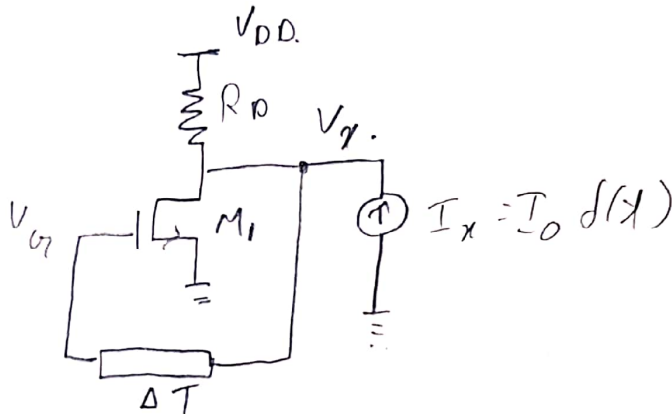


Chapter 1  
 Oscillator Fundamentals

1.1



By KCL we have.

$$I_x = \frac{V_x}{R_D} + g_m V_{GS}$$

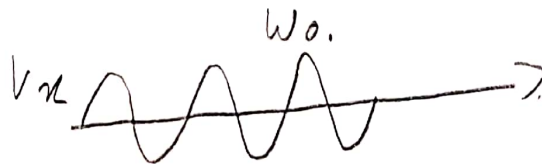
But  $V_{GS} = V_x e^{-s\Delta T}$

$$\therefore I_x = \frac{V_x}{R_D} + g_m V_x e^{-s\Delta T}$$

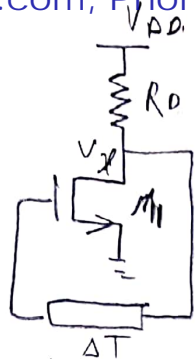
$$I_x = V_x \left( \frac{1}{R_D} + g_m e^{-s\Delta T} \right)$$

$$V_x(t) = \frac{I_0 \delta(t)}{\frac{1}{R_D} + g_m e^{-s\Delta T}} = \frac{I_0 \delta(t) R_D}{1 + g_m R_D e^{-s\Delta T}}$$

If  $g_m R_D = 1$  and  $\Delta T \omega = \pi$ , then system begins to oscillate. We know that application of impulse, the circuit oscillates with a constant amplitude. Hence  $V_x$  will be



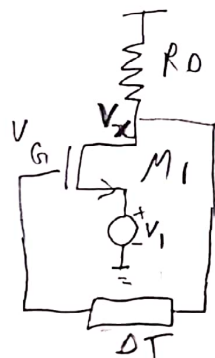
1.2)



If  $V_{DD}$  jumps by  $\Delta V$ , it acts as an impulse i/p to the loop thus giving rise to oscillation.

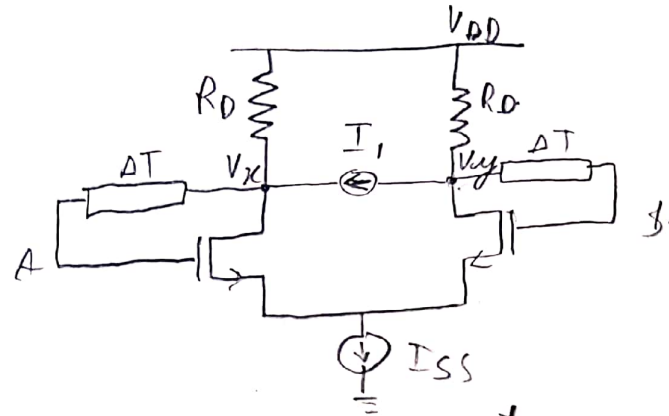
When  $V_{DD}$  increases,  $V_x$  increases and  $V_{DS}$  increases. If we assume it to be operating in saturation, the current doesn't increase immediately. After a delay of  $\Delta T$ ,  $V_G$  increases thus increasing the current through FET and reducing  $V_x$ . This process continues and results in oscillation.

