Homework 1 - Solutions

1 Problem 1-7 (Dally and Poulton)

Scaling of Wire Delay: Suppose a system has a clock cycle of ten gate delays. What clock frequency does this imply in the years 1990, 2000, and 2010? In these same years, how many clocks, or what fraction of a clock, does it take t drive a wire from one corner of the chip to the opposite corner? Assume this wire only runs horizontally and vertically so that its length is twice the linear dimension of the chip. For now, assume that the delay of a wire is its RC time constant. We will see how to do better in Chapter8.

Solutions:

From Table 1-3, we get
Gate Delay = $0.23 * 0.87^{(year-1998)}$ ns
Clock cycle = 10 gate delay.
Clock frequency = 1/ clock cycle.

Chip edge, y = $19 * 1.06^{(year-1998)}$ mm
Length of wire going from one corner of chip to opposite corner is 2y.

From Section 1.3.3.2,
RC of 5mm wire in 1998 = 200ps
RC proportional to L^2
RC of 1mm wire in 1998 = 200 / 5^2 = 8ps

From Table 1-5, RC per unit length scales by $1/x^2 = 1.32$ every year.

RC per 1mm = $8 * 1.32^{(year-1998)}$ ps
Wire delay = RC
Wire delay of a wire of length 2y = RC per 1mm * $(2y)^2$

	1990	2000	2010
Gate Delay	0.7ns	0.17ns	0.043ns
Clock cycle	7ns	1.7ns	0.43ns
Clock Frequency	142.9MHz	588.2MHz	2.3GHz
RC per 1mm	0.86ps	13.9ps	223.9ps
2 X Chip edge	23.8mm	42.6mm	76.4mm
Wire Delay	0.487ns	25.3ns	1306.9ns
Wire Delay in clock cycle	0.07 clocks	14.8 clocks	3039.3 clocks

2 Problem 3-1 (Dally and Poulton)

Example Transmission Lines: Calculate the electrical properties (R_{DC}, C, L, and Z_o) of the following common transmission media: (a) a twisted pair made of "wire-wrap" wire; (b) RG-58 coaxial cable (compare your results with the published values); (c) a wire-wrap wire glued to the surface of a PC board, $\varepsilon_r = 4.5$ and 6 mil dielectric thickness to first plane layer (for this one, you can either ignore that the line is inhomogeneous or find the appropriate empirical formulas in one of the references). For this exercise, go to the manufacturer's data on common wire and cable to find the various physical and electrical parameters.

(a) twisted pair made of "wire-wrap" wire:

Using Coopertools wire-wrap wire with specifications (see http://www.coopertools.com/)

Part No.	AWG	Conductor	Insulator	Outer Diameter (mm)
990222	30	Copper	Teflon	0.5

$$R_{DC} = \frac{\rho l}{A}$$

$$L = \frac{\varepsilon \mu}{C}$$

$$Z_o = \sqrt{\frac{L}{C}}$$

From Table 3-1 on page 82, we see that copper has a resistivity, ρ, of $1.7 \times 10^{-8}\Omega - m$. From Table 2-2 on page 50, we see that 30-Gauge wire has a diameter of 0.2548 mm (2.548×10^{-4} m). Thus, the radius of the wire is $0.5 * 2.548 \times 10^{-4}$m $= 1.274 \times 10^{-4}$m. We calculate the DC resistance per unit length for one wire as:

$$R_{DC} = \frac{\rho}{A} = \frac{1.7 \times 10^{-8}\Omega - m}{\pi * (1.274 \times 10^{-4} m)^2} = 0.333\frac{\Omega}{m}$$

The total DC Resistance of the twisted pair is twice the DC resistance of one wire. Thus, the DC resistance of the twisted pair is:

$$\boxed{R_{DC} = 0.666\frac{\Omega}{m}}$$

Equation 3-5 on page 83 gives us the capacitance equation for 2 wires:

$$C_c = \frac{\pi \varepsilon}{ln(\frac{s}{r})}$$

We find the relative permittivity, ε_r, of the insulator, Teflon, in Table 3-2 on page 84: $\varepsilon_r = 2$. Thus, we have:

$$C_c = \frac{\pi * 2 * 8.854x10^{-12}\frac{F}{m}}{ln(\frac{0.5mm}{0.1274mm})} = 4.07 \times 10^{-11}\frac{F}{m} = \boxed{40.7\frac{pF}{m}}$$

Equation 3-8 on page 84 gives us the equation:

$$C * L = \varepsilon \mu$$

Thus, we have:

$$L = \frac{\varepsilon \mu}{C} = \frac{2 * 8.854x10^{-12}\frac{F}{m} * 4\pi \times 10^{-7}\frac{H}{m}}{4.07 \times 10^{-11}\frac{F}{m}} = 5.47 \times 10^{-7}\frac{H}{m} = \boxed{0.547\frac{\mu H}{m}}$$

Equation 3-31 on page 93 gives us the equation

$$Z_o = \sqrt{\frac{L}{C}}$$

Thus:

$$Z_o = \sqrt{\frac{5.47 \times 10^{-7}\frac{H}{m}}{4.07 \times 10^{-11}}\frac{F}{m}} = \boxed{116\Omega}$$

