Chapter 1

1.1 (a) $x(t) = A\cos 2\pi f_0 t$: Power signal

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2\cos^2 2\pi f_0 t\,dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \frac{A^2}{2}\,dt = \frac{A^2}{2}$$

(b)

$$x(t) = \begin{cases} A\cos 2\pi f_0 t & \text{for } -T_0/2 \le t \le \frac{T_0}{2} \\ 0 & \text{elsewhere} \end{cases}$$

Energy signal

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-T_0/2}^{T_0/2} A^2\cos^2 2\pi f_0 t\,dt = \frac{A^2 T_0}{2}$$

(c)

$$x(t) = \begin{cases} A\exp(-at) & \text{for } t>0,\ a>0 \\ 0 & \text{elsewhere} \end{cases}$$

Energy signal

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{0}^{\infty} A^2\exp(-2at)\,dt$$

$$= \left[\frac{A^2\exp(-2at)}{-2a}\right]_0^{\infty} = \frac{A^2}{2a}$$

(d) $x(t) = \cos t + 5\cos 2t$ for $-\infty < t < \infty$

Power signal

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \cos^2 t + 25\cos^2 2t \, dt; \quad 2\pi f_0 = 1, \quad T_0 = 2\pi$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{1}{2} + \frac{25}{2}\right) dt = \frac{1}{2\pi}(26\pi) = 13$$

1.2 $x(t) = \text{rect}(t/T)$

$$= \begin{cases} 1 & \text{for } -T/2 \le t \le T/2 \\ 0 & \text{elsewhere} \end{cases}$$

ESD $\psi(f) = |X(f)|^2 = T^2 \text{sinc}^2(fT)$

$$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-T/2}^{T/2} dt = T$$

1.3 Using Equations (1.18) and (1.19)

$$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$$

$$P_x = \int_{-\infty}^{\infty} G_x(f)\, df = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)\, df$$

$$P_x = \sum_{n=-\infty}^{\infty} |c_n|^2$$

1.4

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt\,; \qquad 2\pi f_0 = 10,\quad f_0 = 5/\pi,\quad T_0 = \pi/5$$

$$P_x = \frac{5}{\pi}\int_{-\pi/10}^{\pi/10} 100\cos^2 10t + 400\cos^2 20t\,dt$$

$$= \frac{5}{2\pi}\int_{-\pi/10}^{\pi/10} 100\left(1+\cos 20t\right) + 400\left(1+\cos 40t\right)dt$$

$$= \frac{5}{2\pi}\Big[100t + 400t\Big]_{-\pi/10}^{\pi/10} = 250\ \text{W}$$

1.5

$$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2\,\delta(f - nf_0)$$

$$c_1 = c_{-1} = \frac{10}{2} = 5; \qquad c_2 = c_{-2} = \frac{20}{2} = 10$$

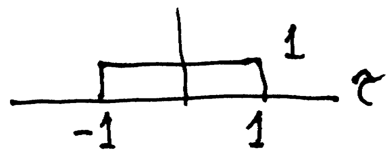
$$c_n = 0 \quad \text{for} \quad n = 0, \pm 3, \pm 4, \cdots$$

$$G_x(f) = (5)^2\delta\left(f - \tfrac{5}{\pi}\right) + (5)^2\delta\left(f + 5/\pi\right) + (10)^2\delta\left(f - 10/\pi\right) + (10)^2\delta\left(f + 10/\pi\right)$$

$$P_x = \int_{-\infty}^{\infty} G_x(f)\,df = 25 + 25 + 100 + 100 = 250\ \text{W}$$

1.6 $\mathcal{F}\{R(\tau)\}$ must be a nonnegative function because $\mathcal{F}\{R(\tau)\} = G(f)$; and, the power spectral density, $G(f)$, must be a nonnegative function.

(a) $x(\tau) = \begin{cases} 1 & \text{for } -1 \le \tau \le 1 \\ 0 & \text{otherwise} \end{cases}$



NO
1. $x(\tau) = x(-\tau)$ ✓
2. $x(0) \ge x(\tau)$ ✓
3. $\mathcal{F}\{x(\tau)\}$ is a positive <u>and</u> <u>negative</u> going function.

