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Chapter 2

Discrete-Time Signals and Systems

P2.1 Generate the following sequences using the basic MATLAB signal functions and the basic MATLAB signal operations discussed in this chapter. Plot signal samples using the `stem` function.

1. $x_1(n) = 3\delta(n+2) + 2\delta(n) - \delta(n-3) + 5\delta(n-7), -5 \le n \le 15$

```
% P0201a: x1(n) = 3*delta(n + 2) + 2*delta(n) - delta(n - 3) +
%                 5*delta(n - 7), -5 <= n <= 15.
clc; close all;
x1 = 3*impseq(-2,-5,15) + 2*impseq(0,-5,15) - impseq(3,-5,15) + 5*impseq(7,-5,15);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201a'); n1 = [-5:15];
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-1,max(x1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n1,'FontSize',8);
print -deps2 ../EPSFILES/P0201a;
```

The plots of $x_1(n)$ is shown in Figure 2.1.

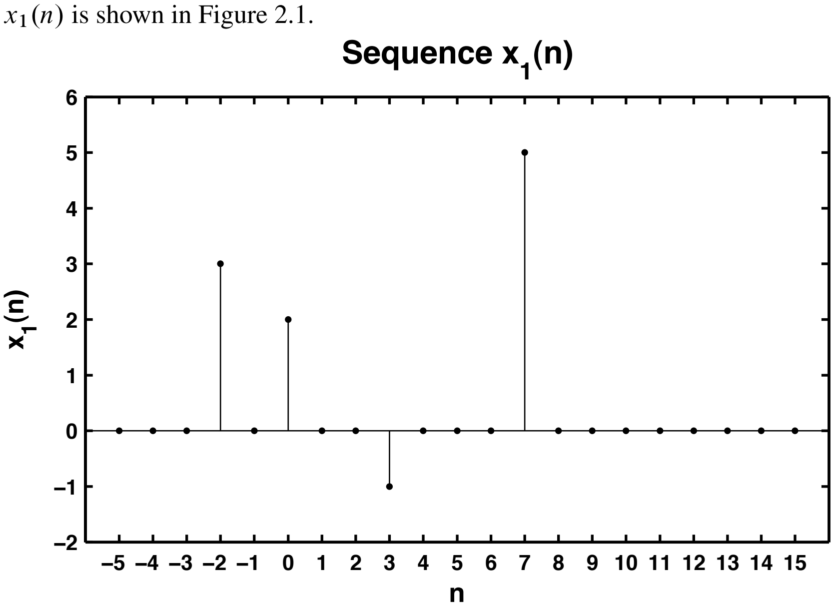

Figure 2.1: Problem P2.1.1 sequence plot

2. $x_2(n) = \sum_{k=-5}^{5} e^{-|k|}\delta(n-2k), -10 \le n \le 10.$

```
% P0201b: x2(n) = sum_{k = -5}^{5} e^{-|k|}*delta(n - 2k), -10 <= n <= 10
clc; close all;

n2 = [-10:10]; x2 = zeros(1,length(n2));
for k = -5:5
    x2 = x2 + exp(-abs(k))*impseq(2*k ,-10,10);
end
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201b');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n2);
print -deps2 ../EPSFILES/P0201b;
```

The plots of $x_2(n)$ is shown in Figure 2.2.

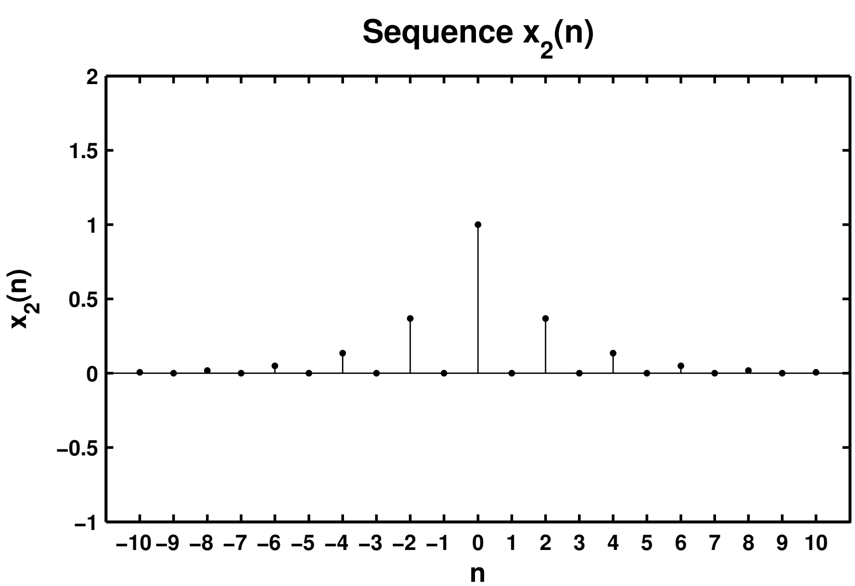

Figure 2.2: Problem P2.1.2 sequence plot

3. $x_3(n) = 10u(n) - 5u(n-5) - 10u(n-10) + 5u(n-15)$.

```
% P0201c: x3(n) = 10u(n) - 5u(n - 5) + 10u(n - 10) + 5u(n - 15).
clc; close all;

x3 = 10*stepseq(0,0,20) - 5*stepseq(5,0,20) - 10*stepseq(10,0,20) ...
     + 5*stepseq(15,0,20);
n3 = [0:20];
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-1,max(x3)+2]);
ytick = [-6:2:12];
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n3);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0201c;
```

The plots of $x_3(n)$ is shown in Figure 2.3.

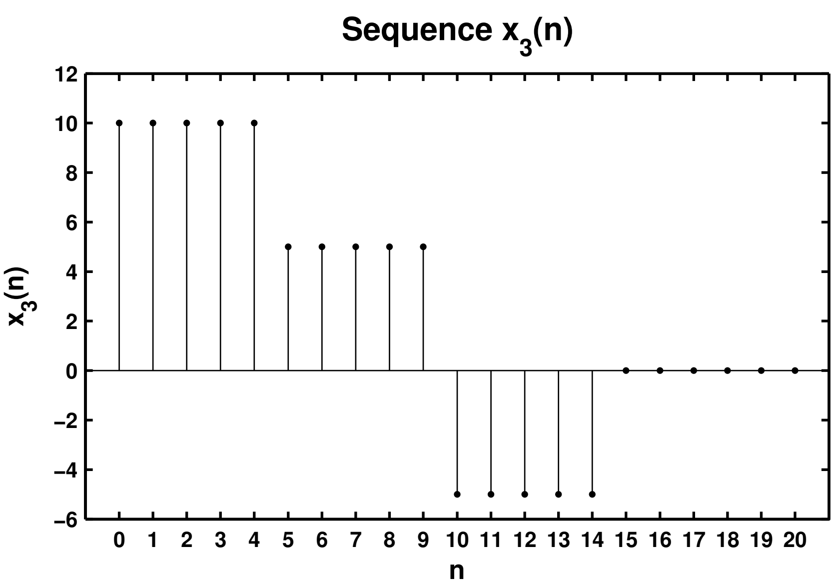

Figure 2.3: Problem P2.1.3 sequence plot

4. $x_4(n) = e^{0.1n}[u(n+20) - u(n-10)]$.

```
% P0201d: x4(n) = e ^ {0.1n} [u(n + 20) - u(n - 10)].
clc; close all;

n4 = [-25:15];
x4 = exp(0.1*n4).*(stepseq(-20,-25,15) - stepseq(10,-25,15));

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-2,max(n4)+2,min(x4)-1,max(x4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_4(n)','FontSize',LFS);
title('Sequence x_4(n)','FontSize',TFS); ntick = [n4(1):5:n4(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201d; print -deps2 ../../Latex/P0201d;
```

The plots of $x_4(n)$ is shown in Figure 2.4.

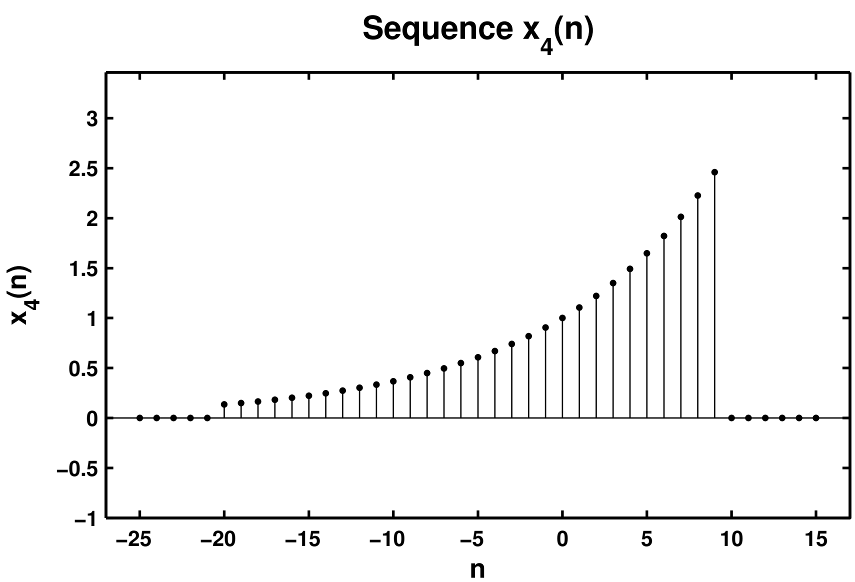

Figure 2.4: Problem P2.1.4 sequence plot

5. $x_5(n) = 5[\cos(0.49\pi n) + \cos(0.51\pi n)]$, $-200 \le n \le 200$. Comment on the waveform shape.

```
% P0201e: x5(n) = 5[cos(0.49*pi*n) + cos(0.51*pi*n)], -200 <= n <= 200.
clc; close all;

n5 = [-200:200]; x5  = 5*(cos(0.49*pi*n5) + cos(0.51*pi*n5));

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201e');
Hs = stem(n5,x5,'filled'); set(Hs,'markersize',2);
axis([min(n5)-10,max(n5)+10,min(x5)-2,max(x5)+2]);
xlabel('n','FontSize',LFS); ylabel('x_5(n)','FOntSize',LFS);
title('Sequence x_5(n)','FontSize',TFS);
ntick = [n5(1): 40:n5(end)]; ytick = [-12 -10:5:10 12];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0201e; print -deps2 ../../Latex/P0201e;
```

The plots of $x_5(n)$ is shown in Figure 2.5.

