Chapter 1   Circuit Variables and Elements

Solutions to Exercises

E1.1.1   Number of electrons/hr $= \dfrac{-1\,\mathrm{A} \times 1\,\mathrm{s} \times 3600}{-1.6 \times 10^{-19}} = 2.25 \times 10^{22}$ electrons/hr.

E1.1.2   (a) $\dfrac{5 \times 10^{18} \times 1.6 \times 10^{-19}}{60} + \dfrac{-2.5 \times 10^{18} \times (-1.6 \times 10^{-19})}{60} = 0.02\,\mathrm{A} \equiv 20\,\mathrm{mA}.$

(b) The sign of the current is reversed to –20 mA

(c) $\dfrac{5 \times 10^{18} \times 1.6 \times 10^{-19}}{60} + \dfrac{2.5 \times 10^{18} \times (-1.6 \times 10^{-19})}{60} = \dfrac{0.4}{60}\,\mathrm{A} \equiv \dfrac{20}{3}\,\mathrm{mA}.$

E1.2.1   (a) Since electrons move to a more positive voltage, they lose potential

energy. Energy loss per electron $= qV = 1.6 \times 10^{-19} \times 10 = 1.6 \times 10^{-18}$ J.

(b) It is converted to K.E.

(c) $\dfrac{1}{2}mv^2 = 1.6 \times 10^{-18}$ J; $v = \sqrt{\dfrac{3.2 \times 10^{-18}}{9.1 \times 10^{-31}}} = 1.88 \times 10^{6}$ m/s.

E1.3.1   (a) $6.25 \times 10^{14} \times 1.6 \times 10^{-18} = 10 \times 10^{-4}$ J $\equiv 1$ mJ.

(b) Current in the conventional positive direction is from B to A and of

magnitude $6.25 \times 10^{14} \times 1.6 \times 10^{-19} = 10 \times 10^{-5}$ A $\equiv 100\ \mu$A.

(c) The kinetic energy is converted to heat.

(d) P = V × I = 10×10^{-4} W $\equiv 1$ mW.

(e) From (a), K.E. given up per second = 1 mW.

E1.4.1   (a) When cruising, battery delivers 15×12 = 180 W.

(b) When charging, battery absorbs 20×12 = 240 W.
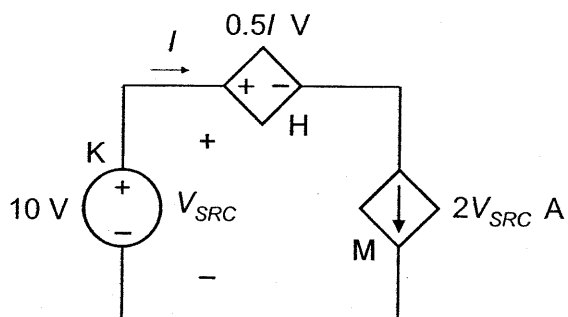
E1.6.1   (a) I is determined by the
current source and is equal
to $2V_{SRC}$, i.e., 20 A.

(b) 0.5×I = 10 V.

(c) K delivers 10×20 = 200 W, H
absorbs 0.5 $I \times I$ = 200 W.
Hence, M neither absorbs
nor delivers power.

(d) Voltage across M must be zero.

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E1.7.1   Each element is rated at 1 kW.

(a) $I = \dfrac{1000}{220} = 4.55$ A, $R = \dfrac{V^2}{P} = \dfrac{(220)^2}{1000} = 48.4$ Ω, $G = \dfrac{1}{R} = \dfrac{1}{48.4}$ S $\equiv 20.7$ mS.

(b) $I = \dfrac{2000}{220} = 9.09$ A, $R = \dfrac{(220)^2}{2000} = 24.2$ Ω, $G = \dfrac{1}{R} = 41.3$ mS.

E1.8.1 $C = \dfrac{\epsilon_0 \epsilon \times A}{d} = 8.85 \times 10^{-12} \times 5{,}000 \times \dfrac{\pi \times (5 \times 10^{-2})^2}{4} \times \dfrac{1}{10^{-3}} = 8.69 \times 10^{-8}$ F.

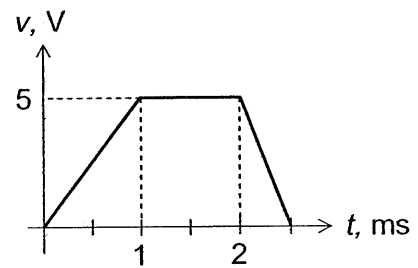$Q = CV = 50 \times 8.69 \times 10^{-8} = 4.34 \times 10^{-6}$ C.

Number of electrons $= \dfrac{4.34 \times 10^{-6}}{1.6 \times 10^{-19}} = 2.7 \times 10^{13}$ electrons.

E1.8.2   (a) $i = C\dfrac{dv}{dt} = 10^{-6} \times \dfrac{5}{10^{-3}} = 5 \times 10^{-3}$ A $\equiv$

5 mA, $0 < t < 1$ ms; $i = 0$, $1 < t < 2$ ms;

$i = -10^{-6} \times \dfrac{5}{0.5 \times 10^{-3}} = -10^{-2}$ A $\equiv -10$ mA,

$2 < t < 2.5$ ms; $i = 0$, $t > 2.5$ ms;

(b) At $t = 1.5$ ms, q due to the applied

voltage equals $Cv = 10^{-6} \times 5 \equiv 5$ μC. The total charge is therefore 10 μC.

E1.9.1   (a) $v = L\dfrac{di}{dt} = 10^{-6} \times \dfrac{5}{10^{-3}} = 5 \times 10^{-3}$ V $\equiv 5$ mV, $0 < t < 1$ ms; $v = 0$, $1 < t < 2$ ms;

$v = -10^{-6} \times \dfrac{5}{0.5 \times 10^{-3}} = -10^{-2}$ V $\equiv -10$ mV, $2 < t < 2.5$ ms; $v = 0$, $t > 2$ ms;

(b) At $t = 1.5$ ms, λ due to the applied voltage equals $\lambda i = 10^{-6} \times 5 \equiv 5$ μWb-

turns. The total flux linkage is therefore 10 μWb-turns.

Solutions to Problems and Exercises

P1.1.1 $2\times(100 \text{ W})\times(6 \text{ h})\times(30 \text{ days}) \equiv 36 \text{ kWh.}$

P1.1.2   (a) $t = 1^{-}: q = \int_{0}^{1^{-}} i\,dt = \int_{0}^{1^{-}} 0\,dt = 0$.

$t = 2: q = \int_{1}^{2} i\,dt = \int_{1}^{2}(1+t)\,dt =$

$\left[t + \dfrac{t^2}{2}\right]_{1}^{2} = 2.5 \text{ mC.}$

$t = 3: q = 2.5 + \int_{2}^{3} i\,dt = 2.5$

$+ \int_{2}^{3} 3\,dt = 5.5 \text{ mC.}$

$t = 4: q = 5.5 + \int_{3}^{4} i\,dt = 5.5$

$+ \int_{3}^{4} 3\,dt = 8.5 \text{ mC.}$

$t = 5: q = 8.5 + \int_{4}^{6} i\,dt = 8.5$

$+ \int_{4}^{6}(-t+7)\,dt = 8.5 + \left[-\dfrac{t^2}{2} + 7t\right]_{4}^{5} = 11 \text{ mC.}$

$t = 6: q = 11 + \int_{5}^{6} i\,dt = \int_{5}^{6} 0\,dt = 11 \text{ mC.}$

(b) $0 \le t \le 1: p = 0$.

$1 \le t \le 2: p = 2t(t+1) = 2t^2 + 2t \text{ mW.}$

$2 \le t \le 3: p = 2t \times 3 = 6t \text{ mW.}$

$3 \le t \le 4: p = (-2t+12)\times 3 = -6t + 36 \text{ mW.}$

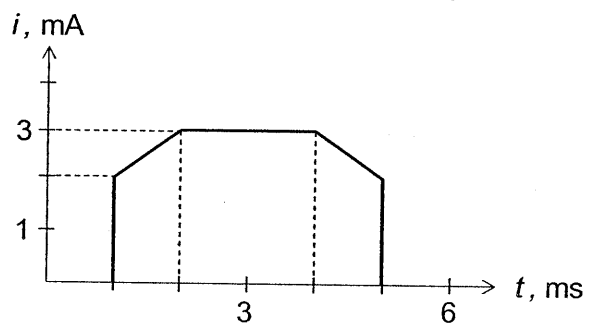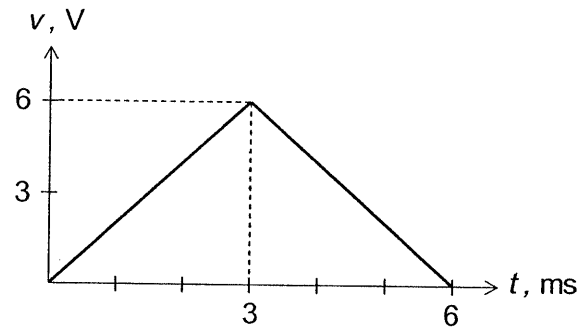$4 \le t \le 5: p = (-2t+12)(-t+7) = 2t^2 - 26t + 84 \text{ mW.}$

$5 \le t \le 6: p = 0$.

(c) $\omega(t) = \int_{0}^{6} p\,dt = \int_{0}^{1} 0\,dt + \int_{1}^{2}(2t^2 + 2t)\,dt + \int_{2}^{3} 6t\,dt + \int_{3}^{4}(-6t+36)\,dt +$

$\int_{4}^{6}(2t^2 - 26t + 84)\,dt + \int_{5}^{6} 0\,dt = \left[\dfrac{2t^3}{3} + t^2\right]_{1}^{2} + \left[3t^2\right]_{2}^{3} + \left[-3t^2 + 36t\right]_{3}^{4} +$

$\left[\dfrac{2t^3}{3} - 13t^2 + 84t\right]_{4}^{5} \; 136/3 = 45.3 \text{ μJ.}$

P1.1.3 $p = vi = (2t + 1)(4 - 2t) = -4t^2 + 6t + 4$.

