

CHAPTER 1

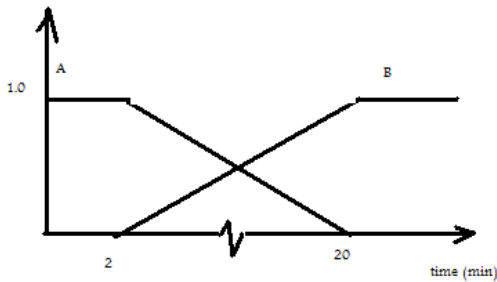
Introduction

1.1 Assume that Extra Fast takes 2 minutes or less (A) Slow Takes 20 minutes or more (B)

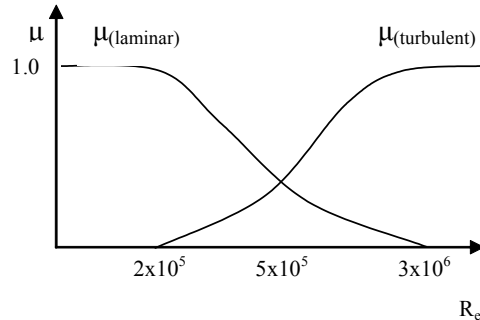
Note that times between 2 min. and 20 min. can be considered as Slow.

$$A = \{1/0 + 1/2 + 0/20 + 0/25\}$$

$$B = \{0/1 + 0/2 + 1/20 + 1/25\}$$



1.2

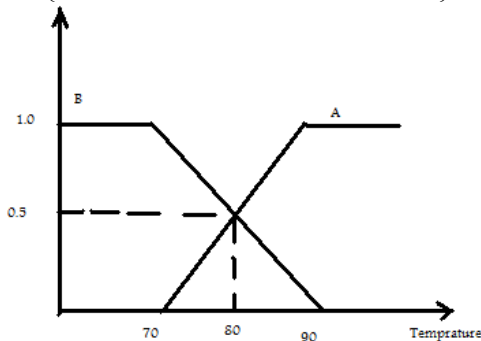


1.3 For $T > 90^\circ$ Curling occurs (A)

For $T < 70^\circ$ Curling does not occur (B)

$$A = \{0/0 + 0/70 + 0.5/80 + 1/90 + 1/100\}$$

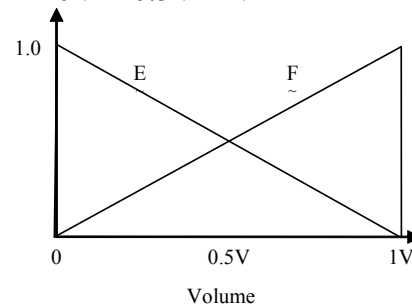
$$B = \{1/0 + 1/70 + 0.5/80 + 0/90 + 0/100\}$$



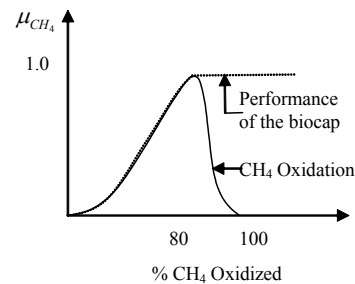
1.4 According to the diameters of base(d) and height (h) ratios, solids can be classified into these types. If $0 < d/h < 1$, it is a rod (its membership is $\mu_{A(n)}$); if $1 < d/h < \infty$, it is a disk ($\mu_{C(n)}$); if $d/h = 1$, it is a right cylinder ($\mu_{B(n)}$).

1.5 Let V be the volume of the glass, then the condition that glass is full is given by, $F = \frac{0}{0V} + \frac{0.5}{0.5V} + \frac{1}{V}$ & empty,

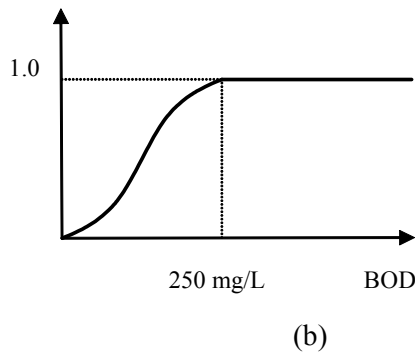
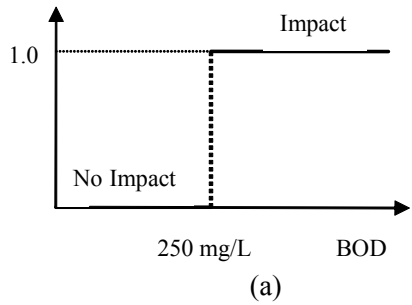
$$E = \frac{1}{0V} + \frac{0.5}{0.5V} + \frac{0}{V}$$