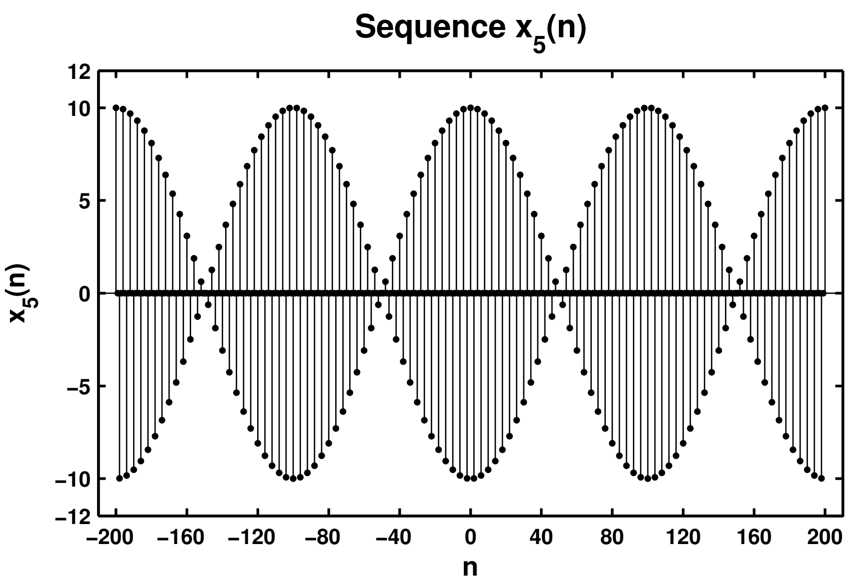

Figure 2.5: Problem P2.1.5 sequence plot

6. $x_6(n) = 2\sin(0.01\pi n)\cos(0.5\pi n),\ -200 \le n \le 200.$

```
%P0201f: x6(n) = 2 sin(0.01*pi*n) cos(0.5*pi*n), -200 <= n <= 200.
clc; close all;

n6 = [-200:200]; x6 = 2*sin(0.01*pi*n6).*cos(0.5*pi*n6);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201f');
Hs = stem(n6,x6,'filled'); set(Hs,'markersize',2);
axis([min(n6)-10,max(n6)+10,min(x6)-1,max(x6)+1]);
xlabel('n','FontSize',LFS); ylabel('x_6(n)','FontSize',LFS);
title('Sequence x_6(n)','FontSize',TFS);
ntick = [n6(1): 40:n6(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201f; print -deps2 ../../Latex/P0201f;
```

The plots of $x_6(n)$ is shown in Figure 2.6.

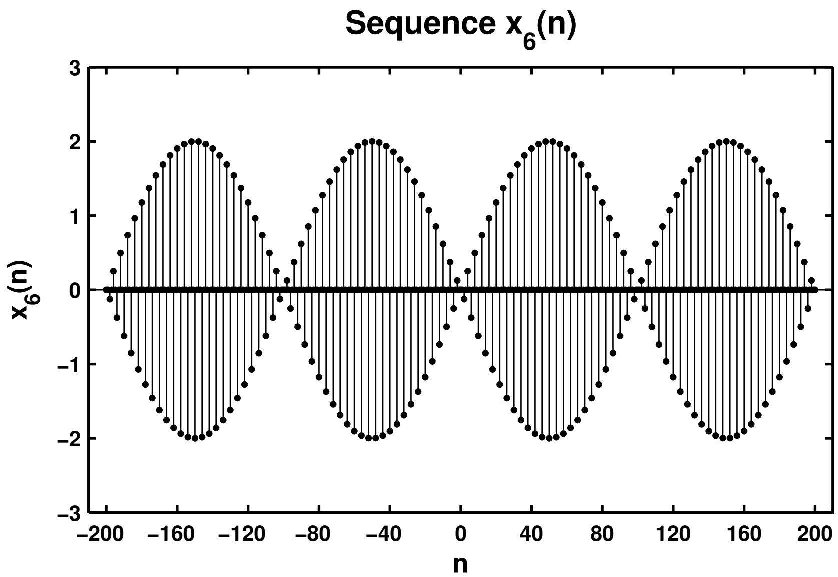

Figure 2.6: Problem P2.1.6 sequence plot

7. $x_7(n) = e^{-0.05n}\sin(0.1\pi n + \pi/3),\ 0 \le n \le 100.$

```
% P0201g: x7(n) = e ^ {-0.05*n}*sin(0.1*pi*n + pi/3), 0 <= n <=100.
clc; close all;

n7 = [0:100]; x7  = exp(-0.05*n7).*sin(0.1*pi*n7 + pi/3);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201g');
Hs = stem(n7,x7,'filled'); set(Hs,'markersize',2);
axis([min(n7)-5,max(n7)+5,min(x7)-1,max(x7)+1]);
xlabel('n','FontSize',LFS); ylabel('x_7(n)','FontSize',LFS);
title('Sequence x_7(n)','FontSize',TFS);
ntick = [n7(1): 10:n7(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201g;
```

The plots of $x_7(n)$ is shown in Figure 2.7.

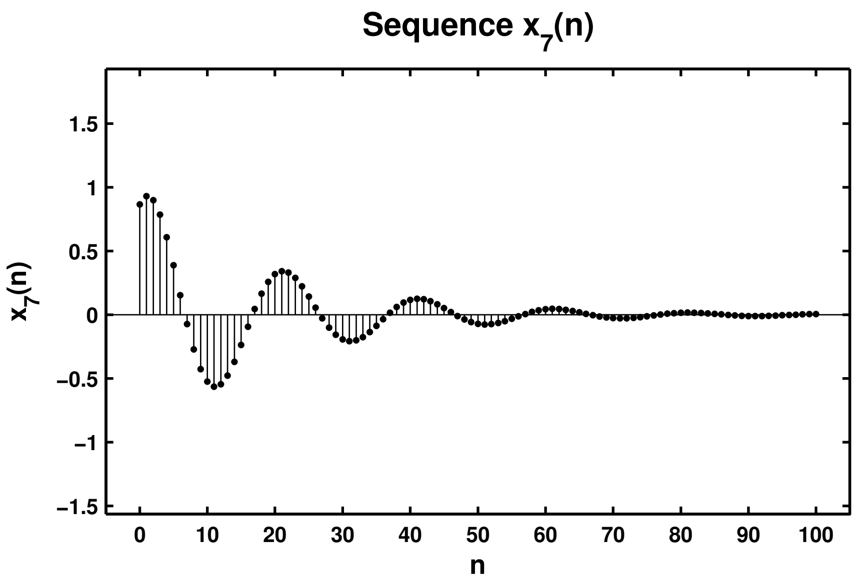

Figure 2.7: Problem P2.1.7 sequence plot

8. $x_8(n) = e^{0.01n} \sin(0.1\pi n),\ 0 \le n \le 100.$

```
% P0201h: x8(n) = e ^ {0.01*n}*sin(0.1*pi*n), 0 <= n <=100.
clc; close all;

n8 = [0:100]; x8  = exp(0.01*n8).*sin(0.1*pi*n8);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201h');
Hs = stem(n8,x8,'filled'); set(Hs,'markersize',2);
axis([min(n8)-5,max(n8)+5,min(x8)-1,max(x8)+1]);
xlabel('n','FontSize',LFS); ylabel('x_8(n)','FontSize',LFS);
title('Sequence x_8(n)','FontSize',TFS);
ntick = [n8(1): 10:n8(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201h
```

The plots of $x_8(n)$ is shown in Figure 2.8.

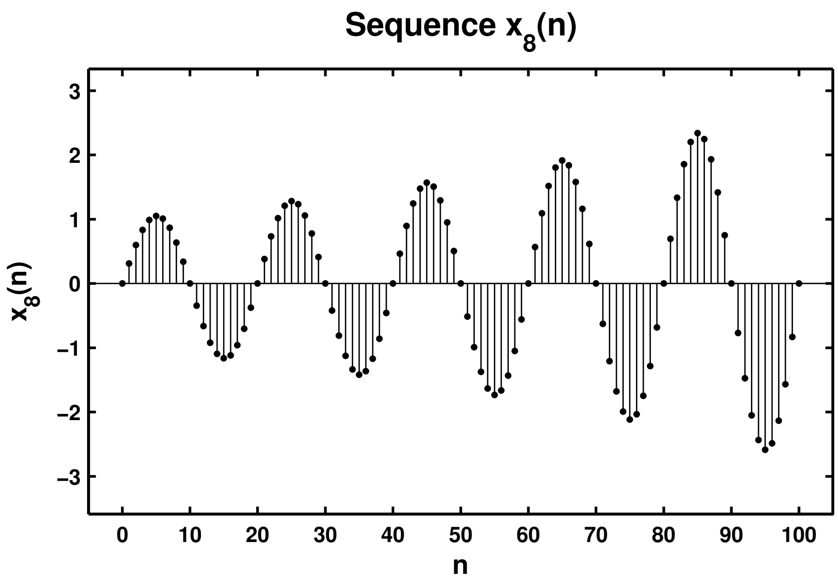

Figure 2.8: Problem P2.1.8 sequence plot

P2.2 Generate the following random sequences and obtain their histogram using the `hist` function with 100 bins. Use the `bar` function to plot each histogram.