(a) $\dfrac{dp}{dt} = -8t + 6$; $p_{max} \Rightarrow \dfrac{dp}{dt} = 0 \Rightarrow t = \dfrac{6}{8} = 0.75s$.

(b) $p = 0 \Rightarrow -4t^2 + 6t + 4 = 0 \Rightarrow t = 2s$; $p = 0$ when either v or i is zero.

(c) At $t = 2s$: $w = \displaystyle\int_0^2 p\,dt = \int_0^2 (-4t^2 + 6t + 4)dt = \left[-\dfrac{4}{3}t^3 + 3t^2 + 4t\right]_0^2 = 9.3$ mJ.

At $t = 4s$: $w = \displaystyle\int_0^4 p\,dt = \int_0^4 (-4t^2 + 6t + 4)dt = \left[-\dfrac{4}{3}t^3 + 3t^2 + 4t\right]_0^4 = -21.3$ mJ;

power is absorbed by device for $t < 2$ s, and is delivered by device for $2 < t < 4$ s.

P1.1.4 (a) Power is absorbed by device during the first and third quarter cycles, when v and i have the same sign, and is delivered during the second and fourth quarter cycles, when v and i have opposite signs.

(b) $p = vi = \sin 2\pi t \times \cos 2\pi t = 0.5\sin 4\pi t$ W; maximum power absorbed or delivered is 0.5 W.
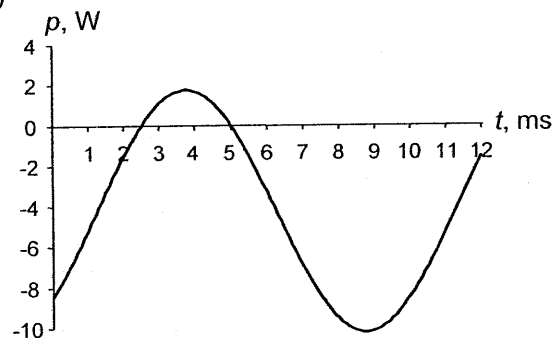
(c) Minimum power absorbed or delivered is 0.5 W.

(d) $P = \displaystyle\int_0^4 0.5\sin 4\pi t = \dfrac{1}{8\pi}\left[-\cos 4\pi t\right]_0^1 = 0$.

P1.1.5 $p = vi = 12\sin(100\pi t - 45°)(\cos 100\pi t)$

$= 6\sin(200\pi t - 45°) + 6\sin(-45°)$

$= 6\sin(200\pi t - 45°) - 3\sqrt{2}$ W.

(a) $P = \dfrac{1}{T}\displaystyle\int_0^T p\,dt$

$= \dfrac{1}{0.01}\displaystyle\int_0^{0.01}\left[6\sin(200\pi t - 45°) - 3\sqrt{2}\right]dt = -3\sqrt{2} = -4.24$ W, since the trigonometric term averages to zero over a complete period.

(b), (c) $\dfrac{dp}{dt} = 1200\pi\cos(200\pi t - 45°)$ $\dfrac{dp}{dt} = 0 \Rightarrow \cos(200\pi t - 45°) = 0 \Rightarrow$

$200\pi t - \dfrac{\pi}{4} = \dfrac{\pi}{2}, \dfrac{3\pi}{2} \Rightarrow t = \dfrac{3}{800} \equiv 3.75$ ms, or $t = \dfrac{7}{800} \equiv 8.75$ ms;

substituting in the expression for p gives p_{max} = 1.76 W at t = 3.75 ms, and p_{min} = −10.24 W at t = 8.75 ms.
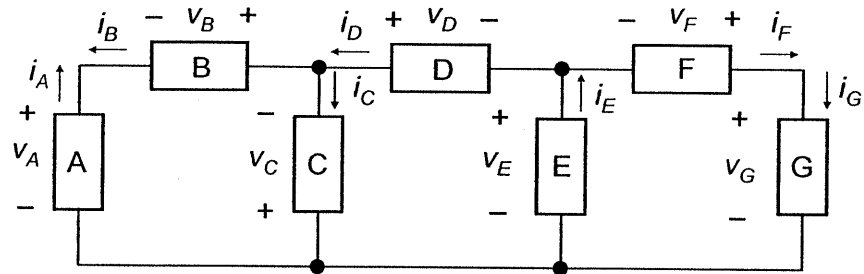
P1.1.6 (a) $0 \le v \le 2$; $p = vi = v(8 - 2v^2) = -2v^3 + 8v$, and $p = 0$, $v \ge 2$.

At v = 1 V; p = 6 W; at v = 2 V, p = 0.

(b) $\dfrac{dp}{dv} = -6v^2 + 8$, $0 \le v \le 2$; $p_{max} \Rightarrow \dfrac{dp}{dv} = 0 \Rightarrow -6v^2 + 8 = 0 \Rightarrow v = \dfrac{2\sqrt{3}}{3}$ V.

(c) $v(t) = 2e^{-t} \Rightarrow i = 8 - 8e^{-2t}$; $q = \int_0^2 i \, dt = \int_0^2 \left(8 - 8e^{-2t}\right) dt = \left[8t + 4e^{-2t}\right]_0^2 = 12.07$ C.

P1.1.7 According to the assigned positive directions, the direction of power flow is indicated in the second row of the table below, where D denotes power delivered and A denotes power absorbed. The signed product of the voltage and current for each element is entered in the fifth row if there is a D in the second row, or is entered in the last row if there is an A in the second row. The remaining entries in the fifth and last rows are made so that they have opposite signs in a given column. The sum of the positive quantities in each of the fifth and last rows is 25 and the sum of the negative quantities is also 25. Thus, the total power delivered is 25 W and the total power absorbed is 25 W.

Element	A	B	C	D	E	F	G
Power flow	D	A	D	D	D	D	A
Voltage, V	5	−3	−2	5	−3	7	4
Current, A	3	−3	1	−2	−1	1	1
Power delivered, W	15	−9	−2	−10	3	7	−4
Power Absorbed, W	−15	9	2	10	−3	−7	4

P1.2.1 Voltage across all elements is 20 V. Hence,

$$20 \times I_A = 40 \Rightarrow I_A = 2\,\text{A}.$$
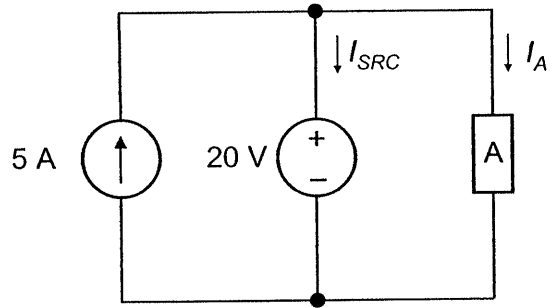
Power delivered by 5 A source is

$$5 \times 20 = 100\,\text{W}.$$

Power absorbed by 20 V source is $100 - 40 = 60$ W.

$$20 \times I_{SRC} = 60 \Rightarrow I_{SRC} = 3\,\text{A}.$$

P1.2.2 Current through all elements is 6 A.

Power absorbed by A is

$$-V_A \times 6 = 240 \Rightarrow V_A = -40 \ \text{V}.$$
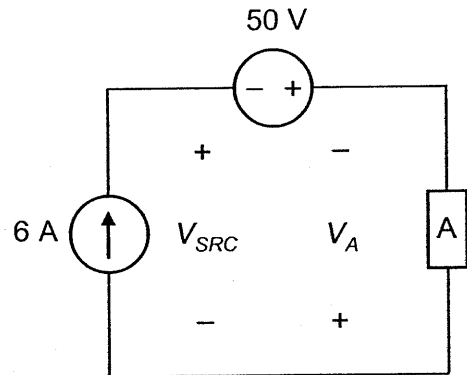
Power delivered by 50 V source is

$$50 \times 6 = 300\,\text{W}.$$

Hence, the 6 A source absorbs $300 - 240 = 60$ W.

It follows that

$$V_{SRC} \times 6 = -60 \Rightarrow V_{SRC} = -10\,\text{V}.$$

P1.2.3 Power absorbed by A is 500 W.

Power delivered by voltage source is $100 I_{SRC}$ W.

Power delivered by current source is $60 \times 0.2 \times 100 = 1200$ W.

Power delivered by B is $40 \times 0.2 \times 100 = 800$ W.

From conservation of power: $500 = 100 I_{SRC} + 800 + 1200$, so $I_{SRC} = -15$ A.

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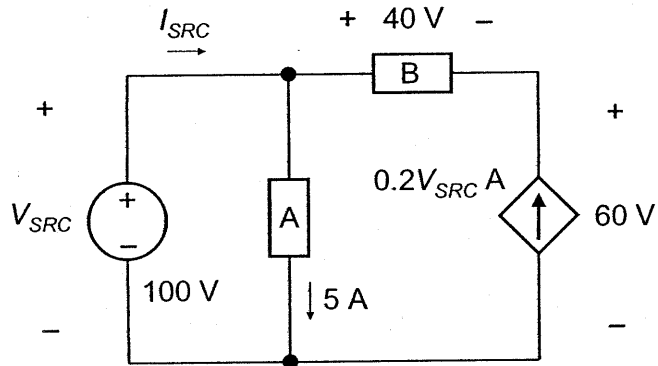
P1.2.4 I_{SRC} = 100A, as determined by the current source.

Voltage across voltage source and B is 0.6×100 = 60 V.

