Chapter 11

11.1

(a) $\dfrac{30}{1.6} = 18.76\ \text{kg}$

(b) $W = mg = 18.75(9.8) = 183.75\ \text{N}$

11.2

$W_{S1} = \rho g V = 7850(9.81)\pi(0.06^2)(0.120) = 104.51\ N$

11.3

(a) $100\ \text{kN/m}^2 = \dfrac{100 \times 10^3\ \text{N}}{\text{m}^2} \times \dfrac{0.2248\ \text{lb}}{1.0\ \text{N}} \times \dfrac{1.0\ \text{m}^2}{1550\ \text{in}^2} = 14.50\ \text{lb/in}^2$

(b) $30\ \text{m/s} = \dfrac{30\ \cancel{\text{m}}}{\cancel{\text{s}}} \times \dfrac{\text{km}}{1000\ \cancel{\text{m}}} \times \dfrac{36008}{1\ \text{h}} = 1080.24\ \text{km/h}$

(c) $800\ \text{slugs} = 800\ \text{slugs} \times \dfrac{14.593\ \text{kg}}{1.0\ \text{slug}} = 11.67 \times 10^3\ \text{kg} = 11.67\ \text{Mg}$

(d) $\dfrac{20\ \text{lb}}{\text{ft}^2} \times \dfrac{4.448\ \text{N}}{1.0\ \text{lb}} \times \dfrac{1.0\ \text{ft}^2}{0.092\ 903\ 04\ \text{m}^2} = 958\ \text{N/m}^2$

11.4

Second law

$$F = ma$$

Law of gravitation

$$F = \frac{G m_A m_B}{R^2}$$

$$ma = \frac{G m_A m_B}{R^2}$$

$$[\cancel{M}]\left[\frac{L}{T^2}\right] = \frac{[G][\cancel{M}][M]}{[L^2]}$$

$$[G] = \left[\frac{L^3}{MT^2}\right]$$

$$= \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$$

11.5

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$$

Since the dimensions each term must be the same, we have

$$[KE] = [M]\left[\frac{L^2}{T^2}\right] = [M]\left[k^2\right]\left[\frac{1}{T^2}\right]$$

Therefore,

$$[k] = [L]$$

In the SI system

$$[KE] = [M]\left[\frac{L^2}{T^2}\right] = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \blacktriangleleft \qquad [k] = \text{m} \blacktriangleleft$$

11.6

$$[g]\,[k]\,[x]\left[\frac{1}{W}\right] = \left[\frac{L}{T^2}\right]\left[\frac{F}{L}\right][L]\left[\frac{1}{F}\right] = \left[\frac{L}{T^2}\right] = [a] \quad \text{Q.E.D.}$$

11.7

$$[F] = [k]\,[x] \qquad \left[\frac{ML}{T^2}\right] = [k][L] \qquad [k] = \left[\frac{M}{T^2}\right] \blacktriangleleft$$

11.8

(a) $\left[mv^2\right] = \left[\frac{FT^2}{L}\right]\left[\frac{L^2}{T^2}\right] = [FL] \blacktriangleleft$

(b) $[mv] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T}\right] = [FT] \blacktriangleleft$

(c) $[ma] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T^2}\right] = [F] \blacktriangleleft$

11.9

Rewrite the equation as $y = 1.0\,x^2$

$$[y] = [1.0]\left[x^2\right] \qquad [L] = [1.0]\left[L^2\right] \qquad [1.0] = \left[\frac{1}{L}\right]$$

$y = x^2$ can be dimensionally correct only if the units of the implied constant 1.0 are mm^{-1}.

11.10

(a) $[I] = [mR^2] = \left[\frac{FT^2}{L}\right][L^2] = [FLT^2]$ ◄

(b) $[I] = [mR^2] = [ML^2]$ ◄

11.11

(a) $[v^3] = [A][x^2] + [B][v][t^2] \qquad \left[\frac{L^3}{T^3}\right] = [A][L^2] + [B]\left[\frac{L}{T}\right][T^2]$

$$[A] = \left[\frac{L}{T^3}\right] \blacktriangleleft \qquad [B] = \left[\frac{L^2}{T^4}\right] \blacktriangleleft$$

(b) $[x^2] = [A][t^2]\left[e^{[B][t^2]}\right] \qquad [L^2] = [A][T^2][1] \qquad [B][T^2] = [1]$

$$[A] = \left[\frac{L^2}{T^2}\right] \blacktriangleleft \qquad [B] = \left[\frac{1}{T^2}\right] \blacktriangleleft$$

11.12

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = P_0 \sin \omega t$$

$$[m]\left[\frac{d^2x}{dt^2}\right] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T^2}\right] = [F]$$

Therefore, the dimension of each term in the expression is $[F]$.

$$+[c]\left[\frac{dx}{dt}\right] = [c]\left[\frac{L}{T}\right] = [F] \qquad [c] = \left[\frac{FT}{L}\right] \blacktriangleleft$$

$$[k][x] = [k][L] = [F] \qquad [k] = \left[\frac{F}{L}\right] \blacktriangleleft$$

$$[P_0][\sin \omega t] = [P_0][1] = [F] \qquad [P_0] = [F] \blacktriangleleft$$

$$[\omega][t] = [\omega][T] = [1] \qquad [\omega] = \left[\frac{1}{T}\right] \blacktriangleleft$$