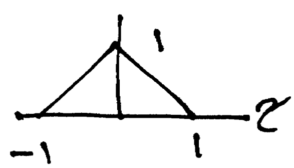
(b) $x(\tau) = \delta(\tau) + \sin 2\pi f_0 \tau$

NO
1. $x(\tau) \ne x(-\tau)$ ×

(c) $x(t) = \exp(|\tau|)$

NO
1. $x(\tau) = x(-\tau)$ ✓
2. $x(0) \not\ge x(\tau)$ ×

(d) $x(\tau) = \begin{cases} -\tau + 1 & \text{for } 0 \le \tau \le 1 \\ \tau + 1 & \text{for } -1 \le \tau \le 0 \end{cases}$



YES
1. $x(\tau) = x(-\tau)$ ✓
2. $x(0) \ge x(\tau)$ ✓
3. $\mathcal{F}\{x(\tau)\} = 2\,\text{sinc}^2 f\tau$ is a nonnegative function. ✓

1.7 (a) $X(f) = \delta(f) + \cos^2 2\pi f$

YES
1. always real ✓
2. $P_x(f) \geq 0$ ✓
3. $P_x(-f) = P_x(f)$ ✓

(b) $X(f) = 10 + \delta(f-10)$

No
1. always real ✓
2. $P_x(f) \geq 0$ ✓
3. $P_x(-f) \neq P_x(f)$ ×

(c) $X(f) = \exp(-2\pi|f-10|)$

No
1. always real ✓
2. $P_x(f) \geq 0$ ✓
3 $P_x(-f) \neq P_x(f)$ ×

(d) $X(f) = \exp[-2\pi(f^2-10)]$

YES
1. always real ✓
2. $P_x(f) \geq 0$ ✓
3. $P_x(-f) = P_x(f)$ ✓

1.8

$$R_x(\tau) = \left\langle A\cos(2\pi f_0 t+\phi)\, A\cos(2\pi f_0 t + 2\pi f_0 \tau + \phi)\right\rangle$$

where $\langle\cdot\rangle$ is the time averaging operator $\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} dt$

Upon expanding (see Appendix D), $R_x(\tau)$ becomes:

$$R_x(\tau) = A^2\Big[\cos 2\pi f_0\tau \left\langle \cos^2(2\pi f_0 t+\phi)\right\rangle$$
$$- \sin 2\pi f_0 \tau \left\langle \cos(2\pi f_0 t+\phi)\sin(2\pi f_0 t+\phi)\right\rangle\Big]$$

The negative term in the above expression goes to zero, and hence

$$R_x(\tau) = \frac{A^2}{2}\cos 2\pi f_0 \tau$$

$$P_x = R_x(0) = A^2/2$$

1.9 (a) $R_x(\tau) = \frac{100}{2}\cos 10\tau + \frac{400}{2}\cos 20\tau$

where $2\pi f_0 = 10$

(b) $P_x = R_x(0) = 50 + 200 = 250\ \text{W}$

1.10 (a) average value of $x(t)$

$$\langle x(t) \rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (1 + \cos 2\pi f_0 t)\, dt = 1$$

(b) the ac power of $x(t)$

$$\langle \sigma^2(t) \rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \cos^2 2\pi f_0 t \, dt = 1/2$$

(c) the rms value of $x(t)$

$$\langle x^2(t) \rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (1 + \cos 2\pi f_0 t)^2 \, dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (1 + 2\cos 2\pi f_0 t + \cos^2 2\pi f_0 t)\, dt = \frac{3}{2}$$

$$x_{rms} = \sqrt{3/2}$$

1.11 (a) $$\langle X(t) \rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A\cos(2\pi f_0 t + \phi)\, dt = 0$$

$$\langle X^2(t) \rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} [A\cos(2\pi f_0 t + \phi)]^2 \, dt$$

$$= \frac{A^2}{2}$$

1.11 (b) $E\{X\} = \int_{-\infty}^{\infty} X(\phi)\, p(\phi)\, d\phi$

$p(\phi) = \frac{1}{2\pi}$ since ϕ is uniformly distributed over $(0, 2\pi)$

$$E\{X\} = \int_0^{2\pi} [A \cos(2\pi f_0 t + \phi)] \frac{1}{2\pi}\, d\phi = 0$$

$$E\{X^2\} = \int_0^{2\pi} [A \cos(2\pi f_0 t + \phi)]^2 \frac{1}{2\pi}\, d\phi$$