Power absorbed by voltage source is 60×60 = 3600 W.
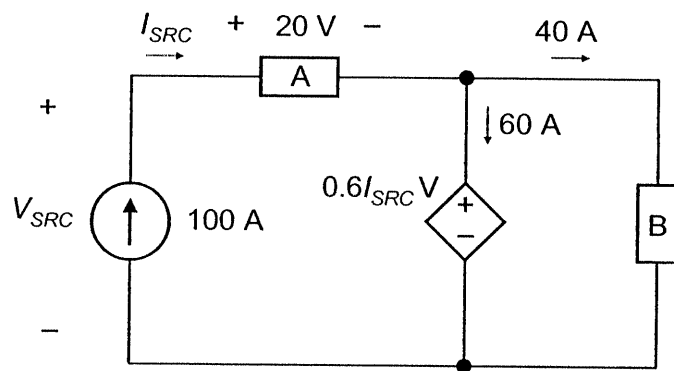
Power absorbed by B is 60×40 = 2400 W.
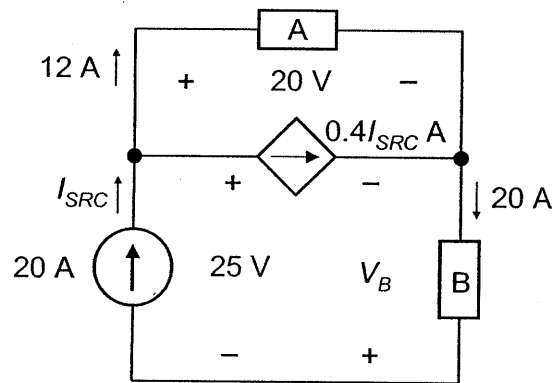
Power absorbed by A is 20×100 = 2000 W. Hence, power delivered by current source is 3600 + 2400 + 2000 = 8000 = $V_{SRC}\times100$, so V_{SRC} = 80 V.

P1.2.5 I_{SRC} = 20 A, as determined by the current source.

Power delivered by independent current source is 25×20 = 500 W.

Power absorbed by A is 20×12 = 240 W.

Power absorbed by dependent current source is $20\times0.4\times20$ = 160 W.

Hence, B must absorb 500 − 240 − 160 = 100 W. It follows that $(-V_B)\times20 = 100$, so V_B = -5 V.
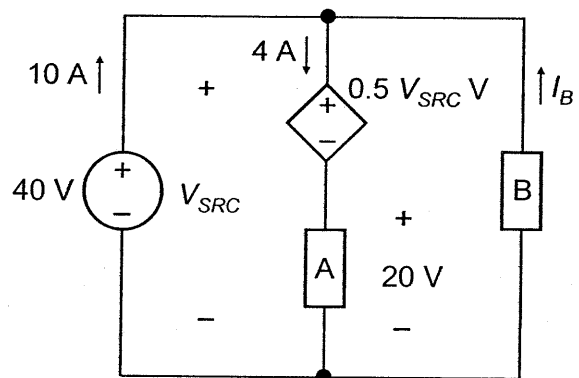
P1.2.6 Power delivered by independent voltage source is 40×10 = 400 W.

Power absorbed by dependent voltage source is $0.5\times40\times4$ = 80 W.

Power absorbed by A is 20×4 = 80 W.

Hence, B must absorb 400 − 80 − 80 = 240 W. It follows that $40\times(-I_B)$ = 240, so I_B = -6 A. Or, 400 + $40I_B$ = 160, so I_B = -6 A.
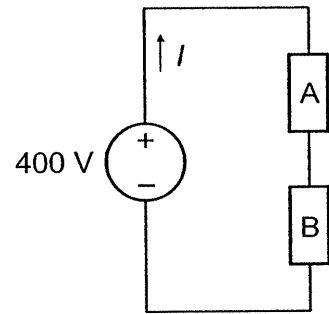
P1.2.7 Power absorbed by each element is $2I^2 \times I = 2I^3$ W.

Total power absorbed by A and B is $4I^3$ W.

Power delivered by source is $400 \times I$ W. Hence, $400I$

$= 4I^3$,so $I = 10$ A, since I should be positive for

power delivery by the source.

Alternatively, we can say that since A and B are

identical, the voltage each of them is 200 V. Hence, $I = \sqrt{\dfrac{200}{2}} = 10$ A.

The power delivered by the source is, therefore, $400 \times 10 \equiv 4$ kW, whereas

each element absorbs $2 \times (10)^3 \equiv 2$ kW, which is also half the power delivered

by the source.

P1.2.8 Since the voltage across each element is

400 V, the current through each element is

$\sqrt{\dfrac{400}{2}} = 10\sqrt{2}$ A. The power absorbed by

each element is, therefore,

$400 \times 10\sqrt{2} \equiv 4\sqrt{2}$ k W. The total power

delivered by the source is $8000\sqrt{2} = 400 I_{SRC}$ W. It follows that $I_{SRC} = 20\sqrt{2}$ A.
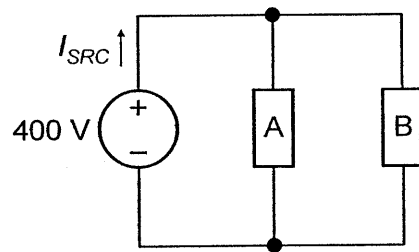
P1.2.9 Voltage across each element is

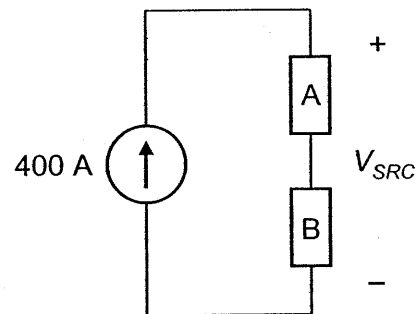$\sqrt{\dfrac{400}{2}} = 10\sqrt{2}$ V. Since A and B are

identical, $V_{SRC} = 20\sqrt{2}$ V. Power delivered by

source is $20\sqrt{2} \times 400 \equiv 8\sqrt{2}$ kW. Power

absorbed by each element

is $10\sqrt{2} \times 2\left(10\sqrt{2}\right)^2 \equiv 4\sqrt{2}$ kW, which is half
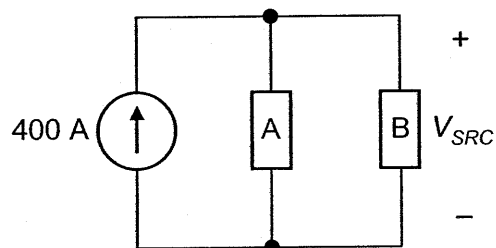
the power delivered by the source.

P1.2.10 Power absorbed by each element is

$V_{SRC} \times 2V_{SRC}^2 = 2V_{SRC}^3$ W.

Total power absorbed by A and B is

$4V_{SRC}^3$ W.

Power delivered by source is

$V_{SRC} \times 400$ W. Hence, $400 V_{SRC} = 4V_{SRC}^3$, so $V_{SRC} = 10$ V.

Power absorbed by each element is $2 \times 10^3 \equiv 2$ kW.

Power delivered by source is 4 kW. Since the power absorbed by each

element is 2 kW, the current in each element is $\dfrac{2000}{10} = 200$ A. As expected,

the source current divides equally between the identical elements A and B.

P1.3.1   (a) $I = \dfrac{10,000}{220} = 45.5$ A.

(b) $R = \dfrac{220 \times 220}{10,000} = 4.84 \ \Omega.$

(c) $G = \dfrac{10,000}{220 \times 220} = \dfrac{1}{4.84} = 0.21$ S.

P1.3.2 $R_2 = R_1 \left[1 + \alpha_m (T_2 - T_1) \right].$

$70 = 60 \left[1 + 0.0039 (T_2 - 20) \right] \Rightarrow T_2 = 62.7 °\mathrm{C}.$

P1.3.3 $P = \dfrac{V^2}{R} \Rightarrow V^2 = R \times P = 1.5 \times 10^6 \times 0.5 = 0.75 \times 10^6 \Rightarrow V = 500\sqrt{3} = 866.0 \,\mathrm{V}.$

P1.3.4 $1(\mathrm{m}\Omega) \times 10(\mathrm{A}) = 10$ mV; $0.1(\mu\mathrm{S}) \times 100(\mathrm{V}) = 10 \ \mu\mathrm{A}.$

P1.3.5 $i = 10^{-9} \left(e^{20v} - 1 \right).$

(a) $v = 0.7V \Rightarrow i = 10^{-9} \left(e^{20 \times 0.7} - 1 \right) \cong 10^{-9} \times e^{20 \times 0.7} \equiv 1.20 \,\mathrm{mA}.$

(b) $v = -0.7V \Rightarrow i = 10^{-9} \left(e^{20 \times (-0.7)} - 1 \right) \cong -10^{-9} \equiv -1\mathrm{nA}.$

P1.3.6 $v = 10t$ V, $\ 0 \le t \le 1$ min;

$= -10t + 20$ V, $\ 1 \le t \le 3$ min;

$= 10t - 40$ V,

    $3 \le t \le 4$ min.

(a) $i = \dfrac{v}{100} = 0.1t$ A,

    $0 \le t \le 1$ min;

$= -0.1t + 0.2$ A,

$1 \le t \le 3$ min;

$= 0.1t - 0.4$ A,

$3 \le t \le 4$ min.
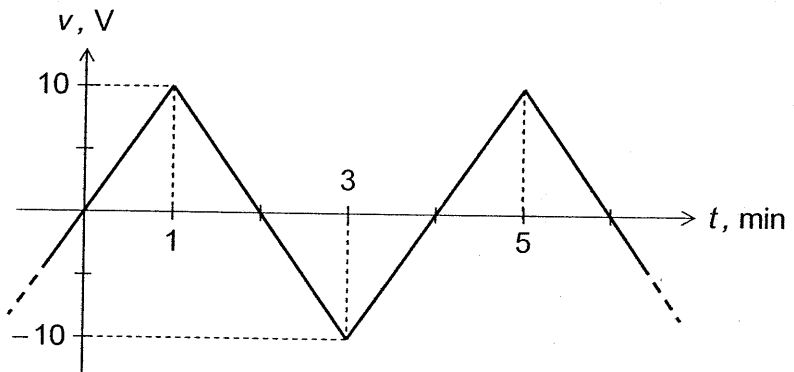
(b) $p = \dfrac{v^2}{R} = t^2$ W, $0 \le t \le 1$ min, $p = \dfrac{(-10t+20)^2}{100}$ W, $1 \le t \le 3$ min,

$p = \dfrac{(10t-40)^2}{100}$ W, $3 \le t \le 4$ min.