(b) RG-58 coaxial cable

Using Belden RG-58 coaxial cable with specifications (see: http://www.belden.com/)

Part No.	AWG	Conductor	Insulator	Outer Diameter (in)	Polyethylene ε
9310	20	Bare Copper	Polyethylene	0.114	2.3

From Table 2-2 on page 50, 20-Gauge wire has a diameter of 0.8118 mm (8.118×10^{-4} m). Thus, the radius of the wire is $0.5 * 8.118 \times 10^{-4}$m $= 4.059 \times 10^{-4}$ m. We calculate the DC resistance per unit length as:

$$R_{DC} = \frac{\rho}{A} = \frac{1.7 \times 10^{-8}\Omega - m}{\pi * (4.059 \times 10^{-4} m)^2} = \boxed{0.0328 \frac{\Omega}{m}}$$

Note: we are ignoring the resistance of the return path, the outer conductor, since its resistance is negligible ($4 \times 10^{-3} \frac{\Omega}{m}$).

Equation 3-4 on page 83 gives us the capacitance equation for 2 wires:

$$C_b = \frac{2\pi\varepsilon}{ln(\frac{r_2}{r_1})}$$

$r_2 = 0.5 * 0.114$ in $= 0.057$ in $= 1.45$ mm

Thus, we have:

$$C_b = \frac{2\pi * 2.3 * 8.854x10^{-12}\frac{F}{m}}{ln(\frac{1.45mm}{0.406mm})} = 1.01 \times 10^{-10}\frac{F}{m} = \boxed{101\frac{pF}{m}}$$

Equation 3-8 on page 84 gives us the equation:

$$C * L = \varepsilon\mu$$

Thus, we have:

$$L = \frac{\varepsilon\mu}{C} = \frac{2.3 * 8.854x10^{-12}\frac{F}{m} * 4\pi \times 10^{-7}\frac{H}{m}}{1.01 \times 10^{-10}\frac{F}{m}} = 2.54 \times 10^{-7}\frac{H}{m} = \boxed{0.254\frac{\mu H}{m}}$$

Equation 3-31 on page 93 gives us the equation

$$Z_o = \sqrt{\frac{L}{C}}$$

Thus:

$$Z_o = \sqrt{\frac{2.54 \times 10^{-7}\frac{H}{m}}{1.01 \times 10^{-10}}\frac{F}{m}} = \boxed{50.3\Omega}$$

Comparing our values with the values published by Belden, we have

Source	R_DC	Capacitance	Inductance	Impedance
Belden	$9.4 \frac{\Omega}{1000ft} = 0.0308 \frac{\Omega}{m}$	$30.8 \frac{pF}{ft} = 101 \frac{pF}{m}$	$0.09 \frac{\mu H}{ft} = 0.295 \frac{\mu H}{m}$	50 Ω
Calculated	$0.0328 \frac{\Omega}{m}$	$101 \frac{pF}{m}$	$0.254\frac{\mu H}{m}$	50.3 Ω

Therefore, our calculated values are different from the published values by, 6.5%, 0%, -13.9%, and 0.6% respectively.

(c) a wire-wrap wire glued to the surface of a PC board, $\varepsilon_r = 4.5$ and 6 mil dielectric thickness to first plane layer. (note: 1000 mil = 1 inch)

We will ignore that the line is inhomogeneous.

From (a) we know that the DC resistance of the wire is:

$$R_{DC} = \boxed{0.333\frac{\Omega}{m}}$$

Note: We are ignoring the resistance of the return path, the ground plane, since its resistance is negligible.

Equation 3-6 on page 83 gives us the capacitance equation for a single wire over a ground plane:

$$C_d = \frac{2\pi\varepsilon}{ln(\frac{2s}{r})}$$

r = 0.1274 mm s = 0.25mm (half the diameter of the entire wire) + 0.006 in. = (0.25 + 0.1524)mm = 0.402 mm

The electric field lines go through Teflon, the dielectric of the board, and air. We ignore the inhomogeneity of the dielectric, and take the relative dielectric constant to be that of Teflon.

Thus, ignoring the inhomogeneity of the insulator, we have:

$$C_d = \frac{2\pi * 2 * 8.854x10^{-12}\frac{F}{m}}{ln(\frac{2*0.402mm}{0.1274mm})} = 6.04 \times 10^{-11}\frac{F}{m} = \boxed{60.4\frac{pF}{m}}$$

Solving for the inductance, we get:

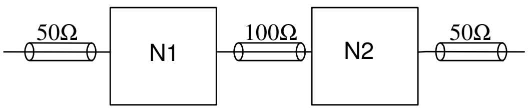
$$L = \frac{\varepsilon\mu}{C} = \frac{2 * 8.854x10^{-12}\frac{F}{m} * 4\pi \times 10^{-7}\frac{H}{m}}{6.04 \times 10^{-11}\frac{F}{m}} = 3.68 \times 10^{-7}\frac{H}{m} = \boxed{0.368\frac{\mu H}{m}}$$