11.13

$$F = G\frac{m_A m_B}{R^2} \qquad G = \frac{FR^2}{m_A m_b} \qquad [G] = \frac{[F][L^2]}{[M^2]}$$

(a) $[G] = \frac{[F][L^2]}{[FT^2/L]^2} = \left[\frac{L^4}{FT^4}\right]$ ◄

(b) $[G] = \frac{[ML/T^2][L^2]}{[M^2]} = \left[\frac{L^3}{MT^2}\right]$ ◄

11.14

(a) $[E] = [m]\left[c^2\right] = \left[\frac{FT^2}{L}\right]\left[\frac{L^2}{T^2}\right] = [FL]$ ◄

(b) $[E] = [m]\left[c^2\right] = [M]\left[\frac{L^2}{T^2}\right] = \left[\frac{ML^2}{T^2}\right]$ ◄

11.15

$$F = G\frac{m^2}{R^2} = 6.67\times10^{-11}\frac{10^2}{0.5^2} = 2.668\times10^{-8}\text{ N}$$
$$W = mg = 10(9.81) = 98.1\text{ N}$$
$$\frac{F}{W}\times100\% = \frac{2.668\times10^{-8}}{98.1}100\% = 2.72\times10^{-8}\ \% \quad ◄$$

11.16

$$F = G\frac{m^2}{R^2} = \frac{6.67\times10^{-11}\times(3)^2}{(1)^2} = 6\times10^{10}\text{ N}$$

11.17

$$m = \frac{WR^2}{GM_e} = \frac{2000(6378+1800)^2\times10^6}{(6.67\times10^{-11})(5.9742\times10^{24})} = 336\text{ kg} \quad ◄$$

11.18

$$g_m = \frac{GM_m}{R_m^2} \qquad g_e = \frac{GM_e}{R_e^2}$$
$$\frac{g_m}{g_e} = \frac{M_m}{M_e}\left(\frac{R_e}{R_m}\right)^2 = \frac{0.07348}{5.9742}\left(\frac{6378}{1737}\right)^2 = 0.1658 \approx \frac{1}{6}\text{ Q.E.D}$$

11.19

$$M_e = 5.9742\times10^{24}\text{ kg}$$
$$R_e = 6378\times10^3\text{ m}$$
$$W = G\frac{M_e m}{(2R_e)^2} = \frac{6.67\times10^{-11}\times(5.9742\times10^{24})\times\frac{150}{9.8}}{(5378\times10^3)^2} = 37.5\text{ N}$$

11.20

$$F = G\frac{M_s m}{R^2} = 6.67\times10^{-11}\frac{(1.9891\times10^{30})(1.0)}{(149.6\times10^9)^2} = 0.00593\text{ N} \quad ◄$$

11.21

$$G\frac{M_e m}{r^2} = G\frac{M_s m}{(R-r)^2}$$

r = distance from the earth

R = distance between the earth and the sun

$$\frac{M_e}{r^2} = \frac{M_s}{(R-r)^2}$$

$$\frac{M_e}{M_s} = \frac{r^2}{R^2 - 2Rr + r^2}$$

$$\frac{5.9742 \times 10^{24}}{1.9891 \times 10^{30}} = \frac{r^2}{(149.6 \times 10^9)^2 - 2(149.6 \times 10^9)r + r^2}$$

$$0 = 2.238 \times 10^{22} - 2.992 \times 10^{11} r - 3.3294 \times 10^5 r^2$$

$$r = 259 \times 10^6 \text{ m} = 259 \times 10^3 \text{ km} \blacktriangleleft$$

Chapter 12

12.1

$y = -0.13t^4 + 4.1t^3 + 0.12t^2 \text{m}$

$v = \dot{y} = -0.52t^3 + 12.3t^2 + 0.24t \text{ m/s}$

$a = \dot{v} = -1.56t^2 + 24.6t + 0.24 \text{ m/s}^2$

At maximum velocity:

$a = 0 \quad -1.56t^2 + 24.6t + 0.24 = 0 \quad t = 15.779 \text{ s}$

$v_{\max} = -0.52(15.779)^3 + 12.3(15.779)^2 + 0.24(15.779)$
$= 1023 \text{ m/s}$

$y = -0.13(15.779)^4 + 4.1(15.779)^3 + 0.12(15.779)^2 = 8080 \text{ m}$

12.2

(a) $x = -\frac{1}{2}gt^2 + v_0 t \quad \therefore v=\dot{x} = -gt+v_0 \quad \therefore a=\ddot{x}=-g$

When t=0, then x=0 and $v=v_0$. Hence v_0 is the initial velocity.
Since gravity is the only source of acceleration in this problem, g must be the gravitational acceleration.