(c) $P = \dfrac{1}{T}\displaystyle\int_0^T p\,dt = \dfrac{1}{4 \times 60}\left[\int_0^1 t^2 dt + \int_1^3 \left(t^2 - 4t + 4\right)dt + \int_3^4 \left(t^2 - 8t + 16\right)dt\right]$

$= \left[\dfrac{t^3}{3}\right]_0^1 + \left[\dfrac{t^3}{3} - 2t^2 + 4t\right]_1^3 + \left[\dfrac{t^3}{3} - 4t^2 + 16t\right]_3^4 = \dfrac{1}{180} \equiv 5.56\ \text{mJ}.$

(d) $V_{avg} = 0$, since the waveform is symmetrical about the horizontal axis. This makes $I_{avg} = 0$ as well. Thus, $V_{avg} \times I_{avg} = 0$, whereas $P \ne 0$. The average power in a resistor is not the product of the average voltage across the resistor and the average current through the resistor, because power, being the product of voltage and current, is a nonlinear quantity.

P1.3.7 $\dfrac{60\sin 100\pi t}{4\sin 100\pi t} = 15\ \Omega;\ P = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} 240\sin^2 \omega t\,d(\omega t) =$

$\dfrac{240}{2\pi}\displaystyle\int_0^{2\pi} \dfrac{1}{2}(1 - \cos 2\omega t)d(\omega t) = \dfrac{240}{2\pi}\left[\dfrac{(\omega t)}{2} - \dfrac{1}{4}\sin 2\omega t\right]_0^{2\pi} = 120\ \text{W}.$

P1.3.8 $v(t) = 10\cos 100\pi t = 10\cos\omega t$, where $\omega = 100\pi$ rad/s, so that the supply

frequency is $\dfrac{100\pi}{2\pi} = 50$ Hz, and

the supply period is $\dfrac{1}{50} \equiv 20$ ms.

(a) $p = \dfrac{v^2}{R} = \dfrac{(10\cos 100\pi t)^2}{10}$

$= 10\cos^2 100\pi t$ as shown.

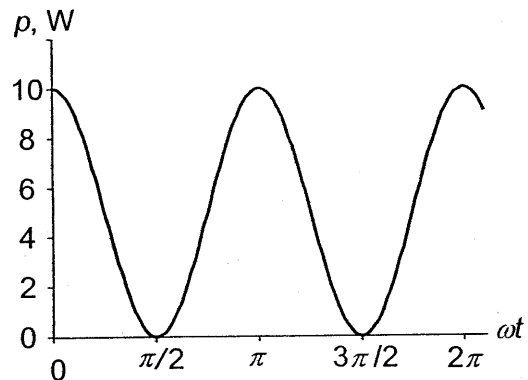(b) $P = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} 10\cos^2 \omega t\,d(\omega t)$

$= \dfrac{10}{2\pi}\displaystyle\int_0^{2\pi}\dfrac{1}{2}(1 + \cos 2\omega t)d(\omega t) = \dfrac{10}{2\pi} \times \dfrac{2\pi}{2} = 5\ \text{W}$

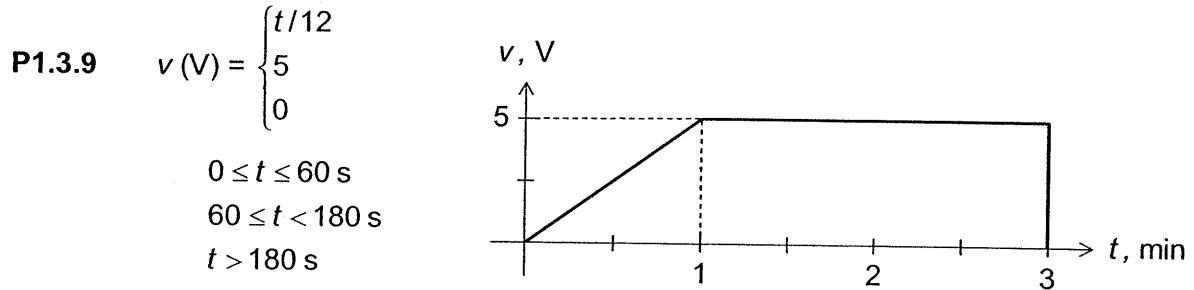$w = \displaystyle\int_0^{0.01} p\,dt = \int_0^{0.01} 10\cos^2 100\pi t\,dt = 5\int_0^{0.01}(1 + \cos 200\pi t)dt$

$= 5\left[t + \dfrac{\sin 200\pi t}{200\pi}\right]_0^{0.01} = 0.05$ J. Since the power dissipated is 5 W, the

energy dissipated during one half cycle is 5(W)×0.01(s) = 0.05 J. Note

that the average power, i.e., average energy per unit time, is independent of the time scale, but the energy is the integral of instantaneous power with respect to time.

P1.3.9
$$v\ (V) = \begin{cases} t/12 \\ 5 \\ 0 \end{cases}$$

$$\begin{aligned} &0 \le t \le 60 \text{ s} \\ &60 \le t < 180 \text{ s} \\ &t > 180 \text{ s} \end{aligned}$$

(a) $0 \le t \le 60$ s; $p = \dfrac{v^2}{R} = \dfrac{1}{5}\left(\dfrac{t}{12}\right)^2 = \dfrac{t^2}{720}$ W, where t is in s.
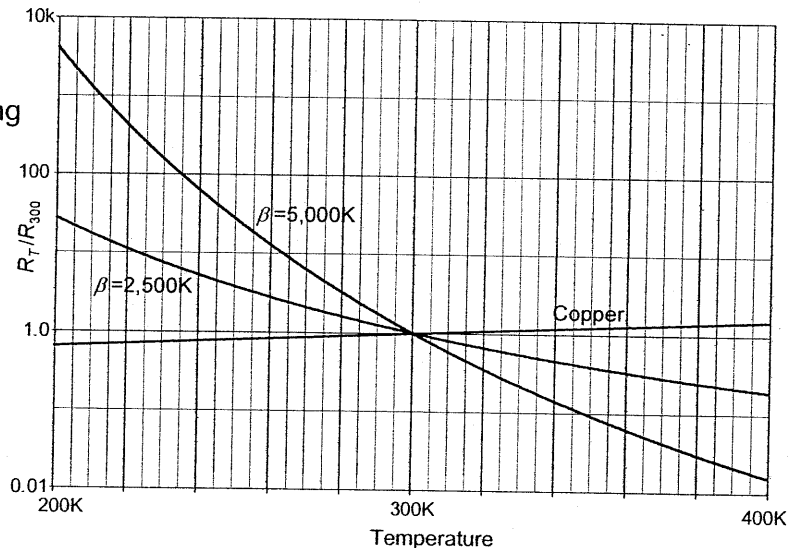
(b) $w = \displaystyle\int_0^{180} p\, dt = \int_0^{60}\dfrac{t^2}{720}\, dt + \int_{60}^{180}\dfrac{25}{5}\, dt = 700$ J.

P1.3.10 $v = 5(\cos\omega t + \cos 2\omega t)^2 = 5(\cos^2\omega t + 2\cos\omega t\, \cos 2\omega t + \cos^2 2\omega t)$

$= 5[\ \frac{1}{2}(1 + \cos 2\omega t) + \cos\omega t + \cos 3\omega t + \frac{1}{2}(1 + \cos 4\omega t)]$ V

$= 5 + 5\cos\omega t + 2.5\cos 2\omega t + 5\cos 3\omega t + 2.5\cos 4\omega t$ V

P1.3.11 Three plots are shown for thermistors having $\beta = 5{,}000$K and $\beta = 2{,}500$K, as well as copper. The large, negative temperature variation for the thermistors is

evident from the values of R_T/R_{300} on a logarithmic scale. The range of values is: 4,160 to 0.0155 ($\beta = 5{,}000$), 64.5 to 0.125, and 0.61 to 1.39 (copper).

P1.4.1 Let the voltage between the plates be V; $\xi = \dfrac{V}{10^{-3}} =$

10^3 V/m; D in space filled with air is $10^3 \varepsilon_0 V$ C/m^2;

charge in this region is $10^3 \varepsilon_0 \times 2 \times 5 \times 10^{-4} V = \varepsilon_0 V$ C; D

in space filled with dielectric is $10^4 \varepsilon_0 V$ C/m^2;

charge in this region is $10^4 \varepsilon_0 \times 3 \times 5 \times 10^{-4} V = 15 \varepsilon_0 V$ C.

Total charge is $16 \varepsilon_0 = 16 \times 8.85 \times 10^{-12} =$

$141.6 \times 10^{-12} \times V$ C; Capacitance $= \dfrac{141.6 \times 10^{-12} V}{V} \equiv$

0.14 nF.
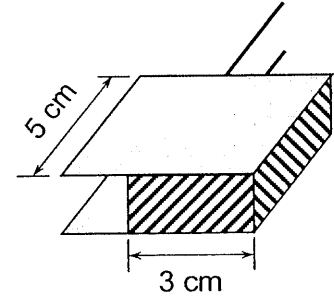
P1.4.2 C/unit area $= \dfrac{\varepsilon}{d}$, or ε = 1 (μF/cm^2)$\times$8 (nm) = $10^{-6} \times 10^4$ (F/m^2)$\times 8 \times 10^{-9}$ (m) =

8×10^{-11} F/m. The relative permittivity is $\dfrac{8 \times 10^{-11}}{8.85 \times 10^{-12}} = 9$.