1. $x_1(n)$ is a random sequence whose samples are independent and uniformly distributed over $[0, 2]$ interval. Generate 100,000 samples.

```
% P0202a: x1(n) = uniform[0,2]
clc; close all;

n1 = [0:100000-1]; x1 = 2*rand(1,100000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202a');
[h1,x1out] = hist(x1,100); bar(x1out, h1);
axis([-0.1 2.1 0 1200]);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_1(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202a;
```

The plots of $x_1(n)$ is shown in Figure 2.9.

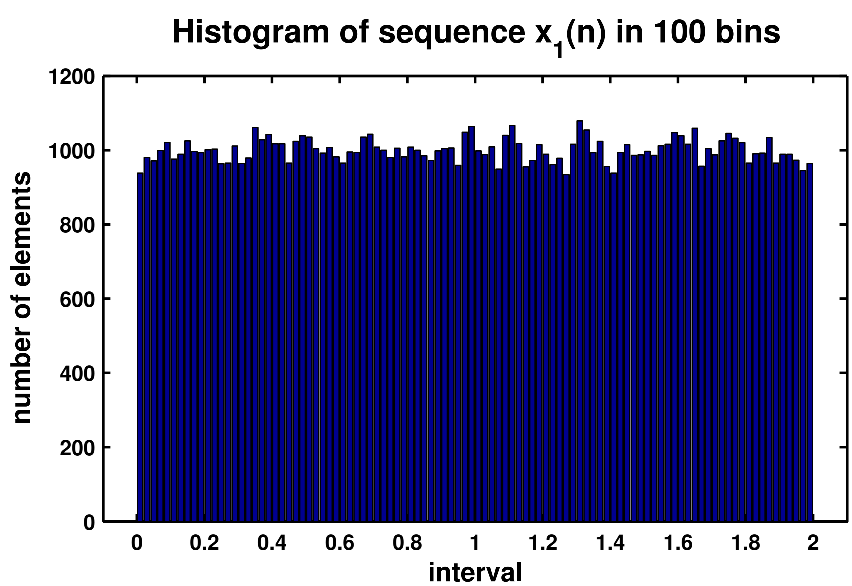

Figure 2.9: Problem P2.2.1 sequence plot

2. $x_2(n)$ is a Gaussian random sequence whose samples are independent with mean 10 and variance 10. Generate 10,000 samples.

```
% P0202b: x2(n) = gaussian{10,10}
clc; close all;

n2 = [1:10000]; x2 = 10 + sqrt(10)*randn(1,10000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202b');
[h2,x2out] = hist(x2,100); bar(x2out,h2);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_2(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202b;
```

The plots of $x_2(n)$ is shown in Figure 2.10.

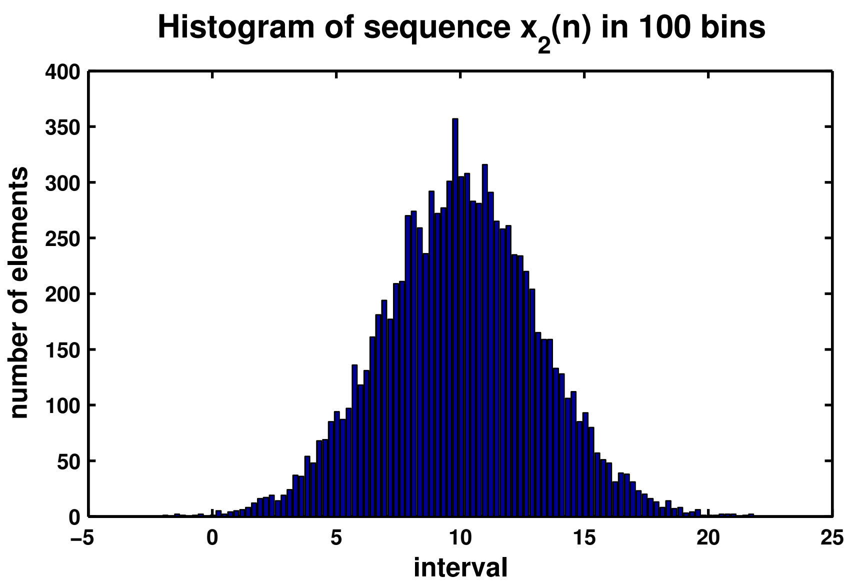

Figure 2.10: Problem P2.2.2 sequence plot

3. $x_3(n) = x_1(n) + x_1(n-1)$ where $x_1(n)$ is the random sequence given in part 1 above. Comment on the shape of this histogram and explain the shape.

```
% P0202c: x3(n) = x1(n) + x1(n - 1) where  x1(n) = uniform[0,2]
clc; close all;

n1 = [0:100000-1]; x1 = 2*rand(1,100000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202c');
[x11,n11] = sigshift(x1,n1,1);
[x3,n3] = sigadd(x1,n1,x11,n11);
[h3,x3out] = hist(x3,100);
bar(x3out,h3); axis([-0.5 4.5 0 2500]);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_3(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202c;
```

The plots of $x_3(n)$ is shown in Figure 2.11.

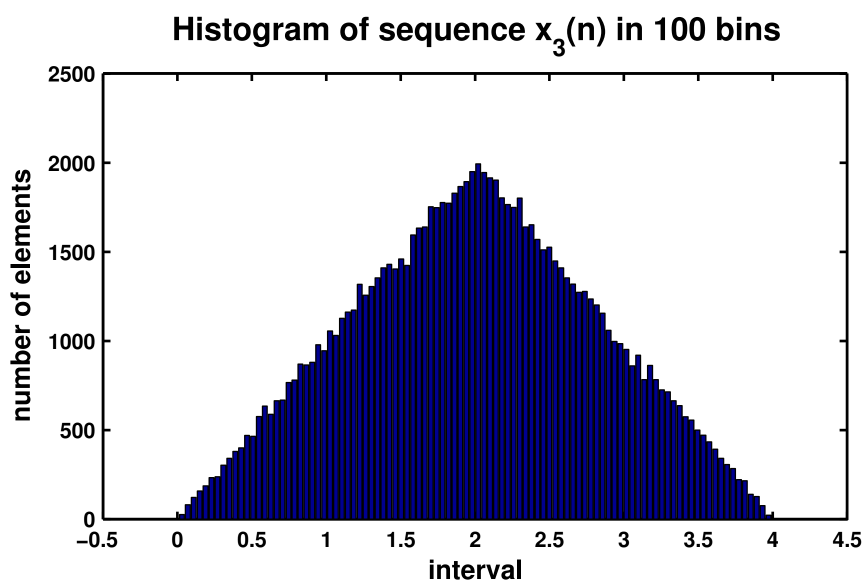

Figure 2.11: Problem P2.2.3 sequence plot

4. $x_4(n) = \sum_{k=1}^{4} y_k(n)$ where each random sequence $y_k(n)$ is independent of others with samples uniformly distributed over $[-0.5, 0.5]$. Comment on the shape of this histogram.

```
%P0202d: x4(n) = sum_{k=1} ^ {4} y_k(n), where each independent of others
%         with samples uniformly distributed over [-0.5,0.5];
clc; close all;

y1 = rand(1,100000) - 0.5; y2 = rand(1,100000) - 0.5;
y3 = rand(1,100000) - 0.5; y4 = rand(1,100000) - 0.5;
x4 = y1 + y2 + y3 + y4;

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202d');
[h4,x4out] = hist(x4,100); bar(x4out,h4);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_4(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202d;
```

The plots of $x_4(n)$ is shown in Figure 2.12.

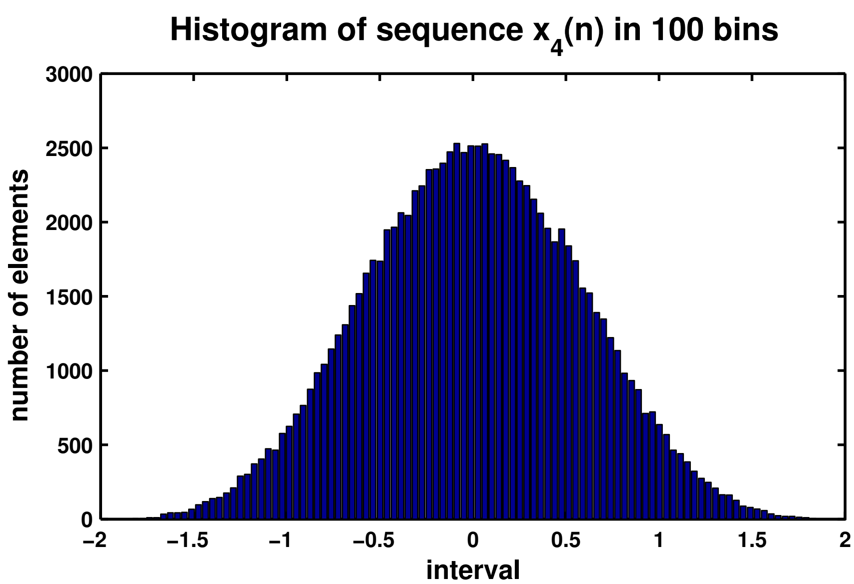

Figure 2.12: Problem P2.2.4 sequence plot

P2.3 Generate the following periodic sequences and plot their samples (using the `stem` function) over the indicated number of periods.