(b) When $x = x_{\max}$, then $v = 0$. $\therefore -gt + v_0 = 0 \quad \therefore t = \frac{v_0}{g}$

$\therefore x_{\max} = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right) = \frac{v_0^2}{2g}$

At the end of the flight $x = 0$ $\quad \therefore -\frac{1}{2}gt^2 + v_0 t = 0 \quad \therefore t = \frac{2v_0}{g}$

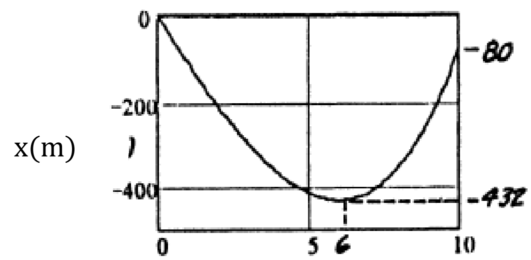
(c) $v_0 = \frac{(25)^2}{2(9.8)} = 31.89 \text{ m} \quad \therefore t = \frac{2(25)}{9.8} = 5.1 \text{ s}$

12.3

(a) $x = t^3 \text{-} 108t$ m

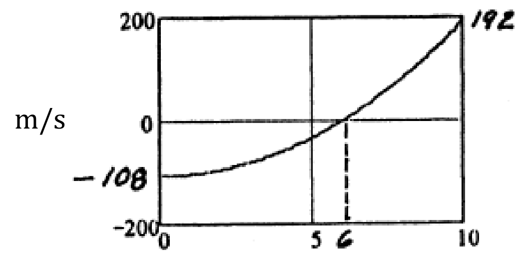
$v = x = 3t^2 - 108$ m/s

$a = \ddot{x} = 6t$ m/s^2



<u>When t = 0:</u>

$x = 0 \;\; v = -108$ m/s $\;\; a = 0$



<u>When t = 10 s:</u>

$x = (10)^3 - 108(10) = -80.0$ m

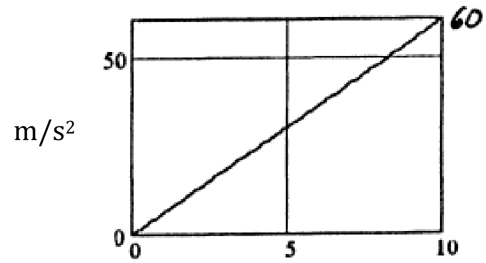
$v = 3(10)^2 - 108 =: 92.0$ m/s

$a = 6(10) = 60:$ m/s^2

<u>When v = 0 (x = x_{min} or x_{max}):</u>

$3t^2 - 108 = 0 \quad \therefore t = 60$ s

$\therefore x = (6)^3 - 108(6) = -432.0$ m



(b) Displacement: $\Delta r = 80.0$ m

(c) Distance travelled: $d = 2(432) - 80 = 784$ m

12.4

$x = t^3 \text{-} 3t^2 - 45t$ m

$v = \dot{x} = 3t^2 - 6t - 45$ m/s

$a = \dot{v} = 6t - 6$ m/s^2

At $t = 8$ s:

$x = 8^3 - 3(8)^2 - 45(8) = -40$ m

$v = 3(8)^2 - 6(8) - 45 = 99$ m/s

$a = 6(8) - 6 = 42$ m/s^2

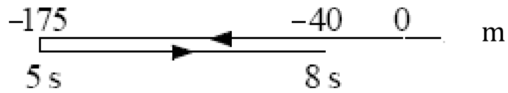
Note that $v(0) < 0$ and $v(8\ s) > 0$. The reversal of velocity occurs when

$v = 3t^2 - 6t - 45 = 0 \quad t = 5.0$ s

$x = 5^3 - 3(5)^2 - 45 - (5) = -175$ m

At t=8 s:

$$s = 175 + (175 - 40) = 310 \text{ m}$$



12.5

(a) $x = t^2 - \dfrac{t^3}{90}\text{m} \quad v = \dot{x} = 2t - \dfrac{t^2}{30}\text{m/s} \quad v = 0 \text{ when } t = 60 \text{ s}$

$$x_{\max} = 60^2 - \frac{60^3}{90} = 1200 \text{ m}$$

(b) $a = \dot{v} = 2 - \dfrac{t}{15}\text{m/s}^2 \quad a = 0 \text{ when } t = 30 \text{ s}$

$$v_{\max} = 2(30) - \frac{30^2}{30} = 30 \text{ m/s}$$

12.6

(a) $x = v_0(t - t_0 + t_0 e^{-t/t_0}) \quad \therefore\ v = \dot{x} = v_0(1 - e^{-t/t_0})$ ◆

Since $v \to v_0$ as $t \to \infty$, v_0 is the limiting or <u>terminal velocity</u>.

(b) $a = \dot{v} = \dfrac{v_0}{t_0} e^{-t/t_0}$ ◆ But from part (a): $v_0 - v = v_0 e^{-t/t_0}$ $\therefore\ a = \dfrac{v_0 - v}{t_0}$ ◆

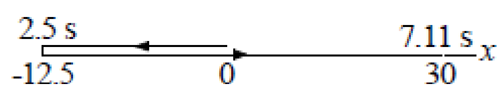
12.7

(a) $x = 2t^2 - 10t \text{ mm} \quad v = \dot{x} = 1t - 10 \text{ mm/s}$

When $x = 30$ mm:

$$2t^2 - 10t = 30 \quad t = 7.110\ s$$

(b) $v = 0$ when $4t - 10 = 0 \quad t = 2.5 \text{ s} \quad x = 2(2.5)^2 - 10(2.5) = -12.5 \text{ mm}$



$$s = 2(12.5) + 0 = 55.0 \text{ mm}$$

12.8

$$x = 4t^2 - 2 \text{ mm}$$
$$y = \frac{x^2}{12} = \frac{16t^4 - 16t^2 + 4}{12} = \frac{4t^4 - 4t^2 + 1}{3} \text{ mm}$$