P1.4.3 Capacitance of a cylindrical shell of radius r,

thickness dr, and length l is $\dfrac{\varepsilon \times 2\pi r l}{dr} = dC$. It is

convenient to consider the reciprocal of the

capacitance. Thus $d\left(\dfrac{1}{C}\right) = \dfrac{1}{2\pi \varepsilon l} \dfrac{dr}{r}$, and

$\dfrac{1}{C} = \dfrac{1}{2\pi \varepsilon l} \int_a^b \dfrac{dr}{r}$; $\dfrac{1}{C} = \dfrac{1}{2\pi \varepsilon l} \ln \dfrac{b}{a}$, or $C = \dfrac{2\pi \varepsilon l}{\ln\left(\dfrac{b}{a}\right)}$.
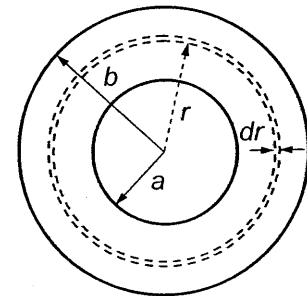
P1.4.4 Minimum capacitance is when dielectric cylinder is

completely out. $C_{min} = \dfrac{2\pi \times 8.85 \times 10^{-12} \times 5 \times 10^{-2}}{\ln(1.2)} =$

15.25 pF; $C_{max} = 100 C_{min} = 1.525$ nF.
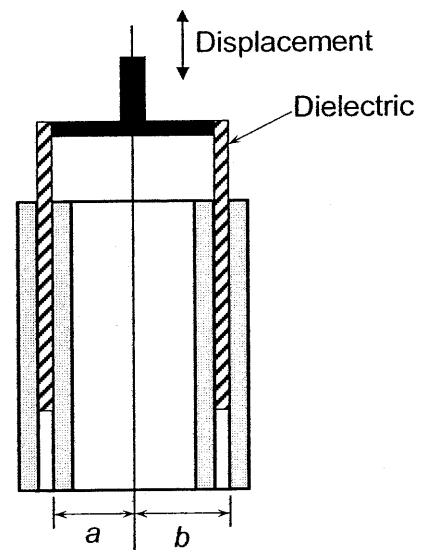
P1.4.5 $i = C\dfrac{dv}{dt} = 2\left[-10Ate^{-10t} + Ae^{-10t} - 10Be^{-10t}\right]$ mA,

$t \geq 0$.

At $t = 0$: $v = 10$ V $\Rightarrow B = 10$ V

$i = 200$ mA $\Rightarrow 2(A - 10B) = 200 \Rightarrow A = 200$ V/s.

P1.4.6 $i = \dfrac{dq}{dt} = \dfrac{d}{dt}(Cv) = \dfrac{d}{dt}\left[C_0\left(1 - e^{-\alpha t}\right)V_0 t^2\right]$

$= C_0 V_0 \dfrac{d}{dt}\left(t^2 - t^2 e^{-\alpha t}\right) = C_0 V_0\left(2t - 2te^{-\alpha t} + \alpha t^2 e^{-\alpha t}\right)$.

P1.4.7 $q = (5\ \mu\text{F})(20\ \text{V}) = 100\ \mu\text{C}$; each pulse supplies 20 μC; hence $\dfrac{100}{20} = 5$ pulses are required.

P1.4.8 $v = \dfrac{1}{C}\displaystyle\int_0^t i\,dt = \dfrac{1}{2\times 10^{-6}}\displaystyle\int_0^t 100\times 10^{-6}dt = 50t$ V; $0 \leq t \leq 200$ ms. At 200 ms, $v = 50\times 200\times 10^{-3} = 10$ V, so $v = 10$ V for $t \geq 200$ ms.

P1.4.9 Differentiating the above, $v = 50$ V for $0 < t < 200$ ms and $v = 0$, $t > 200$ ms. Note that the response at $t = 0$ and at $t = 200$ ms, when the impulses occur, is not defined. Alternatively, the response to the first impulse is

$v = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + V(0) = \dfrac{1}{2}\displaystyle\int_0^{0^+} 100\delta(t)dt = 50$ V, $0 < t < 200$. The response to the

second impulse is $= \dfrac{1}{2}\displaystyle\int_{0.2^-}^{0.2^+} 100\delta(t - 0.2)dt + 50 = 0$, $t > 200$ ms. The complete

response may be written as: $v = 50u(t) - 50u(t - 0.2)$ V.

P1.4.10 <u>Current pulse</u>: $v = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + v(0) = \dfrac{1}{2\times 10^{-6}}\displaystyle\int_0^t 100\times 10^{-6}dt - 10 = 50t - 10$ V;

$0 \leq t \leq 200$ ms. At 200 ms, $v = 10 - 10 = 0$, so $v = 0$ for $t \geq 200$ ms.

<u>Current impulses</u>: Differentiating the above, $v = 50$ V for $0 < t < 200$ ms and $v = 0$, $t > 200$ ms. Adding the initial voltage of -10 V gives: $v = 40$ V; $0 < t < 200$ ms and $v = -10$ V; $t > 200$ ms. Alternatively, the response may be

obtained as: $v = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + V(0) = \dfrac{1}{2}\left(\displaystyle\int_{0^-}^{0^+} 100\delta(t)dt + \displaystyle\int_{0.2^-}^{0.2^+} -\delta(t - 0.2)dt\right) - 10 =$

$= 50u(t) - 50u(t - 0.2) - 10$ V.

P1.4.11 $\quad v = \frac{1}{C}\int_0^t i\,dt + V(t_0)$

(a) $0 \le t \le 10\,\mu s$:

$v = \frac{1}{0.5}\int_0^t 1.5t\,dt =$

$1.5t^2$ mV, where

t is in μs. At $t = 10$

μs, $v = 150$ mV.

$10 \le t \le 40\,\mu s$:

$v = \frac{1}{0.5}\int_{10}^t (-t+25)\,dt + 150$

$\quad = \left[-t^2 + 50t\right]_{10}^t + 150 = -t^2 + 50t - 250$ mV. At $t = 40\,\mu s$, $v = 150$ mV.

$40 \le t \le 60\,\mu s$: $v = \frac{1}{0.5}\int_{40}^t -15\,dt + 150 = \left[-30t\right]_{40}^t + 150 = -30t + 1350$ mV.

At $t = 60\,\mu s$, $v = -450$ mV.

$60 \le t \le 80\,\mu s$: $v = \frac{1}{0.5}\int_{60}^t (0.75t - 60)\,dt - 450$

$\quad = \left[0.75t^2 - 120t\right]_{60}^t - 450 = 0.75t^2 - 120t + 4050$ mV. At $t = 80\,\mu s$, $v =$

-750 mV.

$t \ge 80\,\mu s$: $v = -750$ mV.

Check: Total area = $0.5\times15\times10 + 0.5\times15\times15 - 0.5\times15\times15 - 15\times20 -$

$0.5\times15\times20 = -375$ nC. Hence $v = \frac{-375}{0.5} = -750$ mV.

(b) At $t = 10\,\mu s$, $v = 150$ mV, so $q = CV = 75$ nC.

At $t = 50\,\mu s$, $v = -150$ mV, so $q = CV = -75$ nC.

(c) At $t = 80\,\mu s$, $v = -750$ mV, so $w = \frac{1}{2}Cv^2 = 0.14\,\mu J$.

(d) All the expressions derived above for the voltage are increased by 0.5 V.
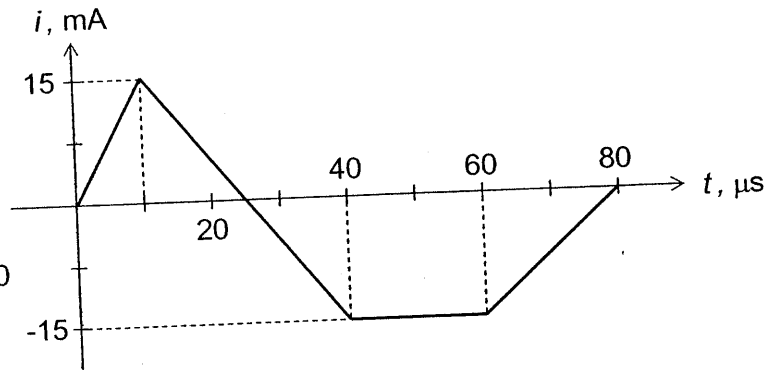
P1.4.12 $v = \dfrac{t}{6}$ V, $0 \le t \le 60$ s;

$$= -\dfrac{t}{18} + \dfrac{40}{3}\, \text{V},$$

$60 \le t \le 240$ s;

$= 0, \; t \ge 240$ s.
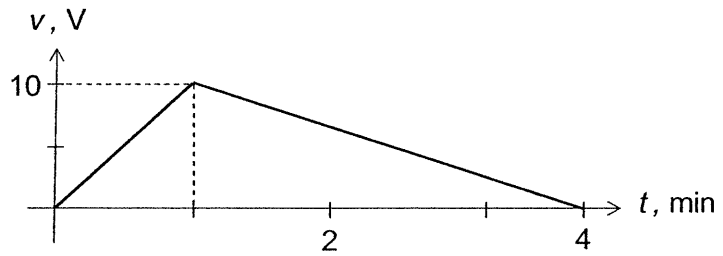
(a) $q = \dfrac{t}{60}\,\mu\text{C},$

$0 \le t \le 60$ s;

$$= -\dfrac{t}{180} + \dfrac{4}{3}\,\mu\text{C},$$
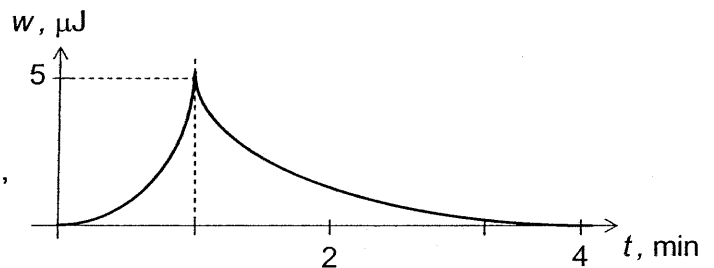
$60 \le t \le 240$ s;

$= 0, \; t \ge 240$ s.
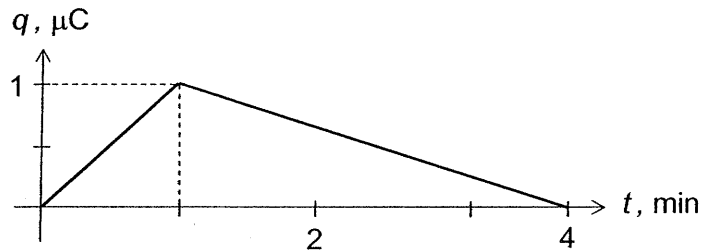
(b) $w = \dfrac{1}{2}qv = \dfrac{t^2}{720}\,\mu\text{J},$

$0 \le t \le 60$ s;

$$= \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right)\mu\text{J},$$

$60 \le t \le 240$ s;

$= 0, \; t \ge 240$ s.