$$= A^2/2$$

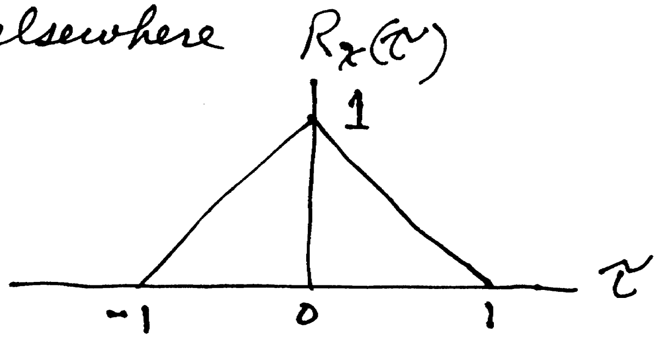
1.12 $X(f) = \text{sinc}\, f$

$$\Psi_x(f) = |X(f)|^2 = \text{sinc}^2 f$$

$$R_x(\tau) = \mathcal{F}^{-1}\{\Psi_x(f)\}$$

From Table A.1, $R_x(\tau)$ is seen to be the following triangular function:

$$R_x(\tau) = \begin{cases} 1 - |\tau| & \text{for } |\tau| < 1 \\ 0 & \text{elsewhere} \end{cases}$$



1.13 (a) $\int_{-\infty}^{\infty} \cos 6t\, \delta(t-3)\, dt = \cos 18$

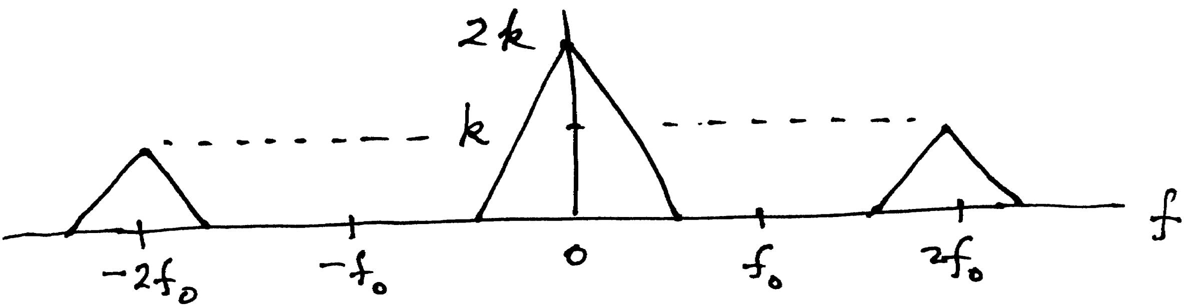
(b) $\int_{-\infty}^{\infty} 10\, \delta(t)\, (1+t)^{-1} dt = 10$

(c) $\int_{-\infty}^{\infty} \delta(t+4)\, (t^2+6t+1)\, dt = -7$

(d) $\int_{-\infty}^{\infty} \exp(-t^2)\, \delta(t-2)\, dt = 0.0183$

1.14 $X_2(f) = k\left[\delta(f-f_0) + \delta(f+f_0)\right]$

$X_1(f) * X_2(f) = X_1(f) * k\left[\delta(f-f_0) + \delta(f+f_0)\right]$



1.15 (a) $P_x = 2\int_0^{10\text{ kHz}} G_x\, df = 2\int_0^{10\text{ kHz}} 10^{-6} f^2\, df$

$= 2\left[\frac{10^{-6} f^3}{3}\right]_0^{10^4} = 667 \text{ kW}$

(b) $P_x = 2\int_{5\text{ kHz}}^{10\text{ kHz}} 10^{-6} f^2\, df = 2\left[\frac{10^{-6} f^3}{3}\right]_{5000}^{10000}$

$= 583 \text{ kW}$

1.16 $10 \log_{10}\left[\frac{100 \times 2 \times \frac{1}{2}}{\frac{1}{2}}\right] = 23\text{ dB}$

1.17 (a) Since $|H(f)|$ decreases monotonically with $|f|$, and $|H(0)| = 1$, we can write the following relationship in terms of the -1 dB frequency, f_1.

$$10 \log_{10} |H(f_1)|^2 = -1\text{ dB}$$

$$\log_{10}\left[\frac{1}{(1+f_1/f_u)^{2n}}\right] = -\frac{1}{10}$$

$$\left[1 + f_1/f_u\right]^{2n} = 10^{1/10}$$

$$\left[f_1/f_u\right]^{2n} = 10^{1/10} - 1 = 0.2584$$

$$\therefore\ n \geq \frac{1}{2}\left[\frac{\log 0.2584}{\log (f_1/f_u)}\right]$$

For $f_1/f_u = 0.9$, $n \geq 6.4$

Thus, $n = 7$

1.17 (b) In the limit, as $n \to \infty$

$$\left(\frac{f}{f_u}\right)^{2n} \to 0, \quad |H(f)| \to 1, \quad \text{for } \left|\frac{f}{f_u}\right| < 1$$

$$\left(\frac{f}{f_u}\right)^{2n} \to \infty, \quad |H(f)| \to 0, \quad \text{for } \left|\frac{f}{f_u}\right| > 1$$

Hence, $|H(f)|$ approaches the transfer characteristic of an ideal low-pass filter with a cut-off frequency at f_u, as n approaches infinity.