1.3)



If we reduce  $V_1$  at source of  $M_1$  from  $100\text{mV}$  to  $0$ , this results in increase in  $V_{DS}$ . If we assume that the FET is operating in saturation, then there is no change in current through FET. Since there is no change in voltage or current in the loop, the circuit does not oscillate, assuming there is no noise in the circuit.

1.4



To obtain the impedance between node  $X$  and  $Y$ , apply a current source between the two nodes.

$$V_A = V_X e^{-s\Delta T}, \quad V_B = V_Y e^{-s\Delta T}$$

$$I_1 = \frac{V_X}{R_D} + g_m V_A = \frac{V_Y}{R_D} - g_m V_B$$

$$= \frac{V_X}{R_D} + g_m V_X e^{-s\Delta T} - \frac{V_Y}{R_D} - g_m V_Y e^{-s\Delta T}$$

$$= V_X \left( \frac{1}{R_D} + g_m e^{-s\Delta T} \right) - V_Y \left( \frac{1}{R_D} + g_m e^{-s\Delta T} \right)$$

$$I_1 = (V_X - V_Y) \left( \frac{1}{R_D} + g_m e^{-s\Delta T} \right)$$

$$Z = \frac{V_X - V_Y}{I_1} = \frac{1}{\frac{1}{R_D} + g_m e^{-s\Delta T}} = \frac{R_D}{1 + g_m R_D e^{-s\Delta T}}$$

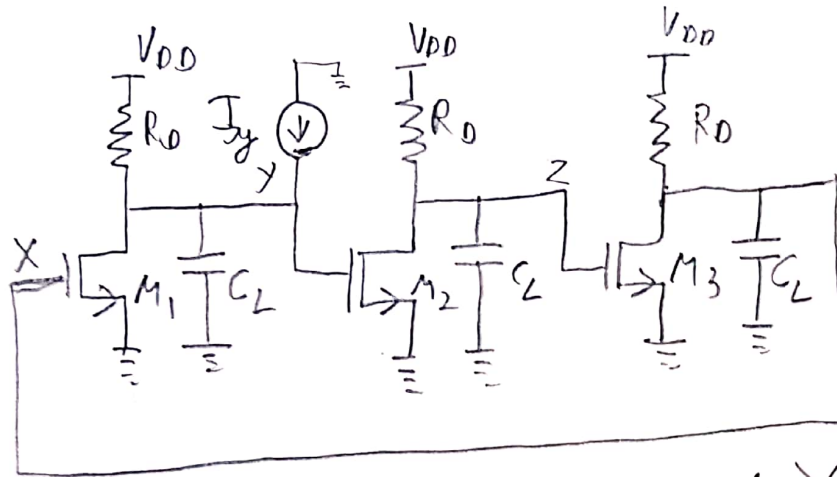
For impedance to go to infinity, denominator should be zero

$$1 + g_m R_D e^{-s\Delta T} = 0 \Rightarrow g_m R_D e^{-s\Delta T} = -1$$

$$\Rightarrow \boxed{g_m R_D = 1} \quad \text{and} \quad \omega_0 \Delta T = \pi$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2\Delta T}}$$

1.5)



Apply current source at node Y.

$$I_y = \frac{V_y}{(R_D \parallel \frac{1}{C_L s})} + g_m V_x \quad \text{--- (1)}$$

$$\begin{aligned} \text{But } V_x &= -g_m (R_D \parallel \frac{1}{C_L s}) V_z \\ &= g_m (R_D \parallel \frac{1}{C_L s}) g_m (R_D \parallel \frac{1}{C_L s}) V_y \quad \text{--- (2)} \end{aligned}$$

From (1) and (2)