When $t = 2$ s:

$$v_x = \dot{x} = 8t = 8(2) = 16 \text{ mm/s}$$
$$v_y = \dot{y} = \frac{16t^3 - 8t}{3} = \frac{16(2)^3 - 8(2)}{3} = 37.33 \text{ mm/s}$$
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{16^2 + 37.33^2} = 40.6 \text{ mm/s} \blacktriangleleft$$

$$a_x = \dot{v}_x = 8 \text{ mm/s}^2$$
$$a_y = \dot{v}_y = \frac{48t^2 - 8}{3} = \frac{48(2)^2 - 8}{3} = 61.33 \text{ mm/s}^2$$
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8^2 + 61.33^2} = 61.9 \text{ mm/s}^2 \blacktriangleleft$$

12.9

(a) $x = R\left(1 + \frac{1}{2}\cos\omega t\right)$ $\quad \therefore v = \dot{x} = -\frac{1}{2}R\omega \sin\omega t$ ◆ $\quad \therefore a = \dot{v} = -\frac{1}{2}R\omega^2 \cos\omega t$ ◆

(b) $|v|_{max} = \frac{1}{2}R\omega \quad |a|_{max} = \frac{1}{2}R\omega^2$

$\therefore$ Doubling ω would <u>double</u> $|v|_{max}$ and <u>quadruple</u> $|a|_{max}$.

12.10

$$x = \sqrt{(v_0 t - b)^2 - b^2}$$

$$\therefore \ v = \dot{x} = \frac{1}{2}\left[(v_0 t - b)^2 - b^2\right]^{-1/2}\left[2v_0(v_0 t - b)\right] = \frac{v_0(v_0 t - b)}{\sqrt{(v_0 t - b)^2 - b^2}} \ \blacklozenge$$

$$\therefore a = \dot{v} = \frac{v_0^2\left[(v_0 t - b)^2 - b^2\right]^{1/2} - v_0(v_0 t - b)\frac{1}{2}\left[(v_0 t - b)^2 - b^2\right]^{-1/2}\left[2v_0(v_0 t - b)\right]}{(v_0 t - b)^2 - b^2}$$

$$= -\frac{v_0^2 b^2}{\left[\sqrt{(v_0 t - b)^2 - b^2}\right]^3} \ \blacklozenge$$

12.11

(a) $v^2 = 2gr_0(r_0/r - 1) + v_0^2$

Differentiate with respect to time: $2v\dot{v} = 2gr_0(-r_0/r^2)\dot{r}$ or $2va = -2g(r_0/r)^2 v$

$\therefore a = -g(r_0/r)^2$ ◆

(b) v_0 is the escape velocity if $v \to 0$ when $r \to \infty$.

$\therefore 0 = \lim_{r \to \infty}\left[2gr_0(r_0/r - 1) + v_0^2\right]$ $\therefore 0 = 2gr_0(0 - 1) + v_0^2$ $\therefore v_0 = \sqrt{2gr_0}$ ◆

(c) For earth: $v_0 = \sqrt{2(9.8)(6400 \times 10^3)} = 11200$ m/s

12.12

$$x = 15 - 2t^2 \text{ m} \qquad v_x = \dot{x} = -4t \text{ m/s} \qquad a_x = \dot{v}_x = -4 \text{ m/s}^2$$
$$y = 15 - 10t + t^2 \text{ m} \qquad v_y = \dot{y} = -10 + 2t \text{ m/s} \qquad a_y = \dot{v}_y = 2 \text{ m/s}^2$$

(a) At $t = 0$: $\mathbf{v} = -10\mathbf{j}$ m/s ◄ $\mathbf{a} = -4\mathbf{i} + 2\mathbf{j}$ m/s² ◄

(b) At $t = 5$ s: $\mathbf{v} = -20\mathbf{i}$ m/s ◄ $\mathbf{a} = -4\mathbf{i} + 2\mathbf{j}$ m/s² ◄

12.13

$$x = 66t \text{ m} \qquad v_x = \dot{x} = 66 \text{ m/s} \qquad a_x = \dot{v}_x = 0$$
$$y = 86t - 4.91t^2 \text{ m} \qquad v_y = \dot{y} = 86 - 9.82t \text{ m/s}$$
$$a_y = \dot{v}_y = -9.82 \text{ m/s}^2$$

(a) $\mathbf{a} = -9.82\mathbf{j}$ m/s² ◄

(b) When $t = 0$: $\mathbf{v} = 66\mathbf{i} + 86\mathbf{j}$ m/s ◄

(c) When $y = h$: $v_y = 0$ $86 - 9.82t = 0$ $t = 8.758$ s

$$h = 86(8.758) - 4.91(8.758)^2 = 377 \text{ m} ◄$$

(d) When $y = -120$ m: $86t - 4.91t^2 = -120$ $t = 18.814$ s

$$L = 66(18.814) = 1242 \text{ m} ◄$$

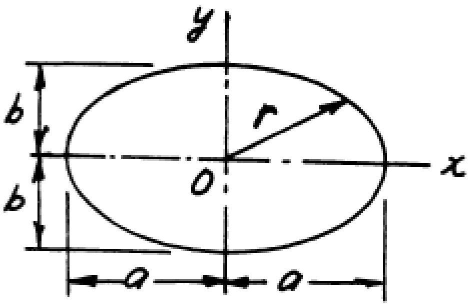
12.14

(a) $v_x = \dot{x} = v_0 \qquad y = h\left(1 - \dfrac{x^2}{b^2}\right) \qquad \therefore\ v_y = \dot{y} = -\dfrac{2h}{b^2}x\dot{x} = -\dfrac{2hv_0}{b^2}x$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + \left(-\frac{2hv_0}{b^2}x\right)^2} = v_0\sqrt{1 + \left(\frac{2h}{b^2}x\right)^2} \ \blacklozenge$$