(c) $i = \dfrac{dq}{dt} = \dfrac{1}{60}\,\mu\text{A},$

$0 < t < 60$ s;

$$= -\dfrac{1}{180}\,\mu\text{A},$$

$60 < t < 240$ s;

$= 0, \; t > 240$ s,
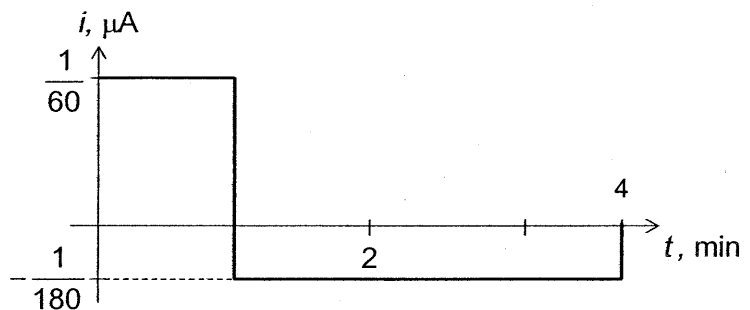
where $\dfrac{1}{60}\,\mu$A may also be expressed as 1 μC/min.

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(d) $p = vi = \dfrac{t}{360}\,\mu W,$

$0 \le t \le 60\,s;$

$= \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)\mu W,$

$60 \le t \le 240\,s;$

$= 0,\ t \ge 240\,s,$ where
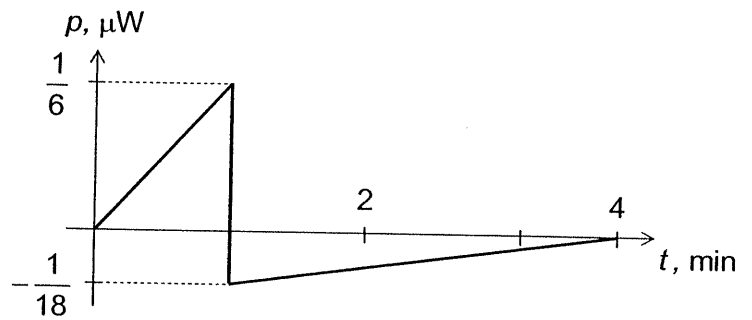
$\dfrac{1}{6}\,\mu W$ may also be expressed as 10 μJ/min.

It is seen that $w = \int p\,dt$. Thus $\displaystyle\int_0^t \dfrac{t}{360}dt = \dfrac{t^2}{720}\,\mu J,\ 0 \le t \le 60\,s.$ At $t = 60\,s,$

$w = 5\,\mu J.$ For $60 \le t \le 240\,s$ $w = \displaystyle\int_{60}^t \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)dt + 5$

$= \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right)\mu J.$

P1.4.13 (a) $q = Cv = 0.1\times10^{-6}\times10\sin(500t) \equiv$

$\sin(500t)\,\mu C,\ 0 \le t \le 2\pi\,ms,$ and $q = 0$

elsewhere.
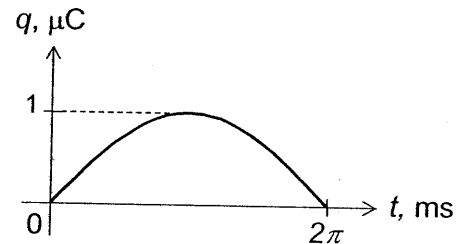
(b) $w = \dfrac{1}{2}qv = 5\sin^2(500t)\,\mu J,$
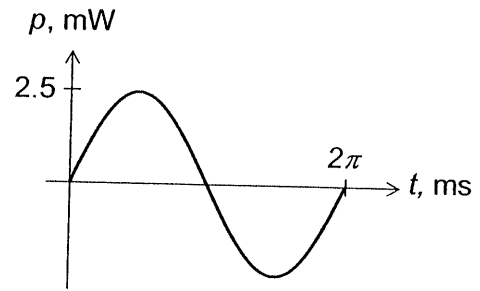
$0 \le t \le 2\pi\,ms,$ and $w = 0$ elsewhere.
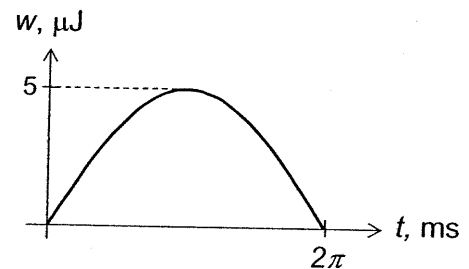
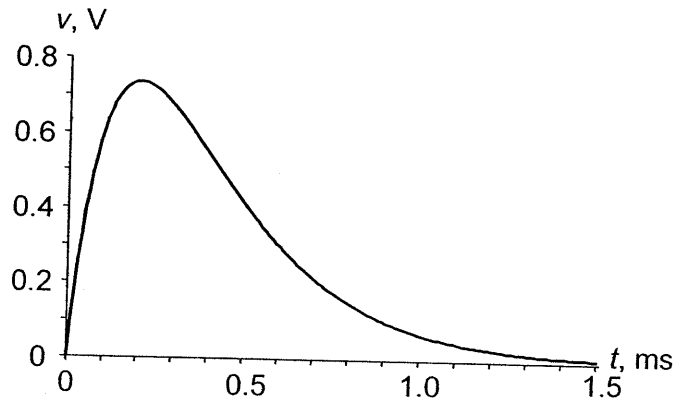(c) $i = \dfrac{dq}{dt} = 500\cos(500t)\,\mu A,$

$0 \le t \le 2\pi\,ms,$ and $i = 0$ elsewhere.
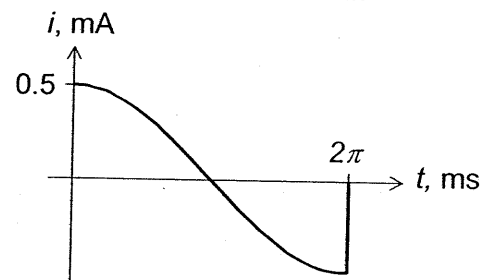
(d) $p = 10\sin(500t) \times 0.5 \times \cos(500t)$

$= 5\sin(500t)\cos(500t) =$

$2.5\sin(1000t)$ mW,

$0 \le t \le 2\pi$ ms, and $p = 0$, elsewhere.

$w = \int p\,dt$. Thus $\int\limits_{0}^{t} 2.5\sin(1000t)\,dt =$

$\dfrac{2.5}{1000}\big[1 - \cos(1{,}000t)\big] \equiv 5\sin^2(500t)$ μJ, as above.

P1.4.14 $v = 10te^{-5t}$ V;

$q = Cv = 50te^{-5t}$ μC;

$i = \dfrac{dq}{dt} = 50\big(e^{-5t} - 5te^{-5t}\big)$

μC/ms, or mA;

$w = \dfrac{1}{2}qv = 250t^2 e^{-10t}$ μJ;

$p = vi =$

$500\left(te^{-10t} - 5t^2e^{-10t}\right)m$

W.
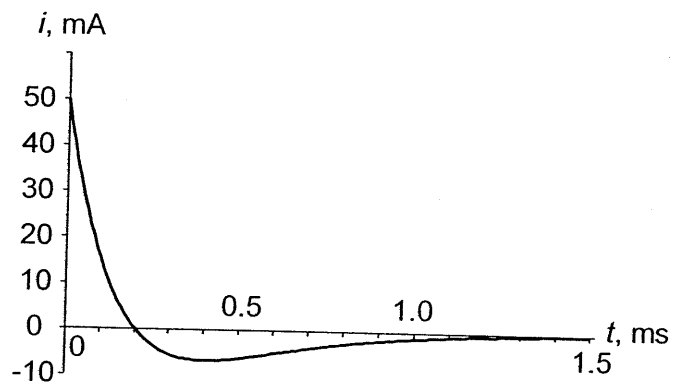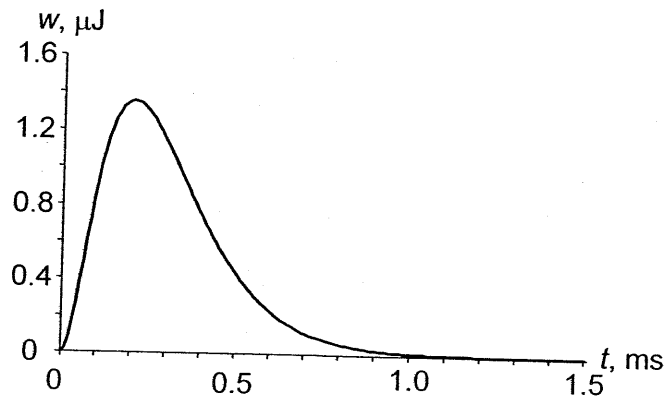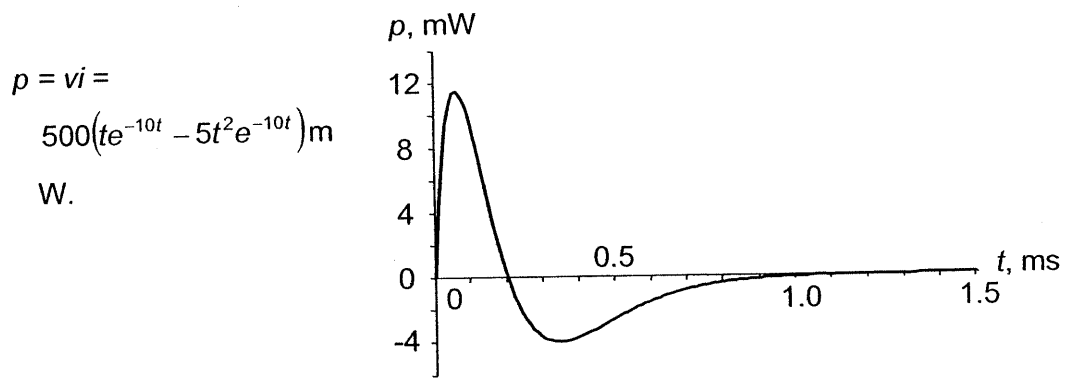
P1.4.15 (a) $q_C = \int_0^t i_C dt = \int_0^t 10te^{-5t}dt = 10\left[-t\frac{e^{-5t}}{5}\right]_0^t + 2\int_0^t e^{-5t}dt = -2te^{-5t} - \frac{2}{5}e^{-5t} + \frac{2}{5}$

μC. $v_C = \frac{q_C}{C} = -\frac{2}{5}te^{-5t} - \frac{2}{25}e^{-5t} + \frac{2}{25}$V.