1. $\tilde{x}_1(n) = \{\ldots, -2, -1, \underset{\uparrow}{0}, 1, 2, \ldots\}_{\text{periodic}}$. Plot 5 periods.

```
% P0203a: x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2...} periodic. 5 periods
clc; close all;

n1 = [-12:12]; x1 = [-2,-1,0,1,2];
x1 = x1'*ones(1,5); x1 = (x1(:))';

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203a');
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-1,max(x1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS);
ntick = [n1(1):2:n1(end)]; ytick = [min(x1) - 1:max(x1) + 1];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0203a
```

The plots of $\tilde{x}_1(n)$ is shown in Figure 2.13.

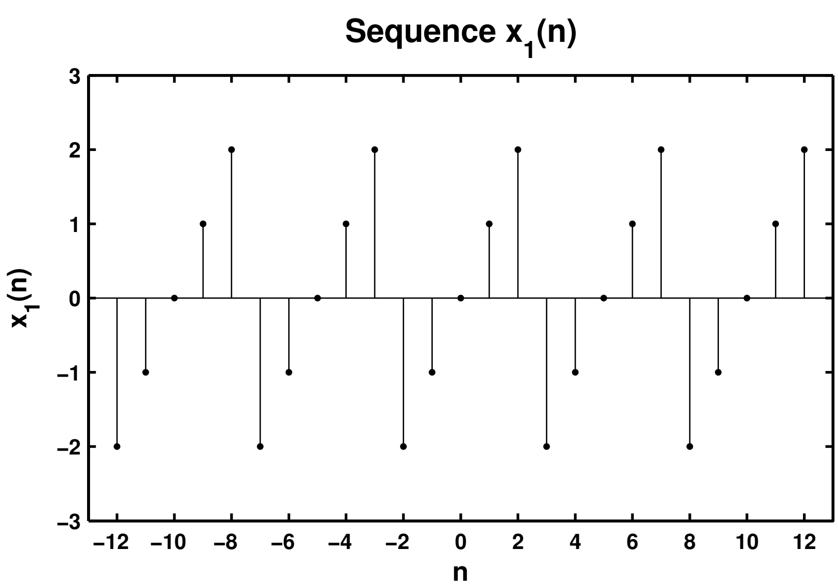

Figure 2.13: Problem P2.3.1 sequence plot

2. $\tilde{x}_2(n) = e^{0.1n}[u(n) - u(n-20)]_{\text{periodic}}$. Plot 3 periods.

```
% P0203b: x2 = e ^ {0.1n} [u(n) - u(n-20)] periodic. 3 periods
clc; close all;

n2 = [0:21]; x2 = exp(0.1*n2).*(stepseq(0,0,21)-stepseq(20,0,21));
x2 = x2'*ones(1,3); x2 = (x2(:))'; n2 = [-22:43];

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203b');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-2,max(n2)+4,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS);
ntick = [n2(1):4:n2(end)-5 n2(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../Chap2_EPSFILES/P0203b;
```

The plots of $\tilde{x}_2(n)$ is shown in Figure 2.14.

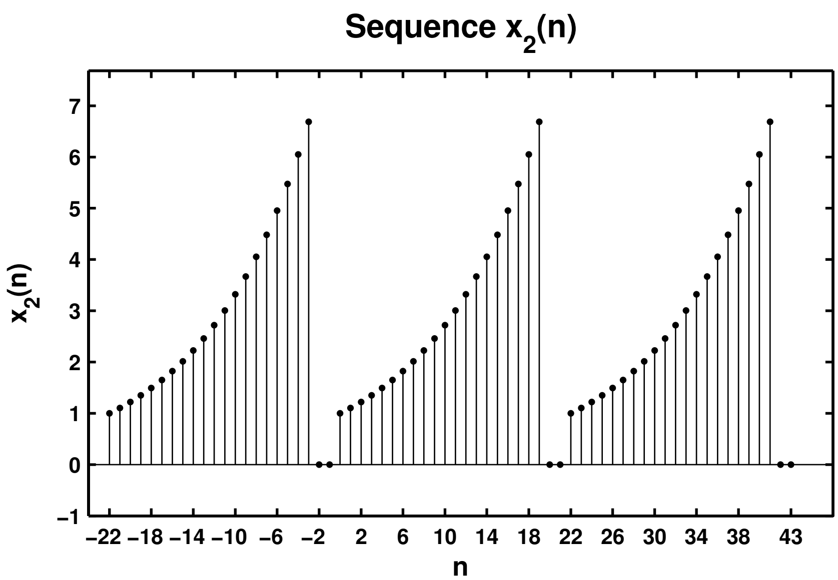

Figure 2.14: Problem P2.3.2 sequence plot

3. $\tilde{x}_3(n) = sin(0.1\pi n)[u(n) - u(n-10)]$. Plot 4 periods.

```
% P0203c: x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2...} periodic. 5 periods
clc; close all;

n3 = [0:11]; x3 = sin(0.1*pi*n3).*(stepseq(0,0,11)-stepseq(10,0,11));
x3 = x3'*ones(1,4); x3 = (x3(:))'; n3 = [-12:35];

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-0.5,max(x3)+0.5]);
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
ntick = [n3(1):4:n3(end)-3 n3(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0203c;
```

The plots of $\tilde{x}_3(n)$ is shown in Figure 2.15.

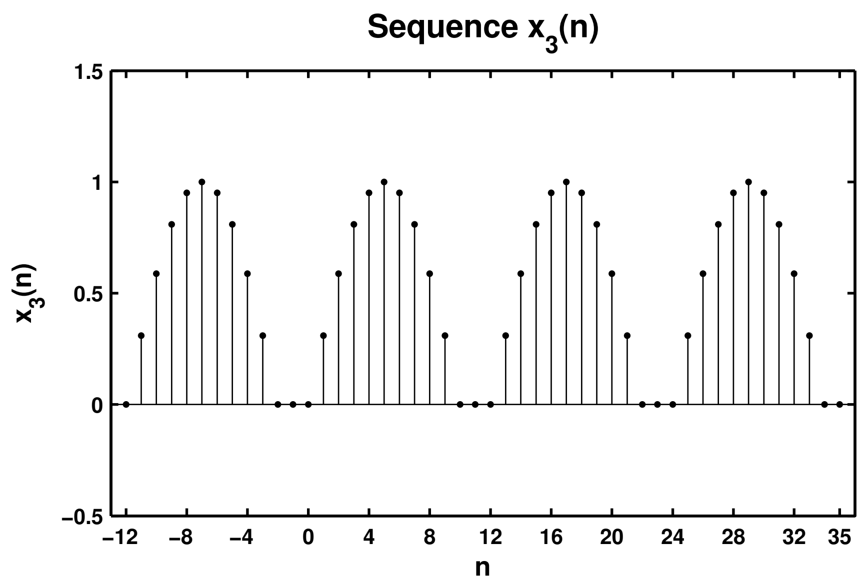

Figure 2.15: Problem P2.3.3 sequence plot

4. $\tilde{x}_4(n) = \{\ldots, \underset{\uparrow}{1}, 2, 3, \ldots\}_{\text{periodic}} + \{\ldots, \underset{\uparrow}{1}, 2, 3, 4, \ldots\}_{\text{periodic}},\ 0 \le n \le 24$. What is the period of $\tilde{x}_4(n)$?

```
% P0203d x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2...} periodic. 5 periods
clc; close all;

n4 = [0:24]; x4a = [1 2 3]; x4a = x4a'*ones(1,9); x4a = (x4a(:))';
x4b = [1 2 3 4]; x4b = x4b'*ones(1,7); x4b = (x4b(:))';
x4 = x4a(1:25) + x4b(1:25);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-1,max(n4)+1,min(x4)-1,max(x4)+1]);
xlabel('n', 'FontSize', LFS); ylabel('x_4(n)','FontSize',LFS);
title('Sequence x_4(n):Period = 12','FontSize',TFS);
ntick = [n4(1) :2:n4(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0203d;
```

The plots of $\tilde{x}_4(n)$ is shown in Figure 2.16. From the figure, the fundamental period of $\tilde{x}_4(n)$ is 12.

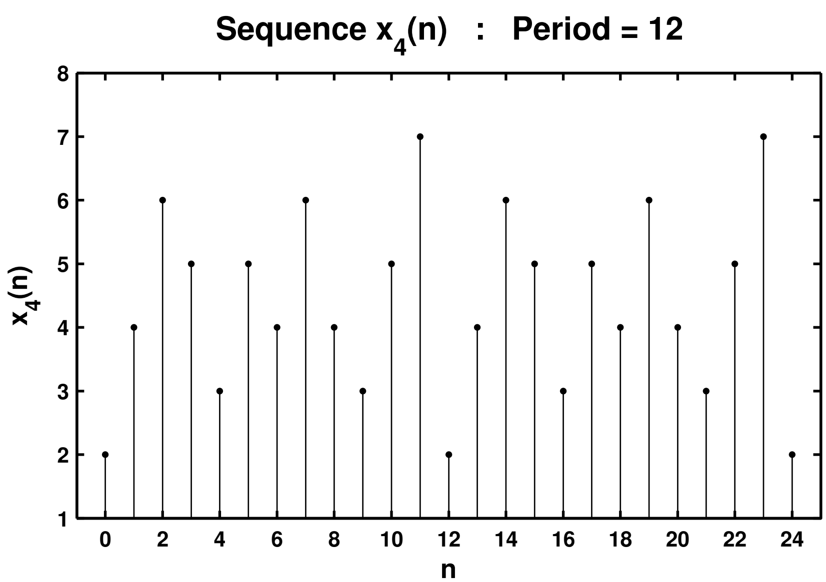

Figure 2.16: Problem P2.3.4 sequence plot

P2.4 Let $x(n) = \{2, 4, -3, \underset{\uparrow}{1}, -5, 4, 7\}$. Generate and plot the samples (use the `stem` function) of the following sequences.