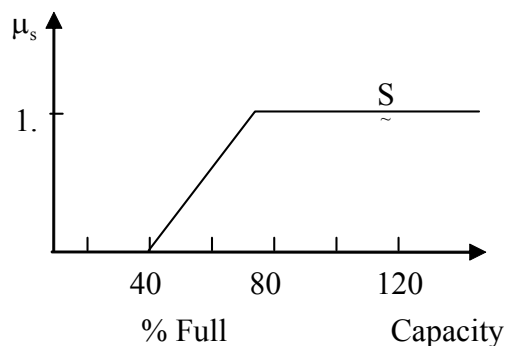
1.6 Since landfills are classified as best if they are capable of oxidizing 80% of the methane that originates, the membership function should achieve a full membership here. A reasonable membership function for the % CH_4 oxidation may be as follows:



1.7 The crisp set Impact membership function, figure (a) above has values of 1 for any BOD greater than or equal to 250mg/L while any value of BOD less than 250mg/L has a value of zero. However the membership function for the fuzzy set, figure (b) above, has some Impact values below 250mg/L.



1.8 .

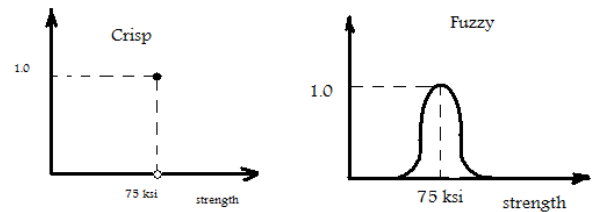


1.9 Crisp:

$$\chi_{LD_{50}} = 1, \text{ for } 0 < LD_{50} \leq 5000 \text{ mg/kg}$$

$$\chi_{LD_{50}} = 0, \text{ for } LD_{50} > 5000 \text{ mg/kg} \text{ and } LD_{50} \leq 0$$

1.10



1.11 Fuzzy Sets can be represented explicitly by families of parameterized functions, the most common being the following:

a) Triangular Functions

$$A(x) = \begin{cases} 0, & x \leq 2 \\ \frac{x-a}{m-a}, & x \in [2, m] \\ \frac{b-x}{b-m}, & x \in [m, b] \\ 0, & x \geq b \end{cases}$$

Where m , a , and b denote the modal value, the lower and upper bound values, respectively, for all nonzero values of $A(x)$.

b) Trapezoidal Function

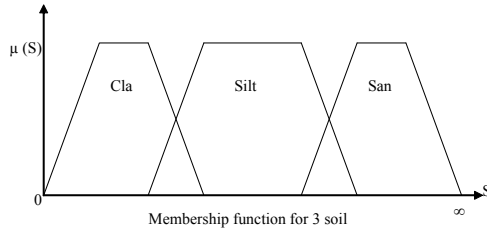
$$A(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{m-a}, & x \in [a, m] \\ 1, & x \in [m, n] \\ \frac{b-x}{b-n}, & x \in [n, b] \\ 0, & x > b \end{cases}$$

c) Gaussian Function

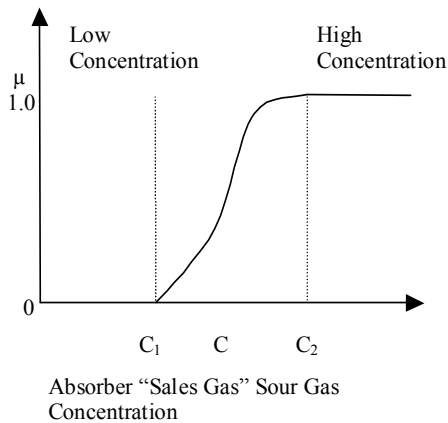
$$A(x) = e^{-k(x-m)^2}$$

Where $k > 0$

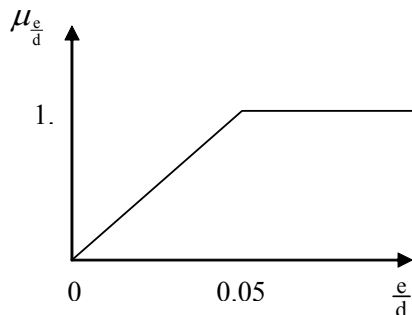
1.12 Fuzzy sets are useful in this situation where there is an inherent overlap among soil types. Clay has a smaller particle size than that of Sand however it is often difficult to distinguish Clay from Silt. This is also true for particle size between sand and silt.



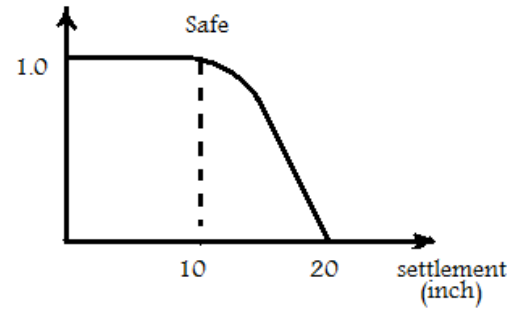
1.13 The membership function for the absorber “sales gas” sour gas concentration as a function of concentration C , with C_1 and C_2 is as follows.