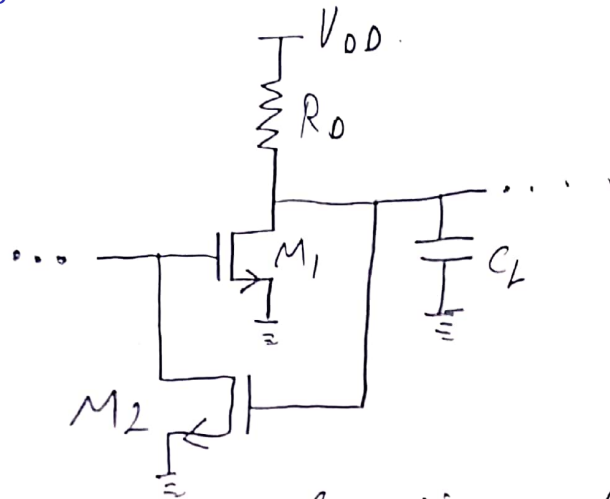
$$I_y = \frac{V_y}{(R_D \parallel \frac{1}{C_L s})} + g_m^3 V_y (R_D \parallel \frac{1}{C_L s})^2$$

$$I_y = V_y \left[ \frac{1}{(R_D \parallel \frac{1}{C_L s})} + g_m^3 (R_D \parallel \frac{1}{C_L s})^2 \right]$$

$$I_y = V_y \left[ \frac{1 + g_m^3 (R_D \parallel \frac{1}{C_L s})^3}{R_D \parallel \frac{1}{C_L s}} \right]$$

$$Z_y = \frac{V_y}{I_y} = \frac{R_D \parallel \frac{1}{C_L s}}{1 + g_m^3 (R_D \parallel \frac{1}{C_L s})^3}$$

1.6



To find Transfer function of single stage, apply  $V_x$  at input of  $M_1$  and check output  $V_F$  at drain of  $M_2$ .  $M_2$  is loaded from previous stage.

$$V_F = -g_m V_{out} \left( R_0 \parallel \frac{1}{C_L s} \parallel \frac{1}{g_m} \right)$$

$$V_F = g_m \left( R_0 \parallel \frac{1}{C_L s} \parallel \frac{1}{g_m} \right) \cdot g_m \left( R_0 \parallel \frac{1}{C_L s} \right) V_x$$

$$H(s) = \frac{-V_F}{V_x} = -g_m^2 \left( R_0 \parallel \frac{1}{C_L s} \parallel \frac{1}{g_m} \right) \left( R_0 \parallel \frac{1}{C_L s} \right)$$

$$\approx -g_m^2 \left( R_0 \parallel \frac{1}{C_L s} \right)^2$$

$$\therefore |H(s)| \equiv 1 \Rightarrow \left| g_m \left( R_0 \parallel \frac{1}{C_L s} \right) \right|^2 = 1 \quad \text{--- (1)}$$

$$H(j\omega) = \frac{g_m^2 R_0^2}{(R_0 C_L j\omega + 1)^2} = \frac{g_m^2 R_0^2}{2 R_0 C_L j\omega + 1 - R_0^2 C_L^2 \omega^2}$$

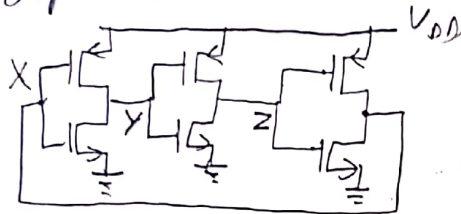
We need a frequency dependant phase shift of  $\frac{-180^\circ}{N}$  per stage

$$\frac{-180^\circ}{N} = -\tan^{-1} \left( \frac{2 R_0 C_L \omega_0}{1 - R_0^2 C_L^2 \omega_0^2} \right) \quad \text{--- (2)}$$

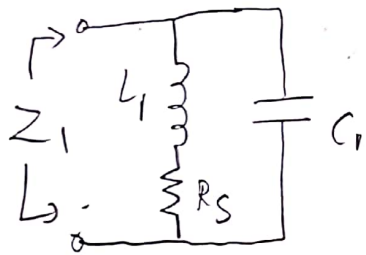
Eq<sup>n</sup>: (1) and (2) give the oscillation condition.



1.7 The frequency of oscillation is dependant on the time delay / propagation delay of the inverter. If the size of each transistor is doubled the capacitance at the node increases. But the resistance offered by the MOS device decreases. The time constant  $RC$  of the inverter stays the same, thus keeping the oscillation frequency constant. Moreover the delay is highly dependant on  $V_{DD}$ , than on sizing.