(b) i_C is max when $\frac{di_C}{dt} = 0$, or $e^{-5t} - 5te^{-5t} = 0$, which gives $t = 0.2$ ms. At

this value of t, $v_C = -\frac{0.4}{5}e^{-1} - \frac{2}{25}e^{-1} + \frac{2}{25} \equiv 21.1$ mV.

To verify using MATLAB, enter syms t, then (1/5)*int(10*t*exp(-5*t),0,0.2).

P1.4.16 $q = 0.5v^2 = 0.5(1+t)^2$; $i = \frac{dq}{dt} = (1+t)$; $p = vi = (1+t)^2$; $w = \int_0^t pdt =$

$\int_0^t (1+t)^2 dt = \frac{1}{3}\left[(1+t)^3\right]_0^3 = \frac{63}{3} = 21$ J.

Alternatively, the work done in increasing the charge on the capacitor by dq is

vdq, and the total energy stored is $w = \int vdq$. But $dq = vdv$. Hence,

$w = \int_1^4 v^2 dv$, where $v = 1$ when $t = 0$ and $v = 4$ when $t = 3$. This gives

$w = \frac{1}{3}\left[(t+1)^3\right]_1^4 = 21$J.

P1.5.1 $\lambda = 100 \times 10^{-6} = 10^{-4}$ Wb-turns; $L = \frac{\lambda}{I} = \frac{10^{-4}}{10^{-2}} = 0.01$ H.

P1.5.2 $v = L\frac{di}{dt} = 2(-10Ate^{-10t} + Ae^{-10t} - 10Be^{-10t})$, $t \geq 0$.

At $t = 0$: $i = 10$ A $\Rightarrow B = 10$ A

$v = 200$ mV $\Rightarrow 2(A - 10B) = 200 \Rightarrow A = 200$ A/s.

P1.5.3
$$v = \frac{d\lambda}{dt} = \frac{d}{dt}(Li) = \frac{d}{dt}\left[L_0\left(1 - e^{-\alpha t}\right)I_0 t^2\right]$$

$$= L_0 I_0 \frac{d}{dt}\left(t^2 - t^2 e^{-\alpha t}\right) = L_0 I_0\left(2t - 2te^{-\alpha t} + \alpha t^2 e^{-\alpha t}\right) \text{ V.}$$

P1.5.4 $\lambda = (5 \ \mu\text{H})(20 \ \text{A}) = 100 \ \mu\text{Wb-turns}$; each pulse supplies 20 μWb-turns; hence

$\frac{100}{20} = 5$ pulses are required.

P1.5.5 $i = \frac{1}{L}\int_0^t v dt = \frac{1}{2 \times 10^{-6}}\int_0^t 100 \times 10^{-6} dt = 50t$ A; $0 \le t \le 200$ ms. At $t = 200$ ms,

$i = 50 \times 200 \times 10^{-3} = 10$ A, so $i = 10$ A for $t \ge 200$ ms.

P1.5.6 $i = \frac{1}{L}\int_0^t v dt + I(0) = \frac{1}{2}\left(\int_{0^-}^{0^+} 100\delta(t)dt + \int_{0.2^-}^{0.2^+} -\delta(t - 0.2)dt\right) =$

$= 50u(t) - 50u(t - 0.2)$ A. Alternatively, we can differentiate the response of

the preceding problem to obtain $i = 50$ A for $0 < t < 200$ ms and $i = 0$, $t > 200$

ms.

P1.5.7   <u>Voltage pulse</u>: $i = \frac{1}{L}\int_0^t v dt + i(0) = \frac{1}{2 \times 10^{-6}}\int_0^t 100 \times 10^{-6} dt - 10 = 50t - 10$ A; $0 \le t$

≤ 200 ms. At 200 ms, $i = 10 - 10 = 0$, so $i = 0$ A for $t \ge 200$ ms.

<u>Voltage impulses</u>: Differentiating the above, $i = 50$ A for $0 < t < 200$ ms and $i =$

0, $t > 200$ ms. Adding the initial current of -10 A gives: $i = 40$ A; $0 < t < 200$

ms and $i = -10$ A; $t > 200$ ms. Alternatively, the response may be obtained as:

$i = \frac{1}{L}\int_0^t v dt + I(0) = \frac{1}{2}\left(\int_{0^-}^{0^+} 100\delta(t)dt + \int_{0.2^-}^{0.2^+} -\delta(t - 0.2)dt\right) - 10 =$

$= 50u(t) - 50u(t - 0.2) - 10$ A.

P1.5.8 $i = \frac{1}{L}\int_0^t v dt + I(t_0)$

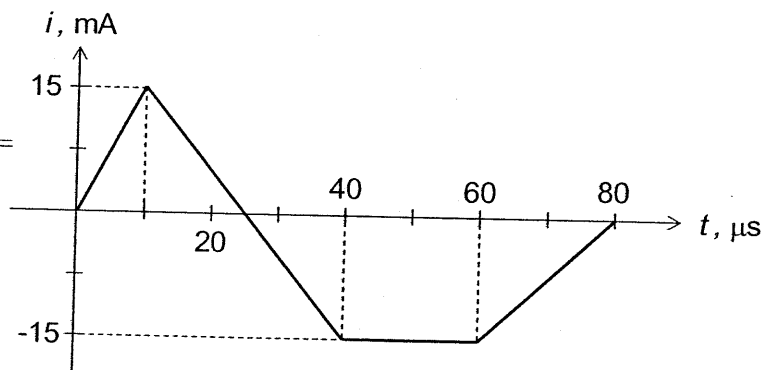(a) $0 \le t \le 10$ μs:

$i = \frac{1}{0.5}\int_0^t 1.5t dt =$

$1.5t^2$ mA,

where t is in μs.

At $t = 10$ μs, $i =$

150 mA.

$10 \le t \le 40 \; \mu s$:

$$i = \frac{1}{0.5} \int_0^t (-t_1 + 25)dt + 150 \quad = \left[-t^2 + 50t\right]_{10}^t + 150$$

$$= -t^2 + 50t - 250 \text{ mA. At } t = 40 \; \mu s, \, v = 150 \text{ mA}$$

$40 \le t \le 60 \; \mu s, \; i = \frac{1}{0.5} \int_{40}^t -15dt + 150 \quad = = \left[-30t\right]_{40}^t + 150 = -30t + 1350$

mA. At $t = 60 \; \mu s$, $i = -450$ mA

$60 \le t \le 80 \; \mu s$: $i = \frac{1}{0.5} \int_{60}^t (0.75t - 60)dt - 450 \quad = \left[0.75t^2 - 120t\right]_{60}^t - 450 =$

$0.75t^2 - 120t + 4050$ mA. At $t = 80 \; \mu s$, $i = -750$ mA.

For $t \ge 80 \; \mu s$, $i = -750$ mA

(b) At $t = 10 \; \mu s$, $i = 150$ mA, so $\lambda = Li = 0.5 \; (\mu H) \times 150$ mA $= 75$ nWb-turns.

At $t = 50 \; \mu s$, $i = -30 \times 50 + 1350 = -150$ mA, so $\lambda = -75$ nWb-turns.

(c) At $t = 80 \; \mu s$, $i = -750$ mA, so $w = \frac{1}{2}Li^2 = 0.14 \; \mu J$.

(d) All the expressions derived above for the current are increased by 0.5 A.

P1.5.9 $i = \dfrac{t}{6}$ A, $0 \le t \le 60$ s;

$= -\dfrac{t}{18} + \dfrac{40}{3}$ A,

$60 \le t \le 240$ s;

$= 0, \; t \ge 240$ s.
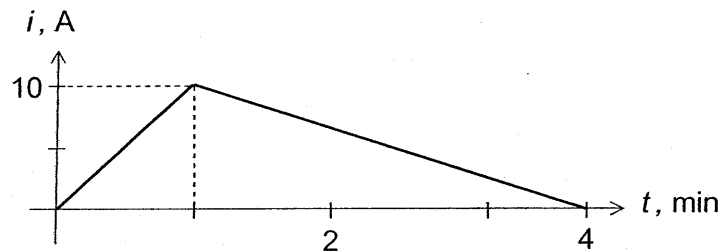
(a) $\lambda = \dfrac{t}{60} \; \mu$Wb-turns,

$0 \le t \le 60$ s;

$= -\dfrac{t}{180} + \dfrac{4}{3} \; \mu$Wb-turns, $60 \le t \le 240$ s $= 0, \; t \ge 240$ s.

(b) $w = \dfrac{1}{2}qv = \dfrac{t^2}{720} \; \mu J$,

$0 \le t \le 60$ s;

$= \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right) \mu J$,

$60 \le t \le 240$ s;

$= 0$, $t \geq 240$ s.