(b) $a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = -\dfrac{2hv_0}{b^2}\dot{x} = -\dfrac{2hv_0^2}{b^2} \qquad \therefore\ a = \sqrt{a_x^2 + a_y^2} = \dfrac{2hv_0^2}{b^2}\downarrow \ \blacklozenge$

12.15



(a) $x = a\cos\omega t \qquad y = b\sin\omega t$

$$\therefore\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2\omega t + \sin^2\omega t = 1$$

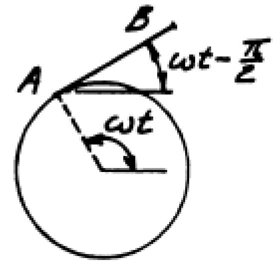
which is the equation of ellipse $\therefore$ Q.E.D.

(b) $v_x = \dot{x} = -a\omega\sin\omega t \qquad a_x = \ddot{x} = -a\omega^2\cos\omega t = -\omega^2 x$

$v_y = \dot{y} = -b\omega\cos\omega t \qquad a_y = \ddot{y} = -b\omega^2\sin\omega t = -\omega^2 y$

$\therefore\ \mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} = -\omega^2(x\mathbf{i} + y\mathbf{j}) = -\omega^2\mathbf{r} \qquad \therefore$ **a** and **r** are <u>collinear</u> $\therefore$ Q.E.D.

12.16



$x = R\cos\omega t + R\omega t\sin\omega t \qquad y = R\sin\omega t - R\omega t\cos\omega t$

$\therefore v_x = \dot{x} = -R\omega\sin\omega t + R\omega\sin\omega t + R\omega^2 t\cos\omega t = R\omega^2 t\cos\omega t$

$\therefore v_y = \dot{y} = R\omega\cos\omega t - R\omega\cos\omega t + R\omega^2 t\sin\omega t = R\omega^2 t\sin\omega t$

$\therefore \mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = R\omega^2 t(\mathbf{i}\cos\omega t + \mathbf{j}\sin\omega t) \qquad \therefore v = R\omega^2 t$ ◆

$\overrightarrow{AB} = \overline{AB}\left[\mathbf{i}\cos(\omega t - \pi/2) + \mathbf{j}\sin(\omega t - \pi/2)\right]$

$= \overline{AB}(\mathbf{i}\sin\omega t - \mathbf{j}\cos\omega t)$

By inspection: $\mathbf{v}\cdot\overrightarrow{AB} = 0 \qquad \therefore \mathbf{v}$ is perpendicular to AB $\quad\therefore$ Q.E.D.

12.17

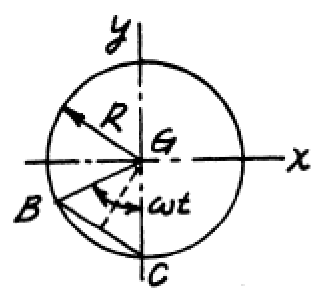
(a) $x = R(\omega t - \sin\omega t) \qquad \therefore v_x = \dot{x} = R\omega(1 - \cos\omega t)$

$y = R(1 - \cos\omega t) \qquad \therefore v_y = \dot{y} = R\omega\sin\omega t$

$\therefore v^2 = v_x^2 + v_y^2 = R^2\omega^2\left[(1-\cos\omega t)^2 + \sin^2\omega t\right] = 2R^2\omega^2(1-\cos\omega t) = 4R^2\omega^2\sin^2\dfrac{\omega t}{2}$

$\therefore v = 2R\omega\sin\dfrac{\omega t}{2}$

But $\overline{BC} = 2R\sin\dfrac{\omega t}{2} \qquad \therefore v = \omega\,\overline{BC} \qquad \therefore$ Q.E.D.



(b) $a_x = \ddot{x} = R\omega^2\sin\omega t \qquad a_y = \ddot{y} = R\omega^2\cos\omega t$

$\therefore \mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} = (R\omega^2\sin\omega t)\mathbf{i} + (R\omega^2\cos\omega t)\mathbf{j}$

$\overrightarrow{BG} = (R\sin\omega t)\mathbf{i} + (R\cos\omega t)\mathbf{j} = \dfrac{1}{\omega^2}\mathbf{a} \qquad \therefore \overrightarrow{BG}$ and $\mathbf{a}$ are parallel $\quad\therefore$ Q.E.D.

12.18

$$x = R\cos\omega t \qquad y = R\sin\omega t \qquad z = -\frac{h}{2\pi}\omega t$$

$$\therefore v_x = \dot{x} = -R\omega\sin\omega t \qquad v_y = \dot{y} = R\omega\cos\omega t \qquad v_z = \dot{z} = -\frac{h\omega}{2\pi}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 = R^2\omega^2\sin^2\omega t + R^2\omega^2\cos^2\omega t + \left(\frac{h\omega}{2\pi}\right)^2 = (R\omega)^2 + \left(\frac{h\omega}{2\pi}\right)^2$$

$$\therefore v = R\omega\sqrt{1 + \left[h/(2\pi R)\right]^2} = \text{constant} \quad \therefore \text{Q.E.D.}$$

$$a_x = \ddot{x} = -R\omega^2\cos\omega t \qquad a_y = \ddot{y} = -R\omega^2\sin\omega t \qquad a_z = \ddot{z} = 0$$

$$a^2 = a_x^2 + a_y^2 + a_z^2 = R^2\omega^4(\cos^2\omega t + \sin^2\omega t) = R^2\omega^4 \qquad \therefore a = R\omega^2 = \text{constant} \quad \therefore \text{Q.E.D.}$$