1. $x_1(n) = 2x(n-3) + 3x(n+4) - x(n)$

```
% P0204a: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
% x1(n) = 2x(n - 3) + 3x(n + 4) - x(n)
clc; close all;

n = [-3:3]; x = [2,4,-3,1,-5,4,7];
[x11,n11] = sigshift(x,n,3);                         % shift by 3
[x12,n12] = sigshift(x,n,-4);                        % shift by -4
[x13,n13] = sigadd(2*x11,n11,3*x12,n12); % add two sequences
[x1,n1] = sigadd(x13,n13,-x,n);                % add two sequences

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204a');
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-3,max(x1)+1]);
xlabel('n','FontSize',LFS);
ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS); ntick = n1;
ytick = [min(x1)-3:5:max(x1)+1];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204a;
```

The plots of $x_1(n)$ is shown in Figure 2.17.

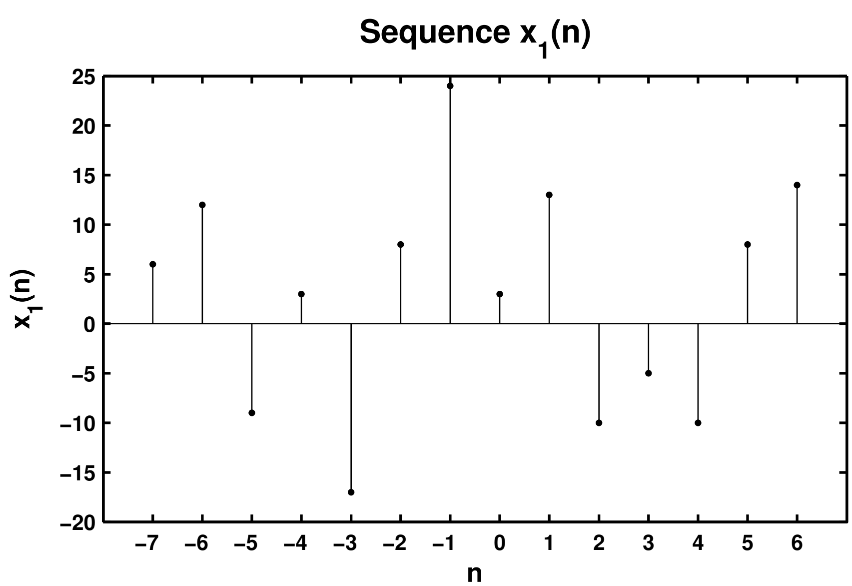

Figure 2.17: Problem P2.4.1 sequence plot

2. $x_2(n) = 4x(4+n) + 5x(n+5) + 2x(n)$

```
% P0204b: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
% x2(n) = 4x(4 + n) + 5x(n + 5) + 2x(n)
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204b');
n = [-3:3]; x = [2,4,-3,1,-5,4,7];

[x21,n21] = sigshift(x,n,-4);                    % shift by -4
[x22,n22] = sigshift(x,n,-5);                    % shift by -5
[x23,n23] = sigadd(4*x21,n21,5*x22,n22); % add two sequences
[x2,n2] = sigadd(x23,n23,2*x,n);           % add two sequences

Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-4,max(x2)+6]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS); ntick = n2;
ytick = [-25 -20:10:60 65];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204b;
```

The plots of $x_2(n)$ is shown in Figure 2.18.

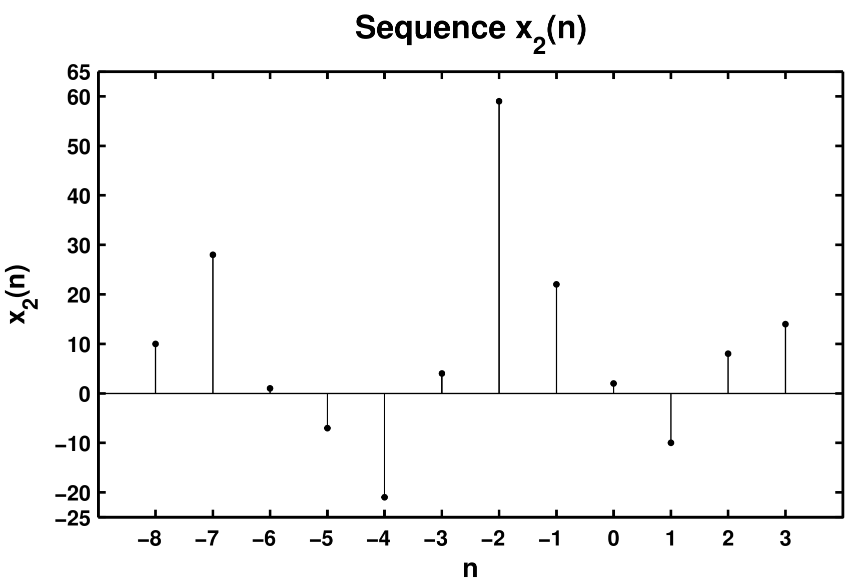

Figure 2.18: Problem P2.4.2 sequence plot

3. $x_3(n) = x(n+3)x(n-2) + x(1-n)x(n+1)$

```
% P0204c: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
%         x3(n) = x(n + 3)x(n - 2) + x(1 - n)x(n + 1)
clc; close all;

n = [-3:3]; x = [2,4,-3,1,-5,4,7];    % given sequence x(n)
[x31,n31] = sigshift(x,n,-3);          % shift sequence by -3
[x32,n32] = sigshift(x,n,2);           % shift sequence by 2
[x33,n33] = sigmult(x31,n31,x32,n32);  % multiply 2 sequences
[x34,n34] = sigfold(x,n);              % fold x(n)
[x34,n34] = sigshift(x34,n34,1);       % shift x(-n) by 1
[x35,n35] = sigshift(x,n,-1);          % shift x(n) by -1
[x36,n36] = sigmult(x34,n34,x35,n35);  % multiply 2 sequences
  [x3,n3] = sigadd(x33,n33,x36,n36);   % add 2 sequences

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-10,max(x3)+10]);
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
ntick = n3; ytick = [-30:10:60];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204c;
```

The plots of $x_3(n)$ is shown in Figure 2.19.

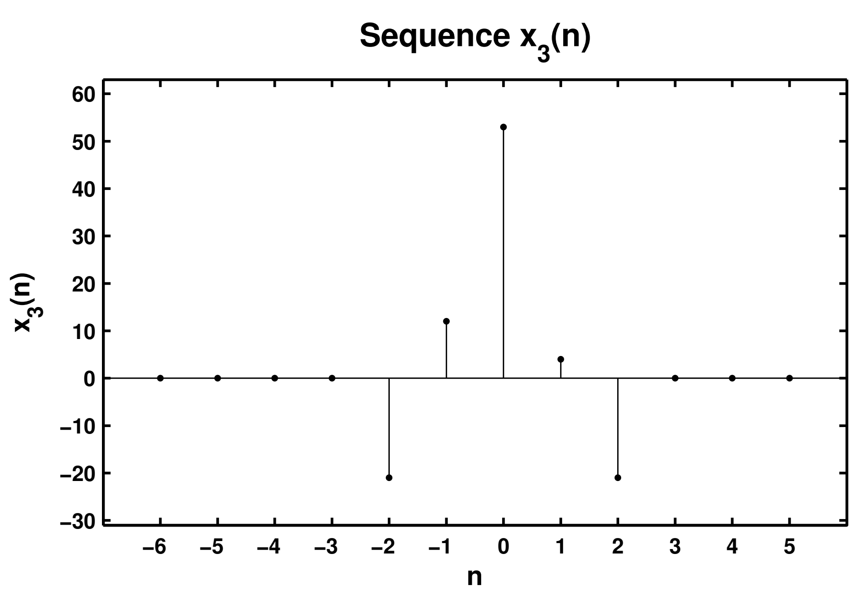

Figure 2.19: Problem P2.4.3 sequence plot

4. $x_4(n) = 2e^{0.5n}x(n) + \cos(0.1\pi n)\, x(n+2)\,, \; -10 \le n \le 10$

```
% P0204d: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
%         x4(n) = 2*e^{0.5n}*x(n)+cos(0.1*pi*n)*x(n+2), -10 <=n< =10
clc; close all;

n = [-3:3]; x = [2,4,-3,1,-5,4,7];        % given sequence x(n)
n4 = [-10:10]; x41 = 2*exp(0.5*n4); x412 = cos(0.1*pi*n4);
[x42,n42] = sigmult(x41,n4,x,n);
[x43,n43] = sigshift(x,n,-2);
[x44,n44] = sigmult(x412,n42,x43,n43);
  [x4,n4] = sigadd(x42,n42,x44,n44);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-1,max(n4)+1,min(x4)-11,max(x4)+10]);
xlabel('n','FontSize',LFS); ylabel('x_4(n)','FontSize',LFS);
title('Sequence x_4(n)','FontSize',TFS);
ntick = n4; ytick = [ -20:10:70];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204d;
```

The plot of $x_4(n)$ is shown in Figure 2.20.

