Chapter 2

2.1 *A manufacturer estimates that its variable cost for manufacturing a given product is given by the following expression:* $C(q) = 25q^2 + 2000q$ [\$] *where C is the total cost and q is the quantity produced.*

a. Derive an expression for the marginal cost of production.

The marginal cost of production is the rate of change of the cost with respect of the quantity produced; therefore:

$$MC(q) = \frac{d\,C(q)}{dq} = 50q + 2000$$

b. Derive expressions for the revenue and the profit when the widgets are sold at marginal cost.

Since the widgets are sold at marginal cost, $\pi = MC(q)$
The revenue is then given by:

$$revenue = \pi\, q = (50q + 2000)\, q = 50q^2 + 2000q$$

Since the profit is the difference between the revenue and the production cost, we have:

$$profit = (50q^2 + 2000q) - (25q^2 + 2000q) = 25q^2$$

2.2 *The inverse demand function of a group of consumers for a given type of widgets is given by the following expression:* $\pi = -10q + 2000$ [\$/unit] *where q is the demand and* π *is the unit price for this product.*

a. Determine the maximum consumption of these consumers

The inverse demand function can be represented by a straight line with a negative slope, as shown below:

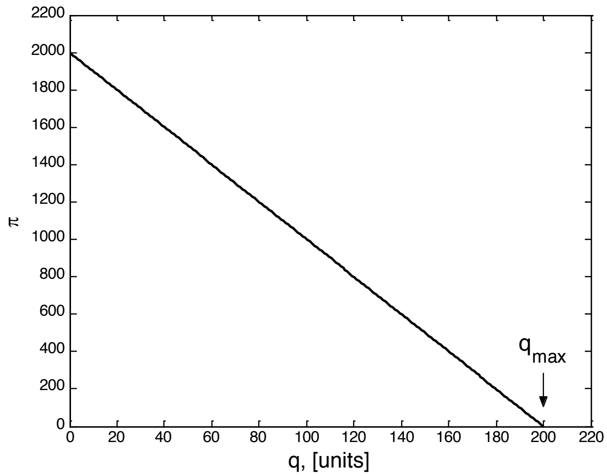

Figure P2.2-a: Price as a function of the quantity produced

The maximum consumption q_{max} is achieved when $\pi = 0$ [\$/unit]. From the inverse demand function, we get:

$$0 = -10q + 2000 \ \$$$

$$q_{max} = \frac{-2000}{-10} = 200 \ \text{units}$$

b. *Determine the price that no consumer is prepared to pay for this product*

The price that no consumer is prepared to pay is such that no widget is sold:

$$\pi = -10 \times 0 + 2000 = 2000 \ \text{\$/unit}$$

c. *Determine the maximum consumers' surplus. Explain why the consumers will not be able to realize this surplus*

Since the area under the inverse demand function represents the consumers' surplus, this surplus will maximum when the area is maximized. This occurs when the number of widgets sold is at its maximum ($q = q_{max}$ and $\pi = 0$). Thus, we have:

$$surplus = \frac{q_{max}\ \pi_{max}}{2} = \frac{200 \times 2000}{2} = 200,000 \ \$$$

This surplus is not achievable because no rational producer would be willing to "sell" its production at a price of 0 \$/unit.

d. *For a price π = 1000* \$/unit*, calculate the consumption, the consumers' gross surplus, the revenue collected by the producers and the consumers' net surplus.*

For a price of 1,000 \$/unit the number of widgets sold would be:

$$q = \frac{\pi}{-10} + 200 = -\frac{1000}{10} + 200 = 100 \text{ units}$$

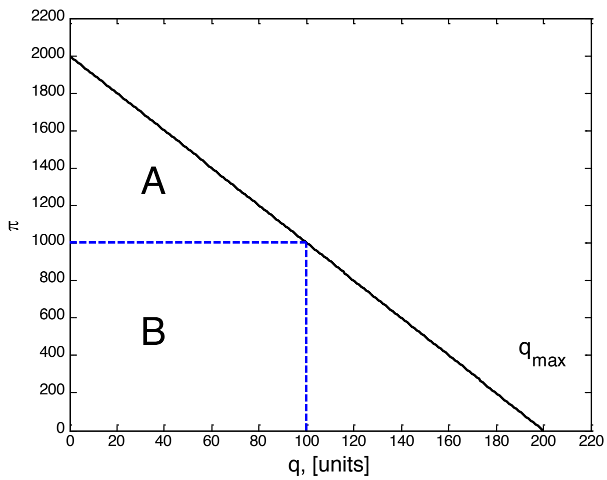

Figure P2.2-b: Price as a function of the number of widgets produced for a price of 1,000 \$/unit

Therefore, the consumers' gross surplus would be given by the sum of areas A and B:

$$A + B = \frac{100 \times 1000}{2} + 100 \times 1000 = 150{,}000 \text{ \$}$$

The revenue collected by the producers is given by area B, which is 100,000 \$

The consumers' net surplus is given by area A, which is 50,000 \$

e. If the price π increases 20%, calculate the change in consumption and the change in the revenue collected by the producers.

