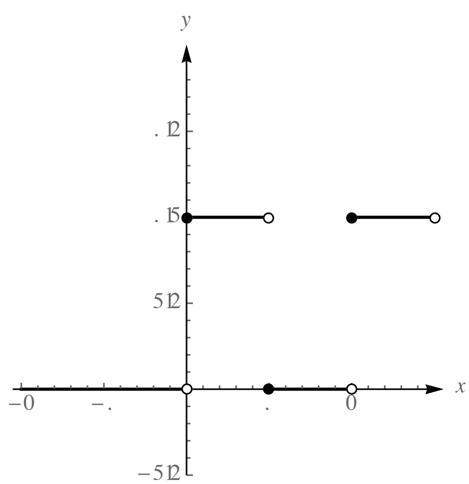
1.2.71



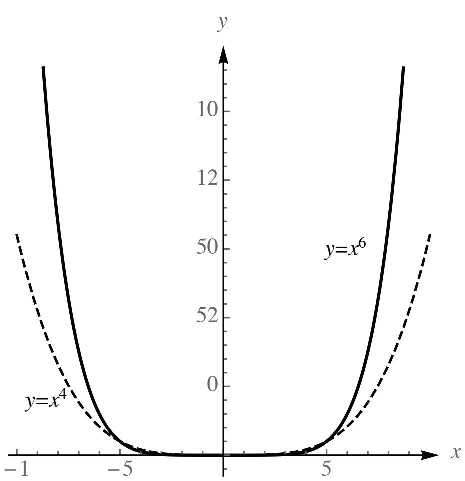
1.2.72



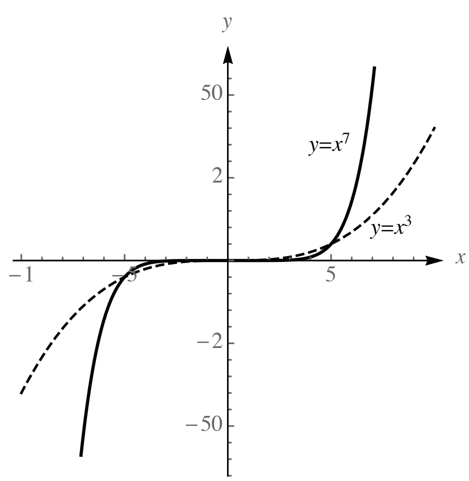
1.2.73



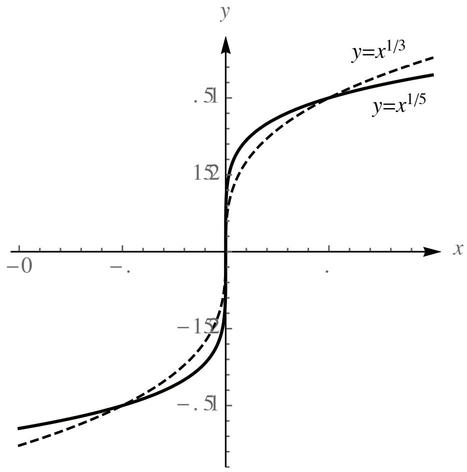
1.2.74



1.2.75



1.2.76



1.2.77

a. $f(0.75) = \frac{.75^2}{1-2(.75)(.25)} = .9$. There is a 90% chance that the server will win from deuce if they win 75% of their service points.

b. $f(0.25) = \frac{.25^2}{1-2(.25)(.75)} = .1$. There is a 10% chance that the server will win from deuce if they win 25% of their service points.

1.2.78

a. We know that the points $(32, 0)$ and $(212, 100)$ are on our line. The slope of our line is thus $\frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$. The function $f(F)$ thus has the form $C = (5/9)F + b$, and using the point $(32, 0)$ we see that $0 = (5/9)32 + b$, so $b = -(160/9)$. Thus $C = (5/9)F - (160/9)$

b. Solving the system of equations $C = (5/9)F - (160/9)$ and $C = F$, we have that $F = (5/9)F - (160/9)$, so $(4/9)F = -160/9$, so $F = -40$ when $C = -40$.

1.2.79

a. Because you are paying \$350 per month, the amount paid after m months is $y = 350m + 1200$.

b. After 4 years (48 months) you have paid $350 \cdot 48 + 1200 = 18000$ dollars. If you then buy the car for \$10,000, you will have paid a total of \$28,000 for the car instead of \$25,000. So you should buy the car instead of leasing it.

1.2.80

a. Note that the island, the point P on shore, and the point down shore x units from P form a right triangle. By the Pythagorean theorem, the length of the hypotenuse is $\sqrt{40000 + x^2}$. So Kelly must row this distance and then jog $600-x$ meters to get home. So her total distance $d(x) = \sqrt{40000 + x^2} + (600 - x)$.