Using the given data:

$$v = (1.2)(4\pi)\sqrt{1 + \left[\frac{0.75}{2\pi(1.2)}\right]^2} = 15.15 \text{ m/s} \;\blacklozenge \qquad a = (1.2)(4\pi)^2 = 189.5 \text{ m/s}^2 \;\blacklozenge$$

12.19

$$x = 0.8v_0 t \qquad y = 0.6v_0 t \qquad z = -0.04v_0^2 t^2$$

$$\therefore v_x = \dot{x} = 0.8v_0 \qquad v_y = \dot{y} = 0.6v_0 \qquad v_z = \dot{z} = -0.08v_0^2 t$$

$$\therefore a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = 0 \qquad a_z = \ddot{z} = -0.08v_0^2$$

(a) At point B: $x = 4$ cm $\quad \therefore 4 = 0.8v_0 t \quad \therefore t = 5/v_0 \quad \therefore v_z = -0.4v_0$

$$\therefore v = \sqrt{v_x^2 + v_y^2 + v_z^2} = v_0\sqrt{0.8^2 + 0.6^2 + 0.4^2} = 1.0770v_0 \;\blacklozenge$$

$$a = |a_z| = 0.08v_0^2 \;\blacklozenge$$

(b) Let θ be the angle between the path and the z-axis at point B.

$$\therefore \cos\theta = \frac{\mathbf{v}\cdot\mathbf{k}}{|\mathbf{v}|} = \frac{-0.4v_0}{1.0770v_0} = -0.3714 \qquad \therefore \theta = 111.8°$$

∴ The angle between the path and the x-y plane is: $\theta - 90° = 21.8°$ ◆

12.20

(a) $\mathbf{r} = (3t^2 + 4t)\mathbf{i} + (-4t^2 + 3t)\mathbf{j} + (-6t + 9)\mathbf{k}$ m

$\therefore \mathbf{v} = \dot{\mathbf{r}} = (6t + 4)\mathbf{i} + (-8t + 3)\mathbf{j} - 6\mathbf{k}$ m/s

$\therefore \mathbf{a} = \dot{\mathbf{v}} = 6\mathbf{i} - 8\mathbf{j}$ m/s^2

(b) The vector normal to the plane formed by **v** and **a** (the instantaneous plane of motion) is

$$\mathbf{v} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6t+4 & -8t+3 & -6 \\ 6 & -8 & 0 \end{vmatrix} = -48\mathbf{i} - 36\mathbf{j} - 50\mathbf{k}$$

and the corresponding unit vector is

$$\mathbf{n} = \pm \frac{48\mathbf{i} + 36\mathbf{j} + 50\mathbf{k}}{\sqrt{48^2 + 36^2 + 50^2}} = \pm(0.615\mathbf{i} + 0.461\mathbf{j} + 0.640\mathbf{k})$$

Since this vector is independent of t, the orientation of the plane does not vary with the location of the particle. Thus the particle is in plane motion on an inclined plane. Q.E.D.

12.21

$x = R\cos\omega t \qquad y = R\sin\omega t \qquad z = \frac{R}{2}\sin 2\omega t$

$\therefore v_x = \dot{x} = -R\omega\sin\omega t \qquad v_y = \dot{y} = R\omega\cos\omega t \qquad v_z = \dot{z} = R\omega\cos 2\omega t$

$\therefore v = \sqrt{v_x^2 + v_y^2 + v_z^2} = R\omega\sqrt{\cos^2\omega t + \sin^2\omega t + \cos^2 2\omega t} = R\omega\sqrt{1 + \cos^2 2\omega t}$

$\therefore v_{max} = \sqrt{2}\,R\omega$ ◆

$a_x = \ddot{x} = -R\omega^2\cos\omega t \qquad a_y = \ddot{y} = -R\omega^2\sin\omega t \qquad a_z = \ddot{z} = -2R\omega^2\sin 2\omega t$

$\therefore a = \sqrt{a_x^2 + a_y^2 + a_z^2} = R\omega^2\sqrt{\cos^2\omega t + \sin^2\omega t + 4\sin^2 2\omega t} = R\omega^2\sqrt{1 + 4\sin^2 2\omega t}$

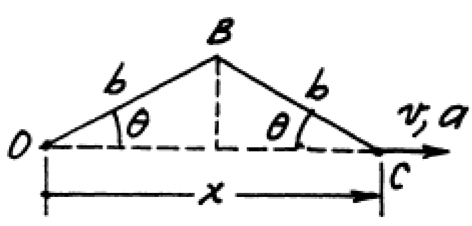
$\therefore a_{max} = \sqrt{5}\,R\omega^2$ ◆

12.22

(a) From geometry: $x = 2b\cos\theta$

$\therefore v = \dot{x} = -2b\dot{\theta}\sin\theta \rightarrow$ ◆

(b) $\therefore a = \dot{v} = -2b(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \rightarrow$ ◆