A 20% increases means that π would be 1,200 \$/unit. The new number of widgets consumed would therefore be:

$$q = \frac{\pi}{-10} + 200 = -\frac{1{,}200}{10} + 200 = 80 \text{ units}$$

The revenue collected by the producers would then be 80 × 1,200 = 96,000 \$. Therefore, the change in consumption is 80 – 100 = –20 units and the change on the revenue collected is 96,000 – 10,0000 = – 4,000 \$.

f. *What is the price elasticity of demand for this product and this group of consumers when the price π is* 1000 $/unit?

The elasticity of demand is given by:

$$\varepsilon = \frac{\pi}{q}\frac{dq}{d\pi}$$

From the inverse demand function, we know that the amount of widgets consumed is given by:

$$q = \frac{\pi}{-10} + 200$$

Therefore, the rate of change of the amount of widgets consumed with respect of the price change is given by:

$$\frac{dq}{d\pi} = -\frac{1}{10}$$

Since the price is 1000 $/unit for 100 units, the price elasticity of demand is:

$$\varepsilon = \frac{1000}{100}\left(-\frac{1}{10}\right) = -1$$

g. *Derive and expression for the gross consumers' surplus and the net consumers' surplus as a function of the demand. Check these expressions using the results of part d.*

The gross consumer surplus is given by the sum of the areas A and B; therefore:

$$gcs = A + B = \frac{q\left(\pi_{\max} - \pi\right)}{2} + q\,\pi$$

Where $\pi_{\max}$ = 2000 $/unit. Since the price is related to the demand by $\pi = -10q + 2000$, we have:

$$gcs = \left(5q^2\right) + \left(-10q^2 + 2000q\right) = -5q^2 + 2000q$$

Evaluating the previous equation for an amount of 100 widgets we obtain a gross consumers' surplus of 150000 $, which is equal to the result obtained in part d.

The net consumer surplus is represented by area A, that is: $ncs = 5q^2$; again, evaluating this equation for 100 widgets we obtain ncs = 50,000 \$.

h. Derive and expression for the net consumers' surplus and the gross consumers' surplus as a function of the price. Check these expressions using the results of part d.

The gross consumer surplus is given by:

$$gcs = \frac{q\left(\pi_{\max} - \pi\right)}{2} + q\,\pi$$

And from the inverse demand function we know that the number of widgets as a function of the price is given by:

$$q = -\frac{\pi}{10} + 200$$

Therefore, the gross consumers' surplus is:

$$gcs = \left(\frac{\pi^2}{20} - 200\pi + 200000\right) + \left(-\frac{\pi^2}{10} + 200\pi\right)$$

$$gcs = \frac{-\pi^2}{20} + 200000$$

For a price of 1000 \$/unit the gcs = 150000 as in part d.

Finally, the net consumers' surplus is represented by area A, therefore:

$$ncs = \left(\frac{\pi^2}{20} - 200\pi + 200000\right)$$

Evaluating this quantity for a price of 1000 \$/unit, we get ncs = 50,000 \$, as in part d.

2.3 *Economists estimate that the supply function for the widget market is given by the following expression:*

$$q = 0.2\,\pi - 40$$

a. *Calculate de demand and the price at the market equilibrium if the demand is as defined in problem 2.2.*

From problem 2.2, we know that the demand function is: $\pi = -10q + 2000$, while the supply function is now given as: $\pi = 5q + 200$. If the market is at its equilibrium, then the market price is such that the values given by the demand function and the supply function are equal. Therefore, we have:

$$5q + 200 = -10q + 2000$$

The number of widgets sold at market equilibrium is thus q = 120 while the market price is: $\pi = 5(120) + 200 = 800$ \$/widget. These results are shown graphically as follows:

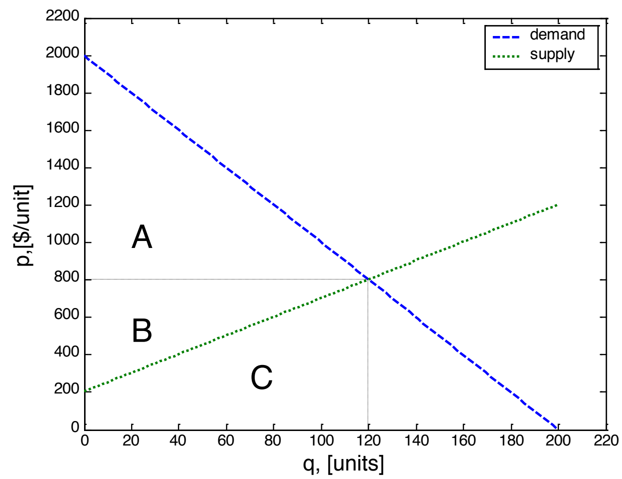

Figure P2.3: Market equilibrium

b. *For this equilibrium, calculate the consumers' gross surplus, the consumers' net surplus, the producers' revenue, the producers' profit and the global welfare.*

The consumers' gross surplus is given by the sum of areas labelled A, B and C on Figure P2.3. Therefore, we have:

$$cgs = \frac{120 \times 1200}{2} + 120 \times 800 = 168,000\,\$$$

The net consumers' surplus is given by area A:

$$ncs = \frac{120 \times 1200}{2} = 72,000\ \ \$$$

Areas B and C give the producers' revenue:

$$pr = 120 \times 800 = 96,000 \ \$$$

And the producers' profit is given by area B

$$pp = \frac{120 \times (800 - 200)}{2} = 36,000 \ \$$$

Finally, the global economic welfare is given by the sum of areas A and B

$$gw = \frac{120 \times 1200}{2} + \frac{120 \times (800 - 200)}{2} = 108,000 \ \$$$

2.4 *Calculate the effect on the market equilibrium of problem* 2.3 *of the following interventions:*
a. A minimum price of \$900 *per widget*
b. A maximum price of \$600 *per widget*
c. A sales tax of \$450 *per widget*
In each case, calculate the market price, the quantity transacted, the consumers' net surplus, the producers' profit and the global welfare. Illustrate your calculations using diagrams.

a) If a minimum price of \$ 900 per widget is imposed on the market, $\pi = -10q + 2000$ we can calculate the amount of widgets transacted from the demand function ($\pi = -10q + 2000$):

$$q = -\frac{\pi}{10} + 200 = -\frac{900}{10} + 200 = 110 \ \text{widgets}$$

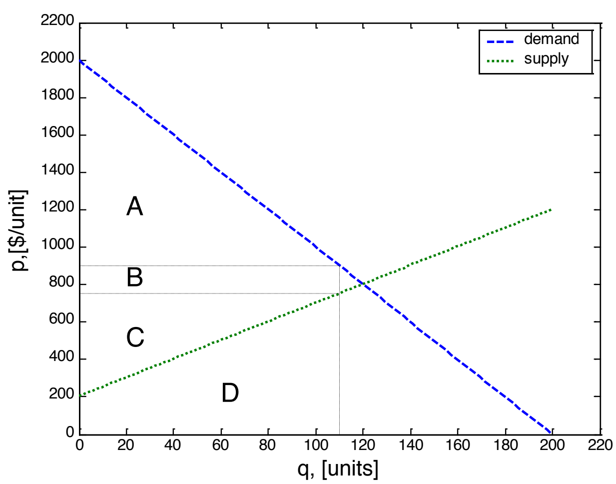

Figure P2.4-a: Effect on the market equilibrium of a minimum price of $900 per widget

The consumers' net surplus is then given by area A:

$$cns = \frac{110 \times 1100}{2} = 60,500 \ \$$$

The producers' profit is given by area B+C. In order to be able to compute area B, we need to know the price at which the producers would be willing to sell 110 widgets without market intervention. We can obtain this information by substitution in the supply function ($\pi_p = 5q + 200$):

$$\pi_p = 5(110) + 200 = 750 \ \$/\text{unit}$$

$$pp = 110 \times (900 - 750) + \frac{110 \times (750 - 200)}{2} = 46,750 \ \$$$

Since the global welfare is the sum of the consumers' surplus and the producers' profit, we have $gw = 107250$ \$

b) If a maximum price of 600 \$/widget is allowed in the market, then the demand is obtained from the supply function as follows:

$$q = 0.2\,\pi - 40 = 0.2(600) - 40 = 80 \ \text{widgets}$$

The price that the customers would be willing to pay for this number of widgets in the absence of market intervention can be computed using the demand function:

$$\pi_d = -10(80) + 2000 = 1,200 \ \$$$

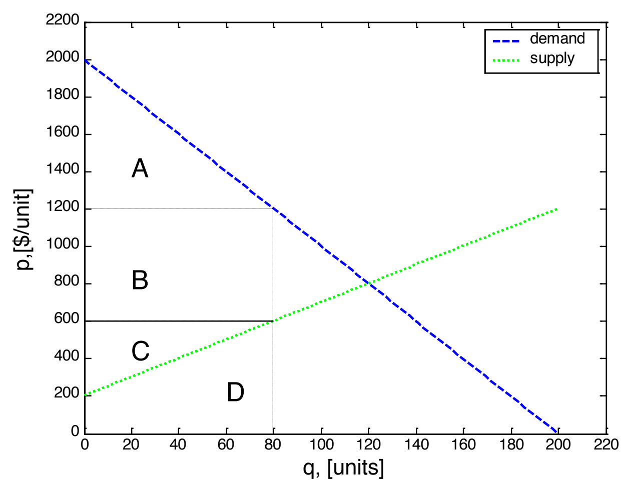

Figure P2.4-b: Effect on the market equilibrium of a maximum price of \$600 per widget

The consumers' net surplus is given by the sum of areas the areas labelled A and B on Figure P2.4-b:

$$cns = \frac{80\times(2000-1200)}{2} + 80\times(1200-600) = 80{,}000 \ \$$$

And the producers' profit is then given by area C:

$$pp = \frac{80\times(600-200)}{2} = 16{,}000 \ \$$$

The global welfare is given by the sum of the consumers' net surplus and the producers' profit: $gw = cns + pp = 96{,}000$ \$.

If a sales tax of 450 \$/widget is applied, then, the supply curve is offset by 450 \$/widget because this tax is passed on to the consumers.

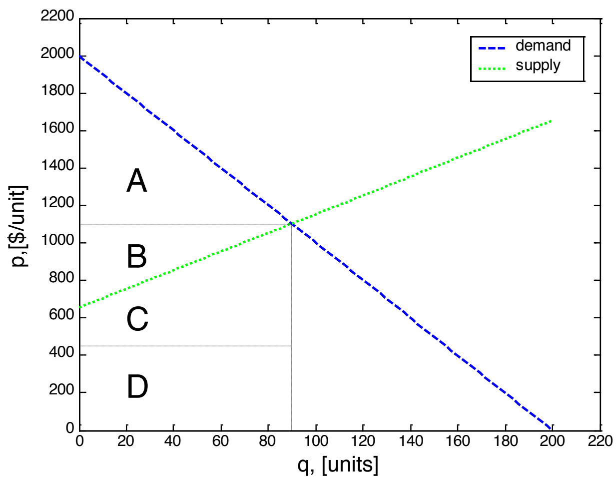

Figure P2.4-c: Effect on the market equilibrium of a sales tax of \$450 per widget

The market equilibrium is then given by:

$$5q + 200 + 450 = -10q + 2000$$

Therefore:

$$q = \frac{2000 - 650}{15} = 90 \text{ widgets}$$

And the market price can be obtained either from the supply function or from the demand function. From the demand function we get:

$$\pi_d = -10(90) + 2000 = 1{,}100 \text{ \$/unit}$$

The consumers' net surplus is given by the area labelled A on Figure P2.4-c:

$$cns = \frac{90 \times (2000 - 1100)}{2} = 40{,}500 \text{ \$}$$

The producers' profit is given by the area labelled B on that same figure:

$$pp = \frac{90 \times (1100 - 650)}{2} = 20{,}250 \text{ \$}$$

The tax revenue is given by area D:

$$tr = 90 \times 450 = 40{,}500 \text{ \$}$$

In this case the global welfare is given by the sum of the consumers' net surplus, the producers' profit and the tax revenue:

$gw = cns + pp + tr = 40{,}500 + 20{,}250 + 40{,}500 = 101{,}250$ \$

2.5 *The demand curve for a product is estimated to be given by the expression:*
$q = 200 - \pi$
Calculate the price and the price elasticity of the demand for the following values of the demand: 0, 50, 100, 150, and 200.
Repeat these calculations for the case where the demand curve is given by the expression:

$$q = \frac{10000}{\pi}$$

The demand elasticity is given by: $\varepsilon = \frac{\pi}{q} \frac{dq}{d\pi}$

For the first demand function: $\frac{dq}{d\pi} = -1$; therefore, the elasticity is given by: $\varepsilon = -\frac{\pi}{q}$.