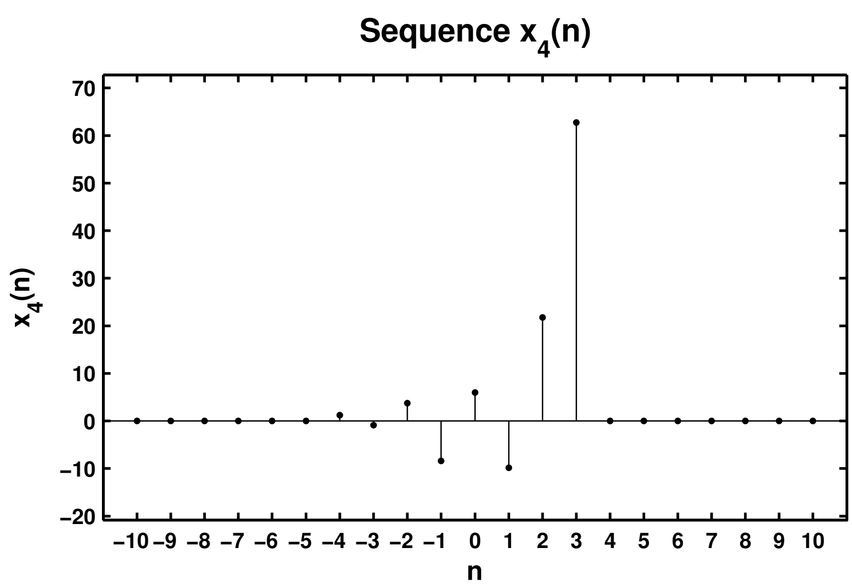

Figure 2.20: Problem P2.4.4 sequence plot

P2.5 The complex exponential sequence $e^{j\omega_0 n}$ or the sinusoidal sequence $\cos(\omega_0 n)$ are periodic if the *normalized* frequency $f_0 \triangleq \dfrac{\omega_0}{2\pi}$ is a rational number; that is, $f_0 = \dfrac{K}{N}$, where K and N are integers.

1. Analytical proof: The exponential sequence is periodic if

$$e^{j2\pi f_0(n+N)} = e^{j2\pi f_0 n} \text{ or } e^{j2\pi f_0 N} = 1 \Rightarrow f_0 N = K \text{ (an integer)}$$

which proves the result.

2. $x_1 = \exp(0.1\pi n)$, $-100 \le n \le 100$.

```
% P0205b: x1(n) = e^{0.1*j*pi*n} -100 <=n <=100
clc; close all;
n1 = [-100:100]; x1 = exp(0.1*j*pi*n1);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0205b');
subplot(2,1,1); Hs1 = stem(n1,real(x1),'filled'); set(Hs1,'markersize',2);
axis([min(n1)-5,max(n1)+5,min(real(x1))-1,max(real(x1))+1]);
xlabel('n','FontSize',LFS); ylabel('Real(x_1(n))','FontSize',LFS);
title(['Real part of sequence x_1(n) = ' ...
       'exp(0.1 \times j \times pi \times n) ' char(10) ...
       ' Period = 20, K = 1, N = 20'],'FontSize',TFS);
ntick = [n1(1):20:n1(end)]; set(gca,'XTickMode','manual','XTick',ntick);
subplot(2,1,2); Hs2 = stem(n1,imag(x1),'filled'); set(Hs2,'markersize',2);
axis([min(n1)-5,max(n1)+5,min(real(x1))-1,max(real(x1))+1]);
xlabel('n','FontSize',LFS); ylabel('Imag(x_1(n))','FontSize',LFS);
title(['Imaginary part of sequence x_1(n) = ' ...
       'exp(0.1 \times j \times pi \times n) ' char(10) ...
       ' Period = 20, K = 1, N = 20'],'FontSize',TFS);
ntick = [n1(1):20:n1(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0205b; print -deps2 ../../Latex/P0205b;
```

The plots of $x_1(n)$ is shown in Figure 2.21. Since $f_0 = 0.1/2 = 1/20$ the sequence is periodic. From the plot in Figure 2.21 we see that in one period of 20 samples $x_1(n)$ exhibits cycle. This is true whenever K and N are relatively prime.

3. $x_2 = \cos(0.1n)$, $-20 \le n \le 20$.

```
% P0205c: x2(n) = cos(0.1n), -20 <= n <= 20
clc; close all;
n2 = [-20:20]; x2 = cos(0.1*n2);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0205c');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title(['Sequence x_2(n) = cos(0.1 \times n)' char(10) ...
       'Not periodic since f_0 = 0.1 / (2 \times \pi)' ...
       ' is not a rational number'], 'FontSize',TFS);
ntick = [n2(1):4:n2(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0205c;
```

The plots of $x_1(n)$ is shown in Figure 2.22. In this case f_0 is not a rational number and hence the sequence $x_2(n)$ is not periodic. This can be clearly seen from the plot of $x_2(n)$ in Figure 2.22.

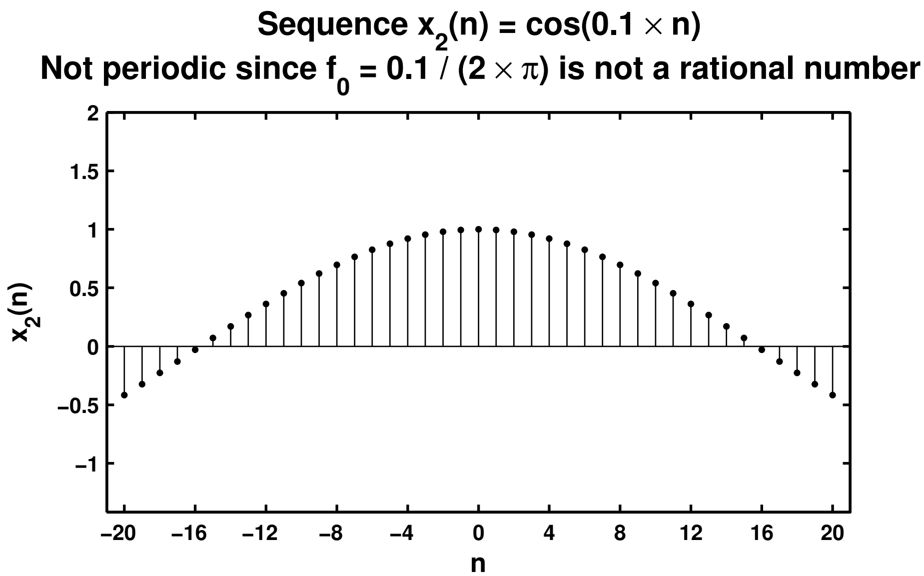

Figure 2.21: Problem P2.5.2 sequence plots

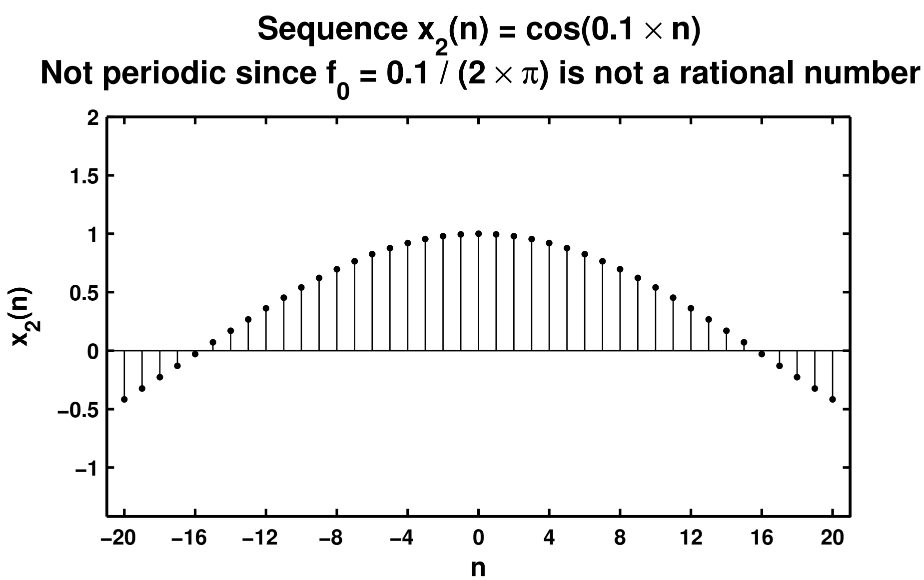

Figure 2.22: Problem P2.5.3 sequence plots

P2.6 Using the evenodd function decompose the following sequences into their even and odd components. Plot these components using the stem function.

1. $x_1(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

```
% P0206a: % Even odd decomposition of x1(n) = [0 1 2 3 4 5 6 7 8 9];
%                                         n  = 0:9;
clc; close all;

x1 = [0 1 2 3 4 5 6 7 8 9]; n1  = [0:9]; [xe1,xo1,m1] = evenodd(x1,n1);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206a');
subplot(2,1,1); Hs = stem(m1,xe1,'filled'); set(Hs,'markersize',2);
axis([min(m1)-1,max(m1)+1,min(xe1)-1,max(xe1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title('Even part of x_1(n)','FontSize',TFS);
ntick = [m1(1):m1(end)]; ytick = [-1:5];
set(gca,'XTick',ntick);set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m1,xo1,'filled'); set(Hs,'markersize',2);
axis([min(m1)-1,max(m1)+1,min(xo1)-2,max(xo1)+2]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of x_1(n)','FontSize',TFS);
ntick = [m1(1):m1(end)]; ytick = [-6:2:6];
set(gca,'XTick',ntick);set(gca,'YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0206a; print -deps2 ../../Latex/P0206a;
```

The plots of $x_1(n)$ is shown in Figure 2.23.