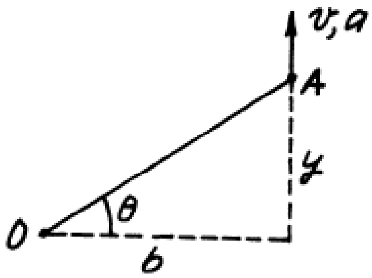


12.23

(a) Geometry: $y = b\tan\theta \quad \therefore v = \dot{y} = b\dot{\theta}\sec^2\theta \uparrow$ ◆

(b) $\therefore a = \dot{v} = b\left[\ddot{\theta}\sec^2\theta + 2\dot{\theta}\sec\theta(\sec\theta\tan\theta\,\dot{\theta})\right]$

$= b\sec^2\theta(\ddot{\theta} + 2\dot{\theta}^2\tan\theta) \uparrow$ ◆



12.24

$$v = \dot{x} = \frac{dx}{d\theta}\frac{d\theta}{dt} = \frac{dx}{d\theta}\omega = R\omega\left[-\sin\theta + \frac{1}{2}(9 - \sin^2\theta)^{-1/2}(-2\sin\theta\cos\theta)\right]$$

$$= -R\omega\sin\theta\left(1 + \frac{\cos\theta}{\sqrt{9 - \sin^2\theta}}\right) \blacklozenge$$

12.25

$$\dot{\theta} = \frac{1200 \text{ rev}}{1.0 \text{ min}} \times \frac{2\pi \text{ rad}}{1.0 \text{ rev}} \times \frac{1.0 \text{ min}}{60 \text{ s}} = 125.66 \text{ rad/s}$$

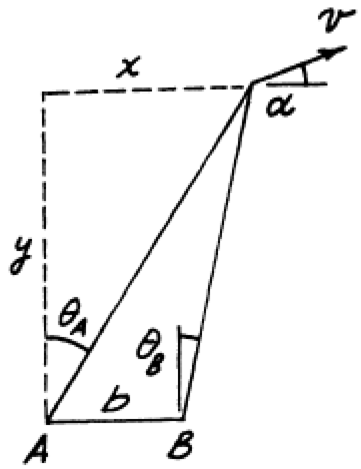
$$r = 55 + 10\cos\theta + 5\cos 2\theta \text{ mm}$$

$$v = \dot{r} = \frac{dr}{d\theta}\dot{\theta} = (-10\sin\theta - 10\sin 2\theta)(125.66) \text{ mm/s}$$

$$a = \dot{v} = \frac{dv}{d\theta}\dot{\theta} = (-10\cos\theta - 20\cos 2\theta)(125.66)^2 \text{ mm/s}^2$$

$$|a|_{max} = 30(125.66)^2 = 474\,000 \text{ mm/s}^2 = 474 \text{ m/s}^2 \text{ (at } t = 0) \blacktriangleleft$$

***12.26**



(a) Geometry: $x = y\tan\theta_A = y\tan\theta_B + b$ .... (a)

$$\therefore y = \frac{b}{\tan\theta_A - \tan\theta_B} = \frac{1000}{\tan 30° - \tan 22°} = 5770 \text{ m} \; ◆$$

(b) Differentiate Eq. (a):

$$\dot{x} = y\dot{\theta}_A \sec^2\theta_A + \dot{y}\tan\theta_A = y\dot{\theta}_B \sec^2\theta_B + \dot{y}\tan\theta_B \quad \text{(b)}$$

$$\therefore \dot{y} = -y\frac{\dot{\theta}_A \sec^2\theta_A - \dot{\theta}_B \sec^2\theta_B}{\tan\theta_A - \tan\theta_B}$$

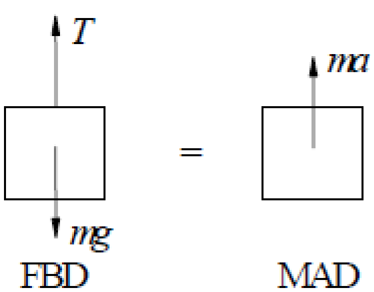
$$= -(5770)\frac{(0.026)\sec^2 30° - (0.032)\sec^2 22°}{\tan 30° - \tan 22°} = 85.11 \text{ m/s}$$

From Eq. (b): $\dot{x} = (5770)(0.026)\sec^2 30° + (85.11)\tan 30° = 249.2$ m/s

$$\therefore v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{249.2^2 + 85.11^2} = 263 \text{ m/s} \; ◆$$

(c) $\therefore \alpha = \tan^{-1}(\dot{y}/\dot{x}) = \tan^{-1}\dfrac{85.11}{249.2} = 18.86° \; ◆$

12.27

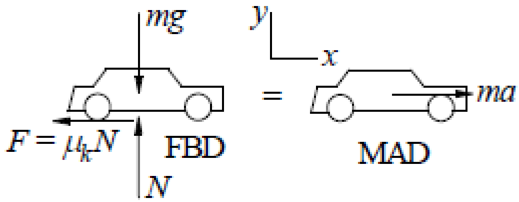


$$v = 4t \text{ m/s} \qquad a = \dot{v} = 4 \text{ m/s}^2$$

$$\Sigma F = ma \quad +\uparrow \qquad T - mg = ma$$

$$T = m(g + a) = 50(9.81 + 4) = 691 \text{ N} \; ◀$$

12.28



$$v_0 = 100 \text{ km/h } = 100 \text{ km/h}\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right) = 27.78 \text{ m/s}$$