(c) $v = \dfrac{d\lambda}{dt} = \dfrac{1}{60}$ µV,

$0 < t < 60$ s;

$= -\dfrac{1}{180}$ µv,

$60 < t < 240$ s;

$= 0$, $t > 240$ s,

where $\dfrac{1}{60}$ µV may also be expressed as 1 µWb-turns/min.

(d) $p = vi = \dfrac{t}{360}$ µW,

$0 \leq t \leq 60$ s;

$= \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)$ µW,

$60 \leq t \leq 240$ s;

$= 0$, $t \geq 240$ s, where

$\dfrac{1}{6}$ µW may also be

expressed as 10 µJ/min.

t is seen that $w = \int p\,dt$. Thus $\int_0^t \dfrac{t}{360}dt = \dfrac{t^2}{720}$ µJ, $0 \leq t \leq 60$ s. At $t = 60$ s,

$w = 5$ µJ. For $60 \leq t \leq 240$ s $w = \int_{60}^t \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)dt + 5$
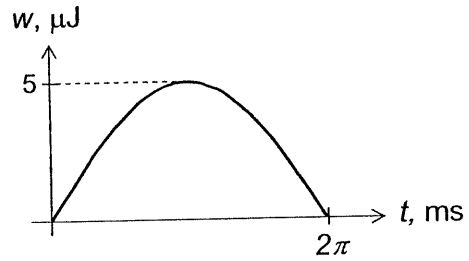
$= \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right)$ µJ.

P1.5.10 (a) $\lambda = Li = 0.1\times10^{-6}\times10\sin(500t) \equiv$

$\sin(500t)$ µWb-turns, $0 \leq t \leq 2\pi$ ms,

and $\lambda = 0$ elsewhere.
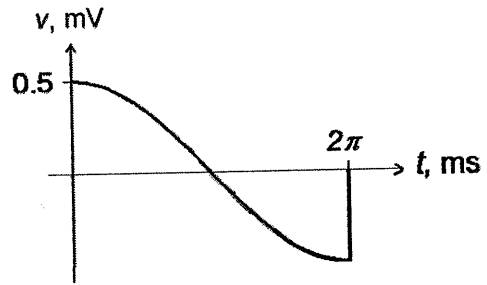
(b) $w = \dfrac{1}{2}\lambda i = 5\sin^2(500t)$ μJ,

$0 \le t \le 2\pi$ ms, and $w = 0$

elsewhere.

(c) $v = \dfrac{d\lambda}{dt} = 500\cos(500t)$ μV,

$0 \le t \le 2\pi$ ms, and $i = 0$

elsewhere.
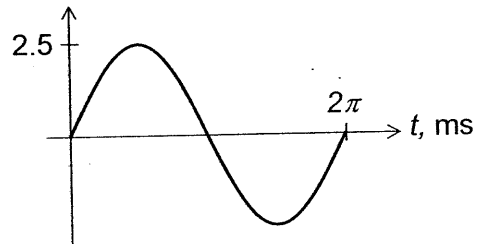

(d) $p = 10\sin(500t) \times 0.5 \times \cos(500t)$

$= 5\sin(500t)\cos(500t) =$

$2.5\sin(1000t)$ mW,

$0 \le t \le 2\pi$ ms, and $p = 0$,

elsewhere.

$w = \int p\,dt$. Thus $\displaystyle\int_0^t 2.5\sin(1000t)\,dt =$

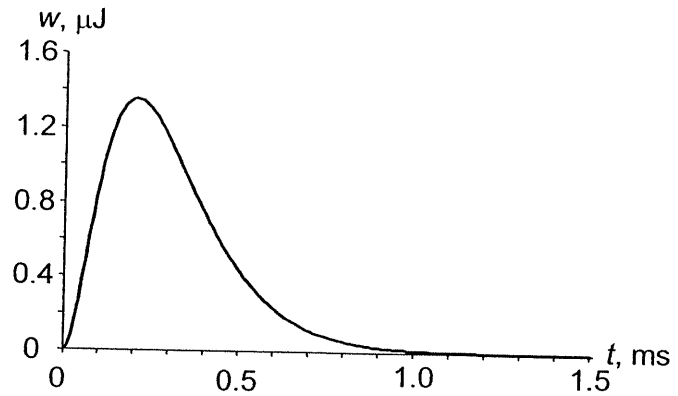$\dfrac{2.5}{1000}\left[1 - \cos(1{,}000t)\right] \equiv 5\sin^2(500t)$ μJ, as above.

P1.5.11 $i = 10te^{-5t}$ A

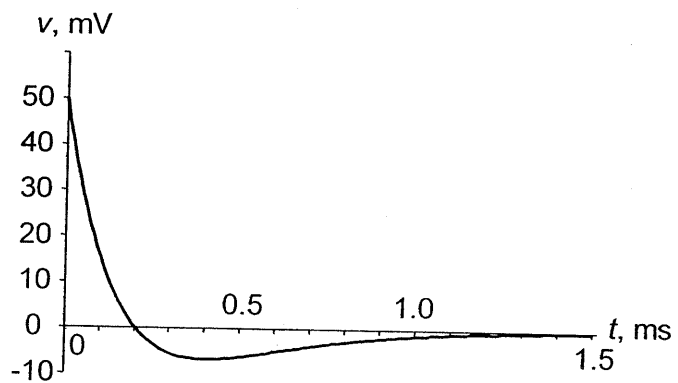$\lambda = Li = 50te^{-5t}$ μWb-

turns;

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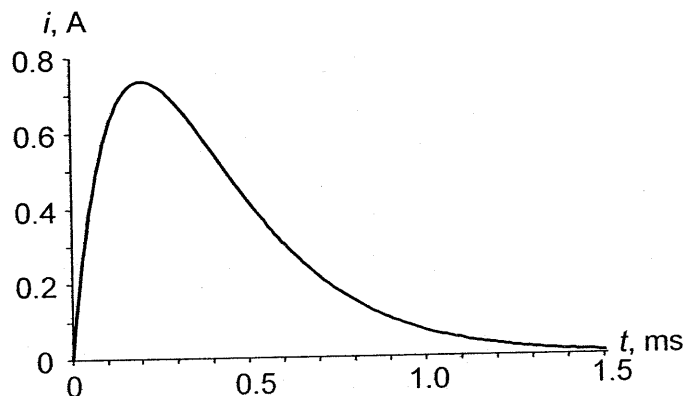
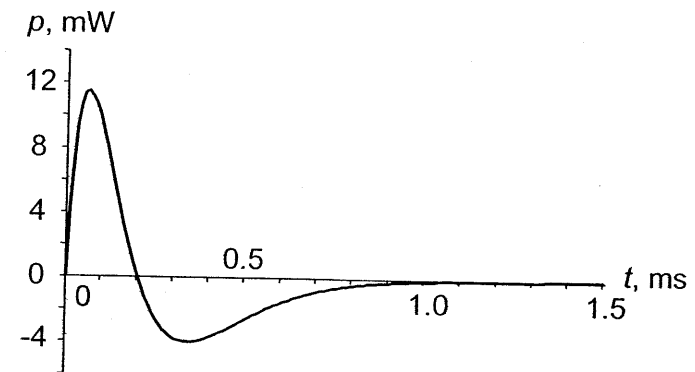$$w = \frac{1}{2}\lambda i = 250t^2 e^{-10t} \; \mu J;$$

$$v = \frac{d\lambda}{dt} = 50\left(e^{-5t} - 5te^{-5t}\right)$$

$$\mu\text{Wb-turns/ms, or mV;}$$

$$p = vi =$$

$$500\left(te^{-10t} - 5t^2 e^{-10t}\right) \; mW.$$

P1.5.12   (a) $\lambda_L = \int_0^t v_L dt = \int_0^t 10te^{-5t}dt = 10\left[-t\frac{e^{-5t}}{5}\right]_0^t + 2\int_0^t e^{-5t}dt = -2te^{-5t} - \frac{2}{5}e^{-5t} + \frac{2}{5}$

μWb-turns. $i_L = \frac{\lambda_L}{L} = -\frac{2}{5}te^{-5t} - \frac{2}{25}e^{-5t} + \frac{2}{25}$ A.

(b) v_L is max when $\frac{dv_L}{dt} = 0$, or $e^{-5t} - 5te^{-5t} = 0$, which gives $t = 0.2$ ms. At

this value of t, $i_L = -\frac{0.4}{5}e^{-1} - \frac{2}{25}e^{-1} + \frac{2}{25} \equiv 21.1$ mA.

P1.5.13 $\lambda = 0.5i^2 = 0.5(1+t)^2$; $v = \dfrac{d\lambda}{dt} = (1+t)$; $p = vi = (1+t)^2$; $w = \int_0^t pdt =$

$$\int_0^t (1+t)^2 dt = \frac{1}{3}\Big[(1+t)^3\Big]_0^3 = \frac{63}{3} = 21 \text{ J.}$$

Alternatively, the work done in increasing the flux linkage in the inductor by $d\lambda$ is $id\lambda$, and the total energy stored is $w = \int id\lambda$. But $d\lambda = idi$. Hence,

$w = \int_1^4 i^2 di$, where $i = 1$ when $t = 0$ and $i = 4$ when $t = 3$. This gives

$w = \dfrac{1}{3}\Big[(t+1)^3\Big]_1^4 = 21\text{J.}$

P1.6.1 The number of molecules in one gram-molecular weight is given by Avogadro's number (6.025×10^{23}). A 0.1 M solution of $CuCl_2$ contains $0.1(6.025 \times 10^{23})$ Cu^{++} ions per liter and $0.2(6.025 \times 10^{23})$ Cl^- ions per liter.

From Eq. (1.4.1), $i = Aune = 1 \times \left(\dfrac{0.2}{60}\right) \times \left(\dfrac{0.1 \times 6.025 \times 10^{23}}{1000}\right) \times \left(2 \times 1.6 \times 10^{-19}\right) +$

$1 \times \left(-\dfrac{0.4}{60}\right) \times \left(\dfrac{0.2 \times 6.025 \times 10^{23}}{1000}\right) \times \left(-1.6 \times 10^{-19}\right) = 0.19 \text{ A.}$