1.18

$$y(t) = \delta(t) * h(t)$$

$$Y(f) = 1 \times H(f)$$

since $\mathcal{F}\{\delta(t)\} = 1$.

Hence, $y(t) = \mathcal{F}^{-1}\{Y(f)\} = \mathcal{F}^{-1}\{H(f)\}$

$$= h(t)$$

1.19 Let $x(t) = \delta(t)$

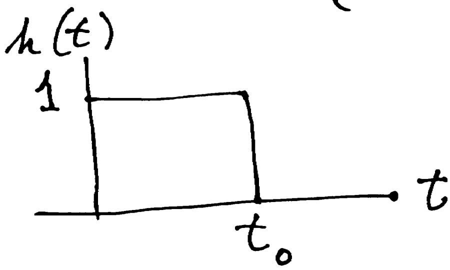
$$g(t) = \delta(t) - \delta(t - t_0)$$

$$h(t) = \int_{-\infty}^{t} [\delta(\tau) - \delta(\tau - t_0)]\, d\tau$$

$$= u(t) - u(t - t_0)$$

where $u(t)$ is the unit step function defined as follows:

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & \text{elsewhere} \end{cases}$$



1.20 (a) Half-power bandwidth is the bandwidth from half-power point to half-power point. $BW = 2f_0$

where $\frac{1}{2} = \left[\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} \right]^2$

$$0.707 = \frac{\sin x_0}{x_0}, \quad x_0 = \pi f_0 10^{-4}$$

$$x_0 \cong 1.4 \Rightarrow f_0 = 4.46 \text{ kHz} \Rightarrow BW \cong 9 \text{ kHz}$$

1.20 (b) Noise equivalent bandwidth

$$BW = 2\int_0^{\infty}\left[\frac{\sin(\pi f 10^{-4})}{\pi f 10^{-4}}\right]^2 df$$

$$= \frac{2\times 10^4}{\pi}\underbrace{\int_0^{\infty}\left[\frac{\sin x}{x}\right]^2 dx}_{\pi/2}$$

$$= 10 \text{ kHz}$$

(c) Null-to-null bandwidth: $BW = 2f_0$ where f_0 is the frequency where

$$\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} = 0$$

The minimum f_0 corresponding to the first null is found by:

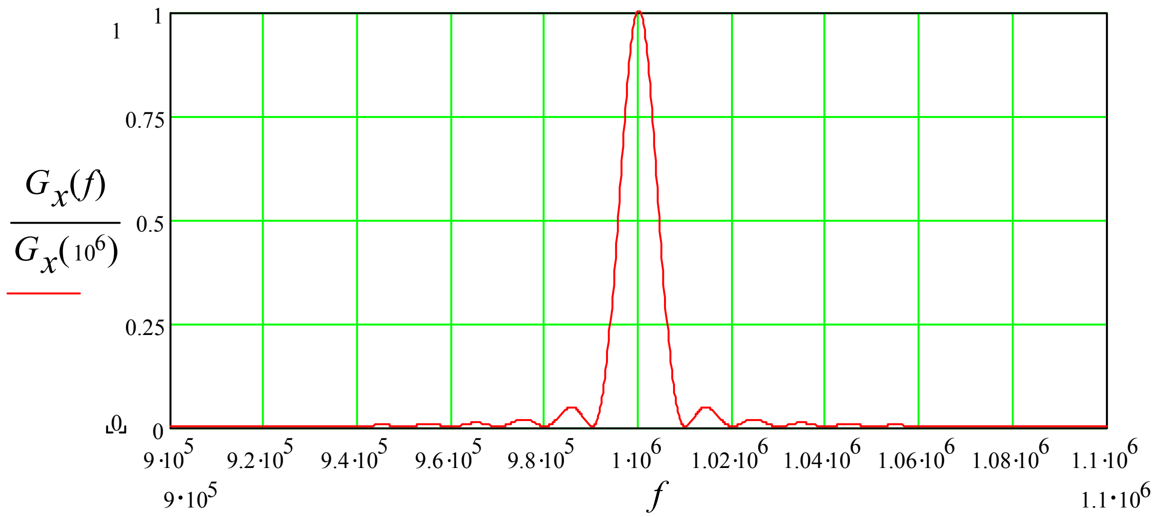
$$\pi f_0 10^{-4} = \pi$$

$$BW = 2f_0 = 20 \text{ kHz}$$

(d) 99% of power bandwidth. $BW = 2f_0$ where

$$0.995 = \frac{10^{-4}\int_0^{f_0}\left(\frac{\sin \pi f 10^{-4}}{\pi f 10^{-4}}\right)^2 df}{10^{-4}\int_0^{\infty}\left(\frac{\sin \pi f 10^{-4}}{\pi f 10^{-4}}\right)^2 df}$$