1.8



$$Z_1 = (R_S + L_1 S) \parallel \frac{1}{C_1 S}$$

$$= \frac{R_S + L_1 S}{R_S + L_1 S + \frac{1}{C_1 S}}$$

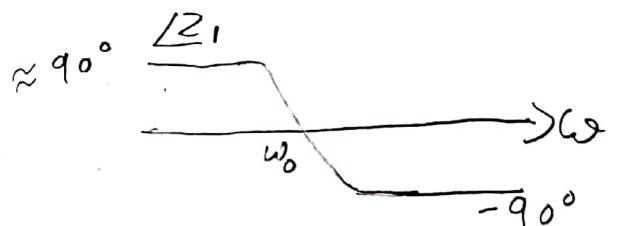
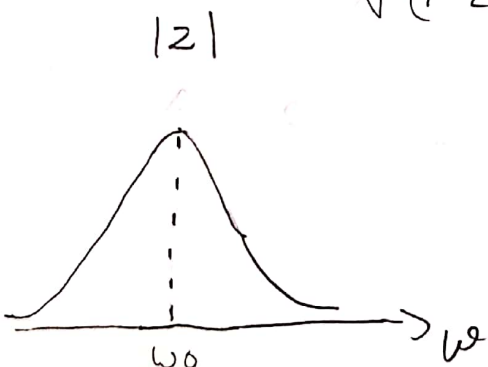
$$Z_1 = \frac{R_S + L_1 S}{R_S C_1 S + L_1 C_1 S^2 + 1}$$

$$Z_1(j\omega) = \frac{R_S + L_1 j\omega}{C_1 R_S j\omega + L_1 C_1 (j\omega)^2 + 1}$$

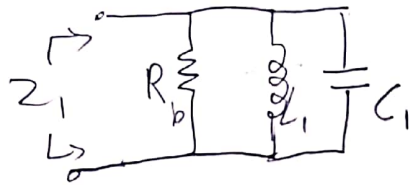
$$= \frac{R_S + L_1 j\omega}{j(R_S C_1 \omega) + 1 - L_1 C_1 \omega^2}$$

$$|Z_1(s=j\omega)| = \sqrt{\frac{R_S^2 + L_1^2 \omega^2}{(1 - L_1 C_1 \omega^2)^2 + R_S^2 C_1^2 \omega^2}}$$

$$\angle Z_1 = -\tan^{-1} \left[ \frac{R_S C_1 \omega}{1 - L_1 C_1 \omega^2} \right]$$



1.9)

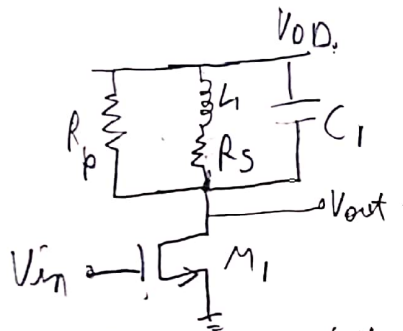


If  $L_1$  doubles and  $R_S$  remains constant, then  $R_p = \frac{L_1^2 \omega^2}{R_S}$  increases by 4 times

Since  $R_p$  increases by 4 times the slope also increases because  $\frac{d|Z_1(j\omega)|}{d\omega} \approx -2R_p C_1$  and  $L$  acts as a better inductor due

If  $L_1$  doubles and  $R_S$  also doubles, then  $R_p$  increases by only twice and thus increase in slope is less severe.