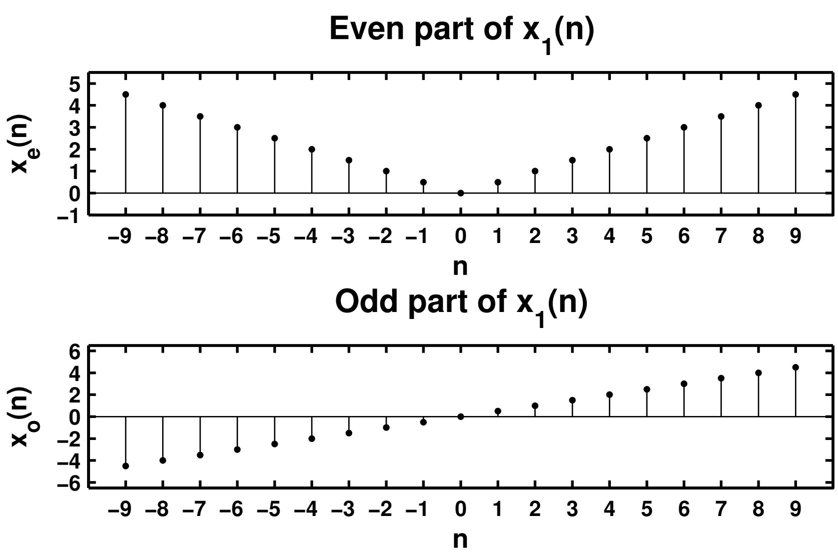

Figure 2.23: Problem P2.6.1 sequence plot

2. $x_2(n) = e^{0.1n}[u(n+5) - u(n-10)]$.

```
% P0206b: Even odd decomposition of x2(n) = e ^ {0.1n} [u(n + 5) - u(n - 10)];
clc; close all;

n2 = [-8:12]; x2 = exp(0.1*n2).*(stepseq(-5,-8,12) - stepseq(10,-8,12));
[xe2,xo2,m2] = evenodd(x2,n2);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206b');
subplot(2,1,1); Hs = stem(m2,xe2,'filled'); set(Hs,'markersize',2);
axis([min(m2)-1,max(m2)+1,min(xe2)-1,max(xe2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title('Even part of x_2(n) = exp(0.1n) [u(n + 5) - u(n - 10)]',...
      'FontSize',TFS);
ntick = [m2(1):2:m2(end)]; set(gca,'XTick',ntick);
subplot(2,1,2); Hs = stem(m2,xo2,'filled'); set(Hs,'markersize',2);
axis([min(m2)-1,max(m2)+1,min(xo2)-1,max(xo2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of x_2(n) = exp(0.1n) [u(n + 5) - u(n - 10)]',...
      'FontSize',TFS);
ntick = [m2(1) :2:m2(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206b; print -deps2 ../../Latex/P0206b;
```

The plots of $x_2(n)$ is shown in Figure 2.24.

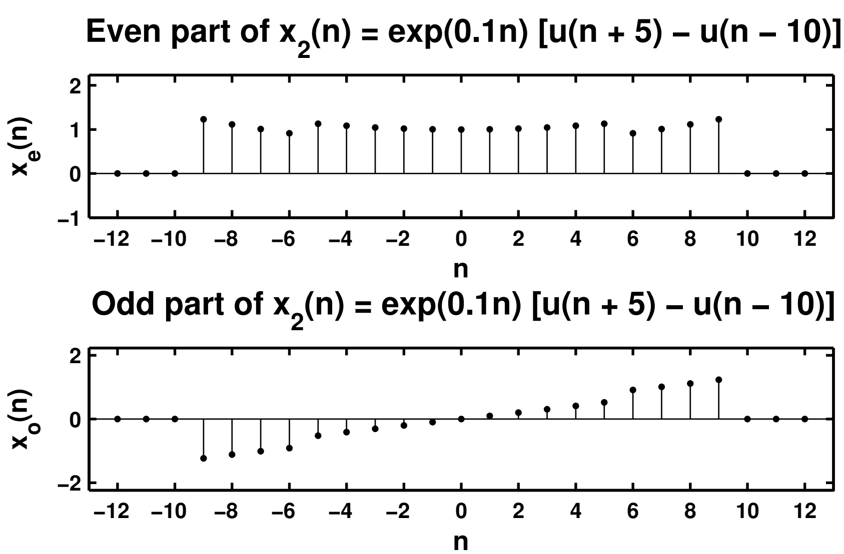

Figure 2.24: Problem P2.6.2 sequence plot

3. $x_3(n) = \cos(0.2\pi n + \pi/4)$, $-20 \le n \le 20$.

```
% P0206c: Even odd decomposition of x2(n) = cos(0.2*pi*n + pi/4);
%                                     -20 <= n <= 20;
clc; close all;

n3 = [-20:20]; x3 = cos(0.2*pi*n3 + pi/4);
[xe3,xo3,m3] = evenodd(x3,n3);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206c');
subplot(2,1,1); Hs = stem(m3,xe3,'filled'); set(Hs,'markersize',2);
axis([min(m3)-2,max(m3)+2,min(xe3)-1,max(xe3)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title('Even part of x_3(n)  =  cos(0.2 \times \pi \times n + \pi/4)',...
      'FontSize',TFS);
ntick = [m3(1):4:m3(end)]; set(gca,'XTick',ntick);
subplot(2,1,2); Hs = stem(m3,xo3,'filled'); set(Hs,'markersize',2);
axis([min(m3)-2,max(m3)+2,min(xo3)-1,max(xo3)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of x_3(n)  =  cos(0.2 \times \pi \times n + \pi/4)',...
      'FontSize',TFS);
ntick = [m3(1):4 :m3(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206c; print -deps2 ../../Latex/P0206c;
```

The plots of $x_3(n)$ is shown in Figure 2.25.

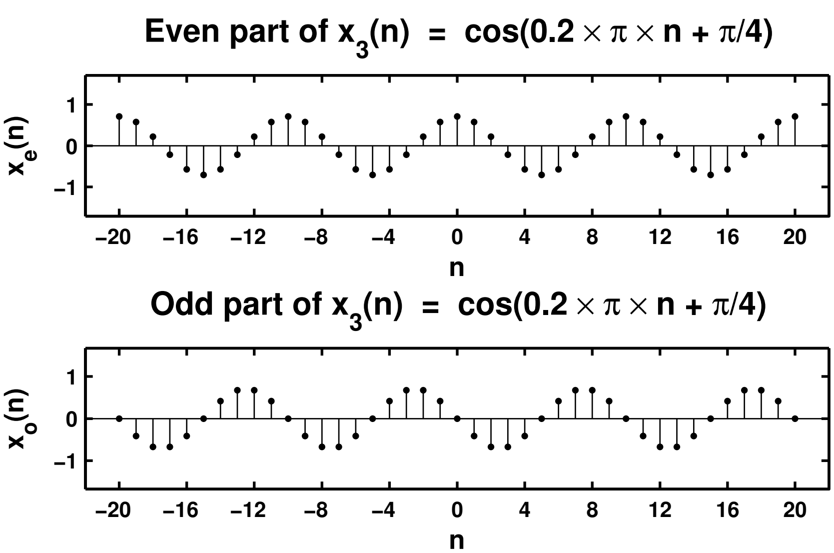

Figure 2.25: Problem P2.6.3 sequence plot

4. $x_4(n) = e^{-0.05n}\sin(0.1\pi n + \pi/3),\ 0 \le n \le 100.$

```
% P0206d: x4(n) = e ^ {-0.05*n}*sin(0.1*pi*n + pi/3), 0 <= n <= 100
clc; close all;

n4 = [0:100]; x4 = exp(-0.05*n4).*sin(0.1*pi*n4 + pi/3);
[xe4,xo4,m4] = evenodd(x4,n4);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206d');
subplot(2,1,1); Hs = stem(m4,xe4,'filled'); set(Hs,'markersize',2);
axis([min(m4)-10,max(m4)+10,min(xe4)-1,max(xe4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title(['Even part of x_4(n) = ' ...
       'exp(-0.05 \times n) \times sin(0.1 \times \pi \times n + ' ...
       '\pi/3)'],'FontSize',TFS);
ntick = [m4(1):20:m4(end)]; set(gca,'XTick',ntick);

subplot(2,1,2); Hs = stem(m4,xo4,'filled'); set(Hs,'markersize',2);
axis([min(m4)-10,max(m4)+10,min(xo4)-1,max(xo4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title(['Odd part of x_4(n) = ' ...
       'exp(-0.05 \times n) \times sin(0.1 \times \pi \times n + ' ...
       '\pi/3)'],'FontSize',TFS);
ntick = [m4(1):20 :m4(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206d; print -deps2 ../../Latex/P0206d;
```

The plots of $x_1(n)$ are shown in Figure 2.26.