Solving for impedance, we get:

$$Z_o = \sqrt{\frac{3.68 \times 10^{-7}\frac{H}{m}}{6.04 \times 10^{-11}}\frac{F}{m}} = \boxed{78.1\Omega}$$

3 Problem 3-6 (Dally and Poulton)

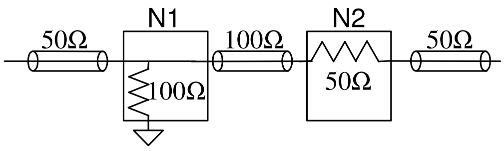
Resistive Matching Networks: One can propagate a signal between transmission lines of differing impedance without reflections by inserting a matching network between the two lines. Consider the situation in Figure 3-56 where a signal is transmitted first over a 50-Ω line, then a 100-Ω line, then back to a 50-Ω line. (a) Using only resistors, design the networks, N1 and N2, so that there are no reflections from a wave traveling from left to right. (b) Now modify your design so that it works for a wave traveling in either direction. (c) How much signal level is lost passing through the two networks? How much signal energy?



(a) Looking into N1, we want to match the impedance of the 50-Ω transmission line. Thus we want the N1 network in parallel with the 100-Ω transmission line that follows it to equal 50 Ωs.

Thus, we put in a 100-Ω resistor connected to ground for N1. The 100-Ω transistor in parallel with the 100-Ω transmission then look like 50 Ωs to the preceding transmission line.

Similarly, we want N2 and the proceeding 50-Ω transmission line to look like 100 Ωs to match the preceeding 100-Ω transmission line. Thus, for N2 we put in a 50-Ω series resistor.



(b)

Now, we solve for no reflections in the forward and reverse directions:

For the first network, we need N1 and the 100-Ω transmission line to look like 50 Ωs (as before). But now we also require that N1 and the 50-Ω transmission line (looking in the reverse direction) look like 100 Ωs.

If we put a resistor connected to ground, R_1, followed by a series resistance, R_2, we can solve for their values given the above constraints, namely:

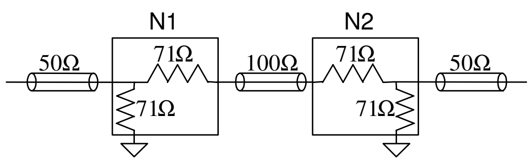
$$(R_1 + 100\Omega)//R_2 = \frac{1}{\frac{1}{R_1+100\Omega} + \frac{1}{R_2}} = 50\Omega$$

$$R_1 + (R_2//50\Omega) = 100\Omega; thus, \frac{(R_1 + 100\Omega)R_2}{R_1 + R_2 + 100\Omega} = 50\Omega$$

Solving these two equations simultaneously we get:

$$\boxed{R_1 = R_2 = \sqrt{5000} = 70.7\Omega}$$

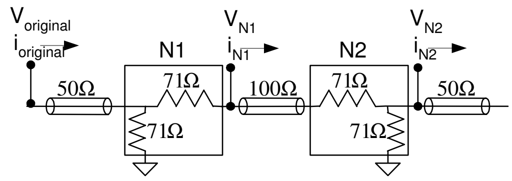
And since N_1 and N_2 are symmetric, we have the symmetric circuit for N2.



(c)

We will solve for the signal levels in the forward direction. For the current, the current continuing onto the 100 Ω transmission line is:

$$i_{N1} = \frac{70.7\Omega}{70.7\Omega + 70.7\Omega + 100\Omega} * i_{original} = 0.292 * i_{original}$$



The current then continuing onto the final 50-Ω transmission line is:

$$i_{N2} = \frac{70.7\Omega}{70.7\Omega + 50\Omega} * i_{N1} = 0.586 * i_{N1} = 0.586 * (0.292 * i_{original}) = 0.172 i_{original}$$

For the voltage, we have V_{N1}, the voltage at the beginning of the 100-Ω transmission line as:

$$V_{N1} = \frac{100\Omega}{100\Omega + 70.7\Omega} * V_{original} = 0.586 * V_{original}$$

The voltage at the beginning of the final 50-Ω transmission line is then:

$$V_{N2} = \frac{(50\Omega//70.7\Omega)}{70.7\Omega + (50\Omega//70.7\Omega)} * V_{N1} = \frac{29.3\Omega}{70.7\Omega + 29.3\Omega} * V_{N1} = 0.292 * V_{N1} = 0.172 * V_{original}$$

Thus, only **17.2%** of our original signal is left after passing through the two networks. Thus, there is an **82.8%** signal loss. We note that, since the network is symmetric, the voltage and current loss is the same.
It follows that the signal energy left after passing through the two networks is:

$$P = i * v = (0.177 i_{original}) * (0.177 V_{original}) = 0.03 * i_{original} V_{original} = 0.03 P_{original}$$

Thus, we lose **97.1%** of our signal energy passing through the two networks.
Since the network is symmetric, the signal level and signal losses will be the same in either direction.

$$\boxed{SignalLevelLoss = 82.8\%\ EnergyLoss = 97.1\%}$$