1.20 (d) The normalized spectrum, $G_x(f)/G_x(10^6)$, appears as:



Applying numerical methods with Mathcad ®, the two-sided 99% bandwidth can be found by numerical integration as:

$$\frac{\int_{10^6-f_1}^{10^6+f_1} G_x(f)\,df}{\int_{-\infty}^{\infty} G_x(f)\,df} = 0.99$$

where f_1 is found to be equal to 103 kHz. Thus, the two-sided 99% bandwidth is equal to 206 kHz.

Since this bandwidth corresponds to the given spectrum (with signaling rate = 10,000 symbols/s), normalizing it relative to one symbol per second, yields the two-sided 99% bandwidth as $(206\times10^3/10^4) = 20.6$ Hz for one symbol per second, or in general the 99% bandwidth in terms of the signaling rate, R, is $20.6\times R$ Hz.

1.20 (e)

35-dB Bandwidth:

35-dB attenuation $\Longrightarrow 10^{-3.5} = 3.16 \times 10^{-4}$

Since $\sin^2 x$ is unity for $x = \frac{\pi}{2}(2k+1)$, $k = 0, 1, \ldots$, the lobe beyond which the attenuation criterion is guaranteed to be met is the minimum k for which

$$10^{-3.5} \geq \frac{1}{\left[\frac{\pi}{2}(2k+1)\right]^2}$$

$$3162.28 \leq \left[\frac{\pi}{2}(2k+1)\right]^2$$

$$56.23 \leq \frac{\pi}{2}(2k+1)$$

$$35.80 \leq 2k+1 \implies k = 18$$

Thus the 18^{th} sidelobe meets the 35-dB criterion, and the actual 35-dB point will be on the falling edge of the 17^{th} sidelobe. $BW = 2f_0$, where f_0 is the minimum value satisfying:

$$\pi f_0 10^{-4} > \frac{\pi}{2}(35)$$

and
$$\left[\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}}\right]^2 = 10^{-3.5}$$

Solving iteratively, we get: $\pi f_0 10^{-4} = 55.171$

$$BW = 2f_0 = 351.2 \text{ kHz}$$

1.20 (f) The absolute bandwidth is <u>infinite</u>, since for any finite test-BW,
$10^{-4}\left[\frac{\sin \pi(f-10^6)10^{-4}}{\pi(f-10^6)10^{-4}}\right]^2$ will have positive measure beyond it.

Chapter 2

Answers to Problems

The pages in Chapter 2 contain a mix of typeset and handwritten text. The handwritten text is comprised of updates and solutions new to this edition.

Chapter 2

2.1 (a) $\underbrace{00010010}_{H}$ $\underbrace{11110011}_{O}$ $\underbrace{11101011}_{W}$

(b) $\underbrace{000}_{0}$ $\underbrace{100}_{4}$ $\underbrace{101}_{5}$ $\underbrace{111}_{7}$ $\underbrace{001}_{1}$ $\underbrace{111}_{7}$ $\underbrace{101}_{5}$ $\underbrace{011}_{3}$

8 symbols

(c) 24 bits / 4 bits per symbol = 6 symbols

(d) 24 bits / 8 bits per symbol = 3 symbols

2.2 (a) 800 char/s × 8 bits/char = 6400 bits/s

(b) $$\frac{6400 \text{ bits/s}}{4 \text{ bits/symbol}} = 1600 \text{ symbols/s.}$$

2.3 (a) 100 char/2 s × 8 bits/char = 400 bits/s.

$$\frac{400 \text{ bits/s}}{5 \text{ bits/symbol}} = 80 \text{ symbols/s}$$

(b) 16-level PCM: 400 bits/s, 100 symbols/s.

8-level PCM: 400 bits/s, 133.3 symbols/s.

4-level PCM: 400 bits/s, 200 symbols/s.

2-level PCM: 400 bits/s, 400 symbols/s.