1.10



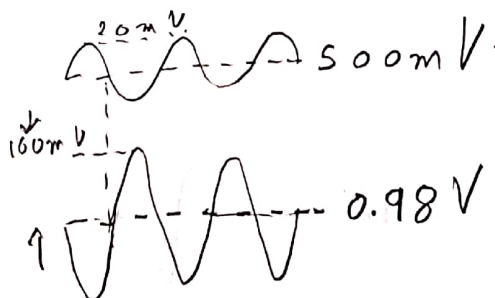
Since there is a resistance in series with  $L_1$ , there will be a voltage drop across  $L$  from  $V_{DD}$  to  $V_{out}$  at DC, but  $R_S \parallel R_p \approx R_S$

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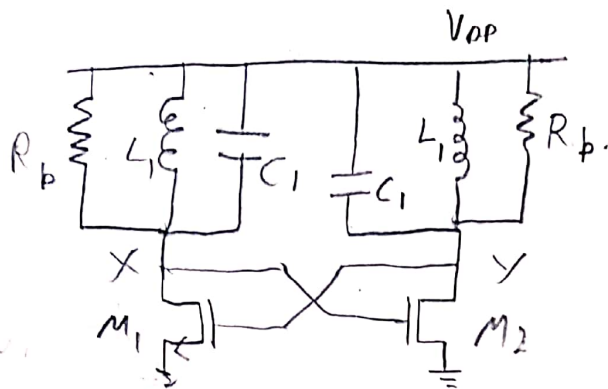
$$V_{out} = V_{DD} - I_D R_S$$

$$= 1V - 0.02$$

$$= 0.98V$$



1.11)



To compute the amplitude of third harmonic we need to find impedance at  $3\omega_0$

$$Z_p = R_p \parallel L_1 \parallel \frac{1}{C_1 s}$$

$$Z_p = R_p \parallel \left( \frac{L_1 s}{L_1 C_1 s^2 + 1} \right) = R_p \parallel \left( \frac{L_1 j\omega}{-L_1 C_1 \omega^2 + 1} \right)$$

$$Z_p|_{\omega=3\omega_0} = R_p \parallel \left( \frac{3L_1 j\omega_0}{-L_1 C_1 (3\omega_0)^2 + 1} \right) = R_p \parallel \left( \frac{3L_1 j\omega_0}{-9 + 1} \right)$$

$$= R_p \parallel \left( \frac{3jL_1 \times \frac{1}{\sqrt{C_1}}}{-8} \right) = R_p \parallel \left( \frac{3j}{-8} \sqrt{\frac{L_1}{C_1}} \right)$$

$$Z_p|_{\omega=3\omega_0} = \frac{3jR_p \sqrt{L_1}}{-8R_p \sqrt{C_1} + 3j\sqrt{L_1}} \quad \text{or} \quad \frac{3jR_p L_1 \omega_0}{-8R_p + 3jL_1 \omega_0}$$

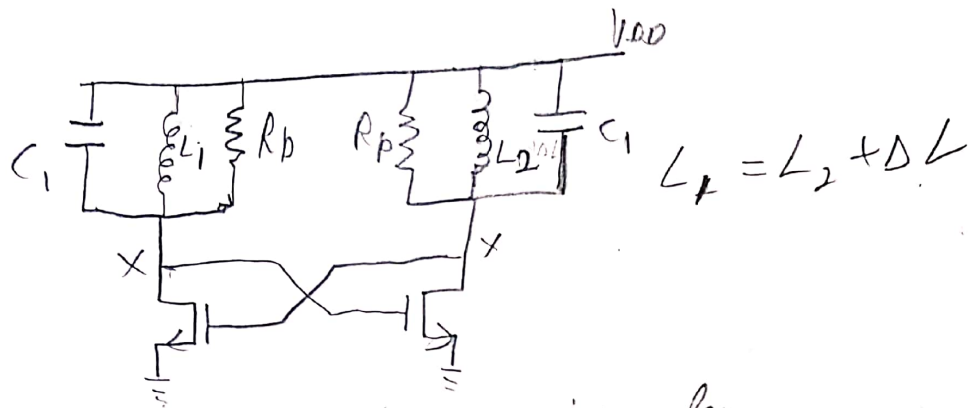
We know that  $Z_p|_{\omega=\omega_0} = R_p$ .

