Chapter 2

2/1 $\lambda = \frac{h}{p}$, $E = \frac{p^2}{2m} \rightsquigarrow p = \sqrt{2Em}$ $\Big\}$ $\boxed{\lambda = \frac{h}{\sqrt{2Em}}}$ Electron wave!

$$E = 4(eV) = 4 \times 1.602 \times 10^{-19} (J) \equiv \left(\frac{Kg\, m^2}{s^2}\right)$$

$$h = 6.625 \times 10^{-34} (J \cdot s)$$

$$m = 9.11 \times 10^{-31} (Kg)$$

$$\lambda = \frac{h}{\sqrt{2Em}} \left[\frac{Kg \cdot m^{\cancel{2}} \cdot \cancel{s} \cdot s}{\cancel{s^2} \cdot Kg^{\frac{1}{2}} \cdot m \cancel{Kg^{\frac{1}{2}}}}\right] \equiv [m]$$

$$\lambda = 0.6131 \times 10^{-9} (m) = \underline{6.13' (\mathring{A})} = 6.13' \times 10^{-10} (m) = 6.13' \times 10^{-8} (cm)$$

2/2 $E = \frac{p^2}{2m}$, $p = \frac{h}{\lambda}$ $\Big\}$ $\boxed{E = \frac{h^2}{2m\lambda^2}}$ $\left[\frac{Kg^2\, m^4\, s^2}{s^4\, m^2}\right] \equiv \left[\frac{Kg\, m^2}{s^2}\right] \equiv [J]$

$\lambda = 600 (nm) = 6 \times 10^{-7} (m)$ Electron wave

h and m see above

$$E = 6.69 \times 10^{-25} [J] = \underline{4.176 \times 10^{-6} (eV)}$$

Note; In case of λ = const: a smaller mass causes a larger energy.

2/3 Electromagnetic waves (light) can be perceived by cones and rods at the retina of the human eye. Electron waves do not stimulate the cones and rods. Electron waves need a "phosphor" (ZnS) or a diffraction device for detection.

2/4 $E = \nu \cdot h$, $c = \nu \cdot \lambda$ $\Big\}$ $\boxed{E = \frac{c}{\lambda} h}$ $\left[\frac{m \cdot Kg \cdot m^2 \cdot s}{s \quad s^2 \quad m}\right] \equiv [J]$

$c = 2.998 \times 10^8 \left(\frac{m}{s}\right)$

$h = 6.625 \times 10^{-34} (J \cdot s)$

$\lambda = 600 (nm) = 6 \times 10^{-7} (m)$

$$\underline{E_{ph} = 2.066 (eV)}$$

$$\underline{= 3.31 \times 10^{-19} (J)}$$

Light wave!

The difference is the mass and the velocity (see also 2/2)

Mass of electron $m_{el} = 9.11 \times 10^{-31}$ Kg ; $v_{el} = \sqrt{\frac{2E}{m}} = 1.2 \times 10^3 \left(\frac{m}{s}\right)$

Apparent mass of photon $(m_{ph})_{eff} = \frac{E}{c^2} = 3.68 \times 10^{-36}$ Kg ; $v_{ph} = c = 3 \times 10^8 \left(\frac{m}{s}\right)$ (photon has no rest mass)

2/5

$$\left.\begin{aligned} E &= \tfrac{1}{2} m v^2 \\ \lambda &= \frac{h}{\sqrt{2Em}} \end{aligned}\right\} \boxed{\lambda = \frac{h}{mv}} \quad \left[\frac{g\,cm^2\,s}{s\,g\,cm}\right] \equiv [cm]$$

$$h = 6.626 \times 10^{-27} \left[\frac{g\,cm^2}{s}\right]$$

$$v = 200\left[\frac{km}{h}\right] = 200 \times 10^5 \left[\frac{cm}{h}\right] = \frac{2 \times 10^7}{3600}\left[\frac{cm}{s}\right]$$

$$m = 50[g]$$

$$\lambda = 2.38 \times 10^{-32}[cm] = 2.38 \times 10^{-24}[\mathring{A}]$$

2/7

$$\left.\begin{aligned} E &= \nu h \\ E &= mc^2 \\ p &= mc \\ c &= \lambda \nu \end{aligned}\right\} \quad \nu h = mc^2 = mc \cdot c = p \cdot c = p \lambda \nu$$

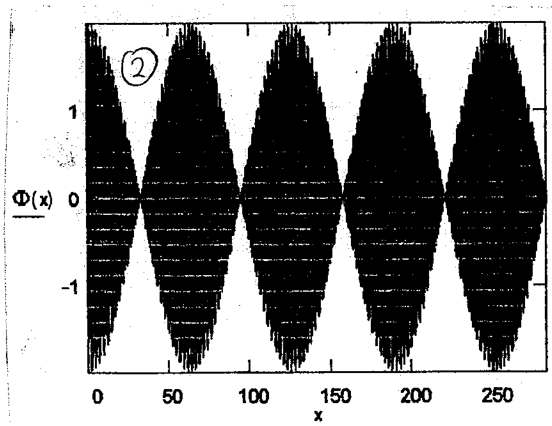
$$\rightsquigarrow \cancel{\nu} h = p \lambda \cancel{\nu} \rightsquigarrow \boxed{h = p \cdot \lambda}$$