$$\Sigma F_y = 0 \quad +\uparrow \qquad N - mg = 0 \qquad \therefore N = mg$$

$$\Sigma F_x = ma \quad \xrightarrow{+} \qquad -\mu_k N = ma \qquad \therefore a = -\frac{\mu_k N}{m} = -\mu_k g$$

$$v = \int a\, dt + C_1 = -\mu_k g t + C_1$$

$$x = \int v\, dt + C_2 = -\frac{1}{2}\mu_k g t^2 + C_1 t + C_2$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \qquad v = v_0 \quad \therefore C_1 = v_0$$

$$\therefore x = -\frac{1}{2}\mu_k g t^2 + v_0 t \qquad v = -\mu_k g t + v_0$$

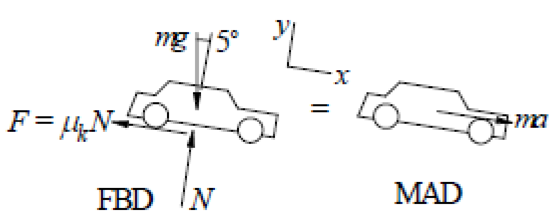
When $v = 0$:

$$-\mu_k g t + v_0 = 0 \qquad \therefore t = \frac{v_0}{\mu_k g}$$

$$x = -\frac{1}{2}\mu_k g\left(\frac{v_0}{\mu_k g}\right)^2 + v_0\left(\frac{v_0}{\mu_k g}\right) = \frac{v_0^2}{2\mu_k g}$$

$$= \frac{27.78^2}{2(0.65)(9.8)} = 60.58 \text{ m}$$

12.29



$$v_0 = 100 \text{ km/h} = 27.78 \text{ m/s}$$

$$\Sigma F_y = 0 + \uparrow \quad N - mg\ \cos 5° = 0 \quad \therefore N = mg\ \cos 5°$$

$$\Sigma F_x = ma \overset{+}{\rightarrow} - \mu_k N + mg \sin 5° = ma$$

$$\therefore a = -\frac{\mu_k N}{m} + g\ \sin 5° = (\sin 5° - \mu_k \cos 5°)g$$

$$= (\sin 5° - 0.65 \cos 5°)9.8 = -5.492\ \text{m/s}^2$$

$$v = \int a\, dt + C_1 = -5.492t + C_1$$

$$x = \int v\, dt + C_2 = -2.746t^2 + C_1 t + C_2$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \quad v = v_0 \quad \therefore C_1 = v_0 = 27.78\ \text{m/s}$$

$$x = -2.746t^2 + 27.78t\ \text{m}$$
$$v = -5.492t + 27.78\ \text{m/s}$$

When $v = 0$:

$$-5.492t + 27.78 = 0 \quad \therefore t = 5.058\ \text{s}$$
$$x = 2.746(5.058)^2 + 27.78(5.058) = 70.3\ \text{m}$$

12.30

$$a = \frac{F}{m} = \frac{-1.2t}{0.1} = -12t\ \text{m/s}^2$$

$$v = \int a_x dt + C_1 = -6t^2 + C_1\ \text{m/s}$$

$$x = \int v_x dt + C_2 = -2t^3 + C_1 t + C_2\ \text{m}$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \qquad v = 64\ \text{m/s} \quad \therefore C_1 = 64\ \text{m/s}$$

$$\therefore x = -2t^3 + 64t\ \text{m} \qquad v = -6t^2 + 64\ \text{m/s}$$

When $t = 4$ s:

$$x = -2(4)^3 + 64(4) = 128\ \text{m}$$

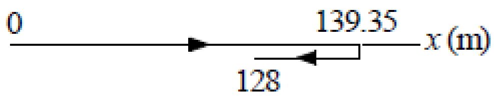
When $v = 0$:

$$-6t^2 + 64 = 0 \qquad t = 3.266\ \text{s}$$

$$x = -2(3.266)^3 + 64(3.266) = 139.35\ \text{m}$$

Distance traveled:

$$d = 2(139.35) - 128 = 150.7 \text{ m} \blacktriangleleft$$



12.31

$$\begin{aligned} a &= \frac{F}{m} = \frac{0.04\sqrt{v}}{0.01} = 4\sqrt{v} \text{ m/s}^2 \\ dt &= \frac{dv}{a} \qquad t = \int \frac{dv}{a} + C_1 = \int \frac{dv}{4\sqrt{v}} + C_1 = \frac{\sqrt{v}}{2} + C_1 \end{aligned}$$

When $t = 0.6$ s (initial condition):

$$v = 0.16 \text{ m/s} \qquad \therefore 0.6 = \frac{\sqrt{0.16}}{2} + C_1 \qquad C_1 = 0.4 \text{ s}$$

$$\therefore t = \frac{\sqrt{v}}{2} + 0.4 \text{ s} \qquad \therefore v = (2t - 0.8)^2 = 4t^2 - 3.2t + 0.64 \text{ m/s}$$

$$x = \int v\, dt + C_2 = \frac{4}{3}t^3 - 1.6t^2 + 0.64t + C_2$$

When $t = 0$ (initial condition):

$$x = 0 \qquad \therefore C_2 = 0 \qquad \therefore x = \frac{4}{3}t^3 - 1.6t^2 + 0.64t \text{ m}$$

When $t = 0.8$ s:

$$x = \frac{4}{3}(0.8)^3 - 1.6(0.8)^2 + 0.64(0.8) = 0.1707 \text{ m} \blacktriangleleft$$