$$\therefore \frac{V_{3\omega_0}}{V_{\omega_0}} = \frac{Z_p|_{\omega=3\omega_0}}{Z_p|_{\omega=\omega_0}} = \frac{3jR_p L_1 \omega_0}{-8R_p + 3jL_1 \omega_0} \cdot \frac{1}{R_p}$$

$$\boxed{\frac{V_{3\omega_0}}{V_{\omega_0}} = \frac{3jL_1 \omega_0}{-8R_p + 3jL_1 \omega_0}}$$



1.12



The phase of each tank is given by

$$\angle Z(j\omega) = \frac{\pi}{2} - \tan^{-1} \frac{L\omega}{R_p(1 - L_1 C_1 \omega^2)}$$

The sum of phases of both stages should be zero

$$\therefore \frac{\pi}{2} - \tan^{-1} \left( \frac{L_1 \omega}{R_p(1 - L_1 C_1 \omega^2)} \right) + \frac{\pi}{2} - \tan^{-1} \left( \frac{L_2 \omega}{R_p(1 - L_2 C_1 \omega^2)} \right) = 0$$

But  $L_1 = L_2 + \Delta L$  and  $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$

$$H(s) \tan^{-1} \left( \frac{\frac{x_1 + y_1}{x_2} \frac{y_2}{y_2}}{1 - \frac{x_1 y_1}{x_2 y_2}} \right) = \tan^{-1} \left( \frac{x_1 y_2 + y_1 x_2}{x_2 y_2 - x_1 y_1} \right)$$

$$\frac{x_1 y_2 + y_1 x_2}{x_2 y_2 - x_1 y_1} = \tan(\pi) = 0$$

$$x_1 y_2 + y_1 x_2 = 0$$

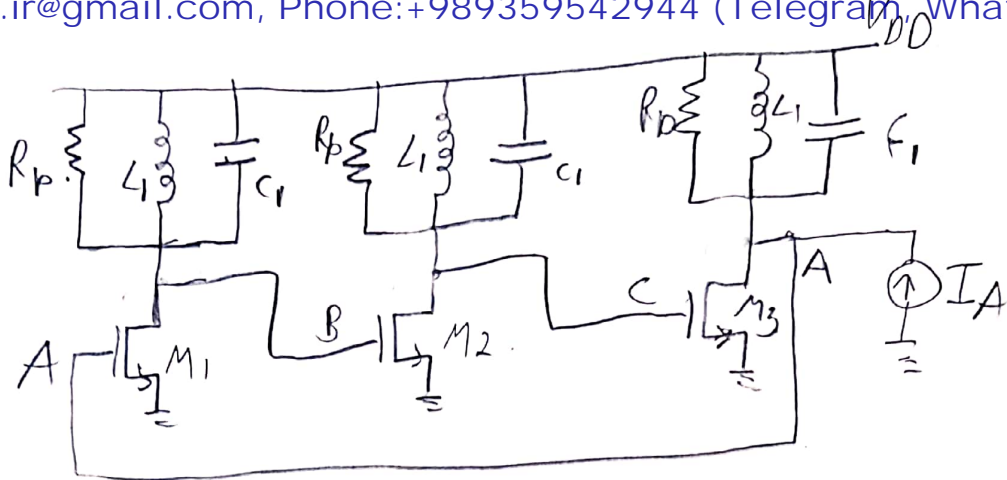
$$R_p [L_2 \omega - L_2 (L_2 + \Delta L) C_1 \omega^3 + (L_2 + \Delta L) \omega - L_2 (L_2 + \Delta L) C_1 \omega^3] = 0$$

$$2L_2 \omega - \Delta L \omega - 2L_2 (L_2 + \Delta L) C_1 \omega^3$$

$$2L_2 + \Delta L = 2L_2 (L_2 + \Delta L) C_1 \omega^2$$

$$\therefore \omega = \sqrt{\frac{2L_2 + \Delta L}{2L_2 (L_2 + \Delta L) C_1}}$$

1.13



$$I_A = \frac{V_A}{R_p \parallel L_1 \parallel S \parallel \frac{1}{C_1}} + g_m V_C$$