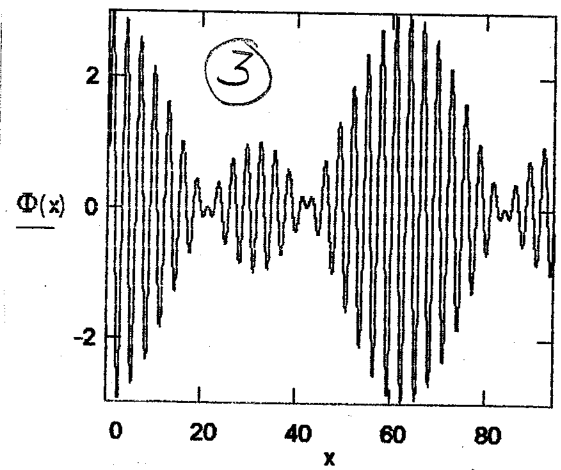
2/8

$$\Phi(x) := \sum_i \sin((k+i) \cdot x)$$

Superposition of ψ waves. The number of ψ-waves is given in the graph.

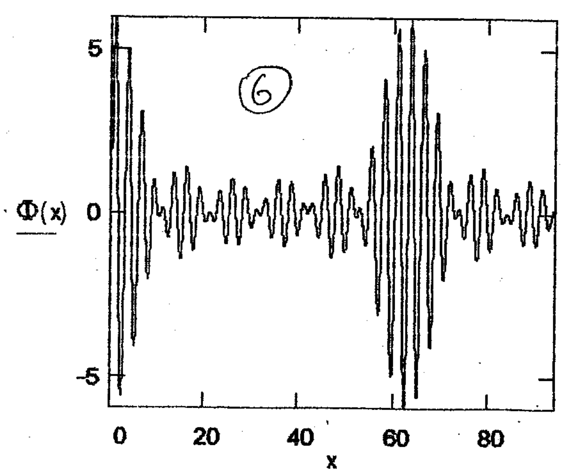
$i = 1, 1.1 \ldots 1.1$
$k = 100$
$x = 0, 01 \ldots 90\pi$

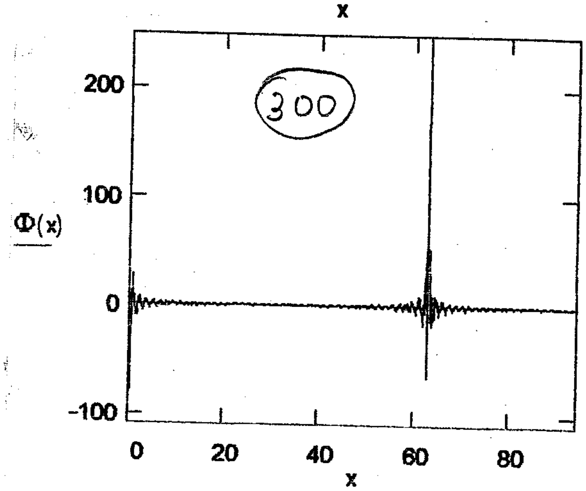




$i = 1, 1.1 \ldots 1.2$
$k = 1$
$x = 0, 0.02 \ldots 30\pi$

$i = 1, 1.1 \ldots 30$
$k = 100$
$x = 0, 0.15 \ldots 30\pi$





$i = 1, 1.1 \ldots 1.5$
$k = 1$
$x = 0, 0.02 \ldots 30\pi$

Chapter 3

3/1 see Appendix A1

3/2 A Vibration is a time or space dependent periodic disturbance.
A Wave is a time and space dependent periodic disturbance.

3/3 The damped vibration has a velocity-dependent term: $m\frac{d^2u}{dt^2} + \gamma\frac{du}{dt} + Ku = 0$

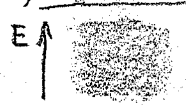
3/4 $\hat{x}^* = a - bi$

$\psi^* = -2Ai\sin\alpha x$

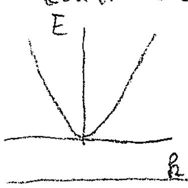
Chapter 4

4/1

a) Free electron case

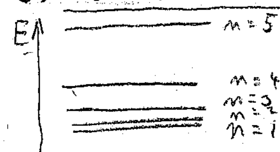


Continuous energy area

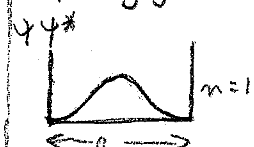


$$\boxed{E = \frac{\hbar^2}{2m} k^2}$$

b) Bound Electron case

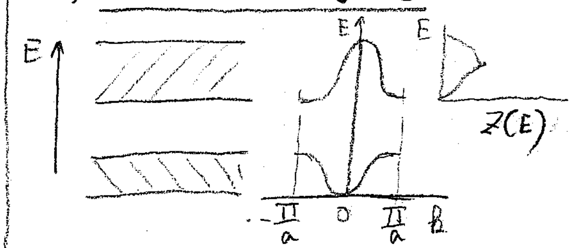


Energy levels



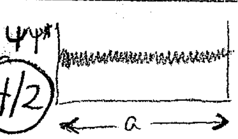
$$\boxed{E = \frac{\hbar^2\pi^2}{2ma^2} n^2}$$

c) Electrons in crystals



Energy bands

4/2



almost classical case

4/3

I) $\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$

Sol. $\psi(x) = Ae^{ikx}$

where $k = \sqrt{\frac{2mE}{\hbar^2}}$

$\longleftrightarrow$

II) $\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$

Sol. $\psi(x) = Ae^{ikx}$

where $k = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

This is possible only for $V_0 = 0$ i.e. $\psi(x) = Ae^{ikx}$ is a particular solution of the problem which is that for free electrons.