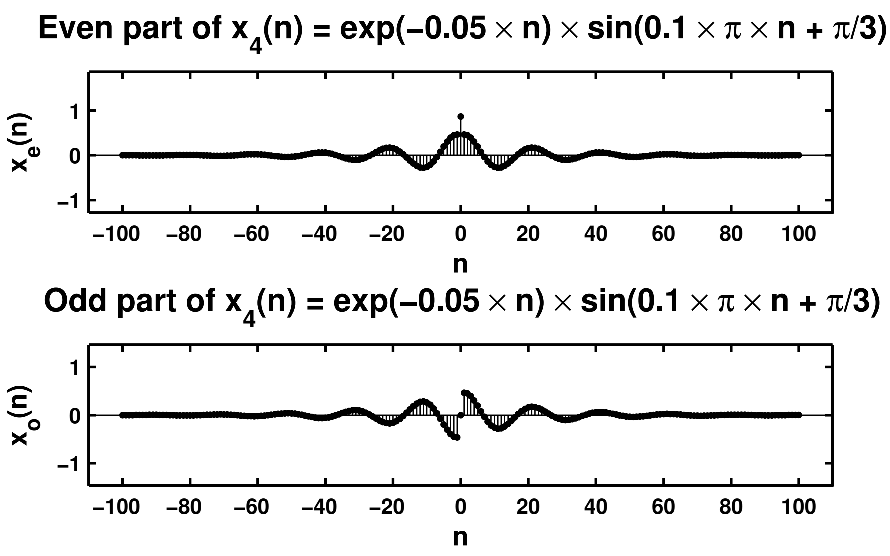

Figure 2.26: Problem P2.6.1 sequence plot

P2.7 A complex-valued sequence $x_e(n)$ is called *conjugate-symmetric* if $x_e(n) = x_e^*(-n)$ and a complex-valued sequence $x_o(n)$ is called *conjugate-antisymmetric* if $x_o(n) = -x_o^*(-n)$. Then any arbitrary complex-valued sequence $x(n)$ can be decomposed into $x(n) = x_e(n) + x_o(n)$ where $x_e(n)$ and $x_o(n)$ are given by

$$x_e(n) = \tfrac{1}{2}\left[x(n) + x^*(-n)\right] \qquad \text{and} \qquad x_o(n) = \tfrac{1}{2}\left[x(n) - x^*(-n)\right] \tag{2.1}$$

respectively.

1. Modify the evenodd function discussed in the text so that it accepts an arbitrary sequence and decomposes it into its conjugate-symmetric and conjugate-antisymmetric components by implementing (2.1).

```
function [xe , xo , m] = evenodd_c(x , n)
% Complex-valued signal decomposition into even and odd parts (version-2)
% -----------------------------------------------------------------------
%[xe , xo , m] = evenodd_c(x , n);
%
[xc , nc] = sigfold(conj(x) , n);
[xe , m] = sigadd(0.5 * x , n , 0.5 * xc , nc);
[xo , m] = sigadd(0.5 * x , n , -0.5 * xc , nc);
```

2. $x(n) = 10\exp([-0.1 + \jmath 0.2\pi]n), \quad 0 \le n \le 10$

```
% P0207b: Decomposition of x(n) = 10*e ^ {(-0.1 + j*0.2*pi)*n},
%                                                0 < = n <= 10
% into its conjugate symmetric and conjugate antisymmetric parts.
clc; close all;

n = [0:10]; x = 10*exp((-0.1+j*0.2*pi)*n); [xe,xo,neo] = evenodd(x,n);
Re_xe = real(xe); Im_xe = imag(xe); Re_xo = real(xo); Im_xo = imag(xo);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0207b');
subplot(2,2,1); Hs = stem(neo,Re_xe); set(Hs,'markersize',2);
ylabel('Re[x_e(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-5,12]);
ytick = [-5:5:15]; set(gca,'YTick',ytick);
title(['Real part of' char(10) 'even sequence x_e(n)'],'FontSize',TFS);

subplot(2,2,3); Hs = stem(neo,Im_xe); set(Hs,'markersize',2);
ylabel('Im[x_e(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-5,5]);
ytick = [-5:1:5]; set(gca,'YTick',ytick);
title(['Imaginary part of' char(10) 'even sequence x_e(n)'],'FontSize',TFS);

subplot(2,2,2); Hs = stem(neo,Re_xo); set(Hs,'markersize',2);
ylabel('Re[x_o(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-5,+5]);
ytick = [-5:1:5]; set(gca,'YTick',ytick);
title(['Real part of' char(10) 'odd sequence x_o(n)'],'FontSize',TFS);

subplot(2,2,4); Hs = stem(neo,Im_xo); set(Hs,'markersize',2);
ylabel('Im[x_o(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-5,5]);
ytick = [-5:1:5]; set(gca,'YTick',ytick);
title(['Imaginary part of' char(10) 'odd sequence x_o(n)'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0207b;%print -deps2 ../../Latex/P0207b;
```

The plots of $x(n)$ are shown in Figure 2.27.

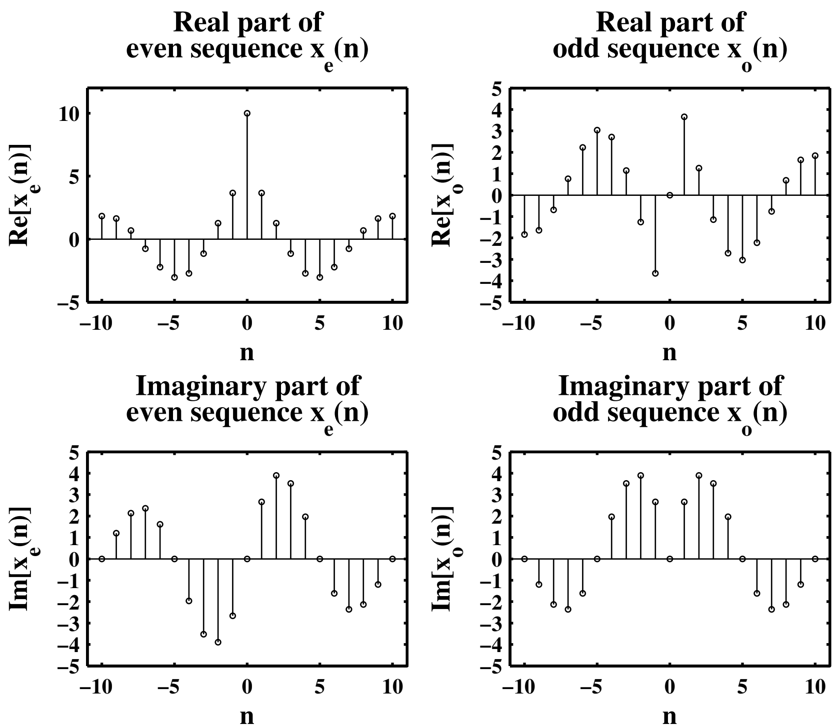

Figure 2.27: Problem P2.7.2 sequence plot

P2.8 The operation of *signal dilation* (or *decimation* or *down-sampling*) is defined by $y(n) = x(nM)$ in which the sequence $x(n)$ is down-sampled by an integer factor M.

1. MATLAB function:

```
function [y,m] = dnsample(x,n,M)
% [y,m] = dnsample(x,n,M)
% Downsample sequence x(n) by a factor M to obtain y(m)
mb = ceil(n(1)/M)*M; me = floor(n(end)/M)*M;
nb = find(n==mb); ne = find(n==me);
y = x(nb:M:ne); m = fix((mb:M:me)/M);
```

2. $x_1(n) = \sin(0.125\pi n), \quad -50 \le n \le 50$. Decimation by a factor of 4.

```
% P0208b: x1(n) = sin(0.125*pi*n),-50 <= n <= 50
%         Decimate x(n) by a factor of 4 to obtain y(n)
clc; close all;
n1 = [-50:50]; x1 = sin(0.125*pi*n1); [y1,m1] = dnsample(x1,n1,4);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0208b');
subplot(2,1,1); Hs = stem(n1,x1); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title('Original sequence x_1(n)','FontSize',TFS);
axis([min(n1)-5,max(n1)+5,min(x1)-0.5,max(x1)+0.5]);
ytick = [-1.5:0.5:1.5]; ntick = [n1(1):10:n1(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m1,y1); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n) = x(4n)','FontSize',LFS);
title('y_1(n)  =  Original sequence x_1(n) decimated by a factor of 4',...
      'FontSize',TFS);
axis([min(m1)-2,max(m1)+2,min(y1)-0.5,max(y1)+0.5]);
ytick = [-1.5:0.5:1.5]; ntick = [m1(1):2:m1(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0208b;
```

The plots of $x_1(n)$ and $y_1(n)$ are shown in Figure 2.28. Observe that the original signal $x_1(n)$ can be recovered.

3. $x(n) = \sin(0.5\pi n), \quad -50 \le n \le 50$. Decimation by a factor of 4.

```
% P0208c: x2(n) = sin(0.5*pi*n),-50 <= n <= 50
%         Decimate x2(n) by a factor of 4 to obtain y2(n)
clc; close all;
n2 = [-50:50]; x2 = sin(0.5*pi*n2); [y2,m2] = dnsample(x2,n2,4);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0208c');
subplot(2,1,1); Hs = stem(n2,x2); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
axis([min(n2)-5,max(n2)+5,min(x2)-0.5,max(x2)+0.5]);
title('Original sequence x_2(n)','FontSize',TFS);
ytick = [-1.5:0.5:1.5]; ntick = [n2(1):10:n2(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m2,y2); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n) = x(4n)','FontSize',LFS);
axis([min(m2)-1,max(m2)+1,min(y2)-1,max(y2)+1]);
title('y_2(n) = Original sequence x_2(n) decimated by a factor of 4',...
      'FontSize',TFS);
ntick = [m2(1):2:m2(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0208c; print -deps2 ../../Latex/P0208c;
```