$$= \frac{V_A}{Z} + g_m^3 Z^2 V_A$$

where  $Z = R_p \parallel L_1 \parallel S \parallel \frac{1}{C_1}$

$$I_A = V_A \left( \frac{1}{Z} + g_m^3 Z^2 \right)$$

$$\boxed{\frac{V_A}{I_A} = \frac{V_A}{\frac{1}{Z} + g_m^3 Z^2}}$$

For the circuit to oscillate, impedance should go to  $\infty$

$$\text{i.e. } \frac{1}{Z} + g_m^3 Z^2 = 0$$

$$1 + g_m^3 Z^3 = 0$$

$$g_m^3 Z^3 = -1$$

$$g_m^3 \left( R_p \parallel L_1 \parallel S \parallel \frac{1}{C_1} \right)^3 = -1$$

$$g_m^3 \left( \frac{j R_p L_1 \omega_1}{R_p (1 - L_1 C_1 \omega_1^2) + j L_1 \omega_1} \right)^3 = -1$$

By finding magnitude on both sides and squaring, we have

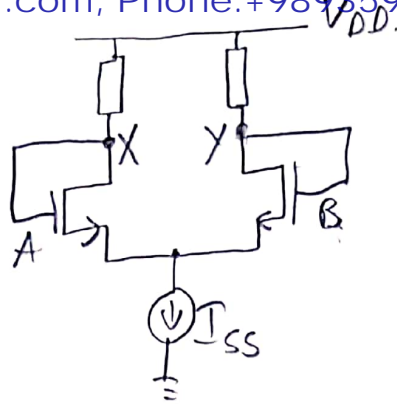
$$\left[ \frac{g_m^2 R_p^2 L_1^2 \omega_1^2}{R_p^2 (1 - L_1 C_1 \omega_1^2)^2 + L_1^2 \omega_1^2} \right]^3 = 1^2, \text{ use } \omega_1 = \alpha \omega_0 \text{ in this eq}^n$$

$$\Rightarrow g_m^2 R_p^2 L_1^2 \alpha^2 \omega_0^2 = R_p^2 (1 - \alpha^2)^2 + L_1^2 \alpha^2 \omega_0^2$$

$$\boxed{g_m R_p = \sqrt{1 + \frac{R_p^2}{L_1^2 \omega_0^2} \left( \frac{1}{\alpha} - \alpha \right)^2}}$$

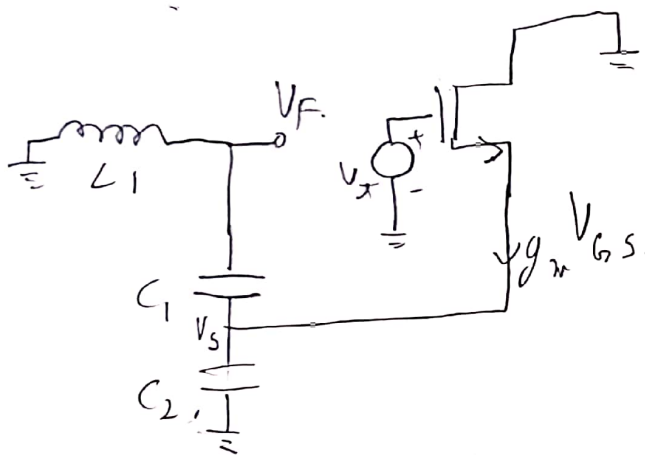
where  $\alpha = \frac{\omega_1}{\omega_0}$

1.14)



The CS stage provides a  $180^\circ$  phase shift at its output. We would need another  $180^\circ$  phase shift for oscillations to occur. Hence we need a delay line of  $180^\circ$  in series with the gate of the two transistors.

1.15



By applying a voltage  $V_x$  at input we get

$$\frac{V_s}{V_x} = \frac{g_m Z}{1 + g_m Z}, \text{ where } Z \text{ is the impedance seen at the source.}$$

We can see that

$$V_F = V_s \frac{L_1 s}{L_1 s + \frac{1}{C_1 s}}$$

$$V_F = \frac{g_m Z}{1 + g_m Z} \left( \frac{L_1 s}{L_1 s + \frac{1}{C_1 s}} \right) V_x.$$