1.14 The relation shows that the load becomes more eccentric as it approaches $\frac{e}{d} = 0.05$, it remains eccentric thereafter

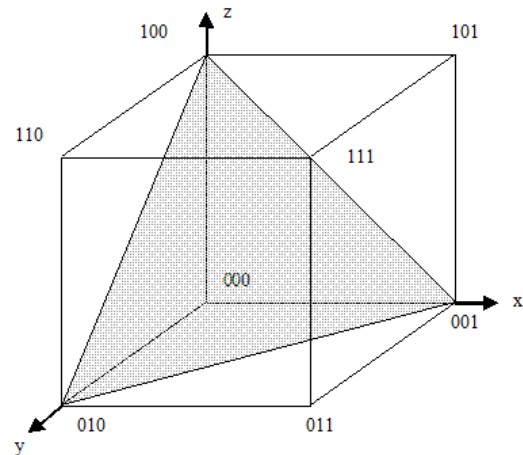


1.15



1.16 The geometric shape can resemble a disk, a cylinder, or a rod depending on the aspect ratio of d/h . For $d/h \ll 1$ the shape of the object approaches a long rod; in fact, as d/h approaches 0 the shape approaches a line. For $d/h \gg 1$ the object approaches the shape of a flat disk; as d/h approaches infinity the object approaches a circular area. For other values of this aspect ratio, e.g. for $d/h \approx 1$, the shape is typical of what we would call a “right circular cylinder.”

1.17



CHAPTER 2

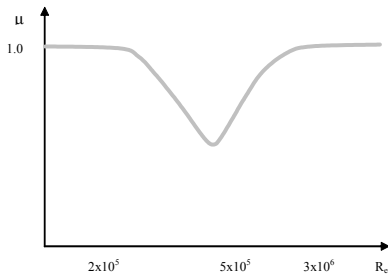
Classical Sets and Fuzzy Sets

2.1

a)

$$\widetilde{T \cup L}$$

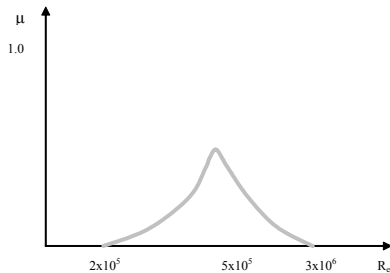
$$\mu_{\widetilde{T \cup L}}(R_e) = \mu_{\widetilde{T}}(R_e) \vee \mu_{\widetilde{L}}(R_e)$$



b)

$$\widetilde{T \cap L}$$

$$\mu_{\widetilde{T \cap L}}(R_e) = \mu_{\widetilde{T}}(R_e) \vee \mu_{\widetilde{L}}(R_e)$$



c)

$$\widetilde{T} = \left\{ \frac{0}{0} + \frac{0.1}{2 \times 10^5} + \frac{0.75}{5 \times 10^5} + \frac{1}{3 \times 10^6} \right\}$$

$$\widetilde{L} = \left\{ \frac{1.0}{0} + \frac{0.9}{2 \times 10^5} + \frac{0.25}{5 \times 10^5} + \frac{0}{3 \times 10^6} \right\}$$

2.2 (a) Select ponds number 4 and 5

(b) $A \cup B = \{0.5/1 + 0.6/2 + 0.8/3 + 1/4 + 1/5\}$

2.3

$$A \cup B = \{0.2/1 + 0.3/2 + 0.6/3 + 0.9/4\}$$

$$A \cap B = \{0.15/1 + 0.25/2 + 0.5/3 + 0.8/4\}$$

$$A|B = A \cap \bar{B}, \bar{B} = \left\{ \frac{0.8}{1} + \frac{0.7}{2} + \frac{0.5}{3} + \frac{0.2}{4} \right\}$$

$$A|B = \{0.15/1 + 0.85/2 + 0.5/3 + 0.2/4\}$$

$$B|A = B \cap \bar{A},$$

$$\bar{A} = \{0.85/1 + 0.75/2 + 0.4/3 + 0.1/4\}$$

$$B|A = \{0.2/1 + 0.3/2 + 0.4/3 + 0.1/4\}$$

2.4 a)

$$\widetilde{D_1 \cup D_2} = \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1.0}{3.0}$$

b)

$$\widetilde{D_1 \cap D_2} = \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}$$

c) $\widetilde{D_1} = \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1.0}{3.0}$

d) $\widetilde{D_2} = \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1.0}{3.0}$

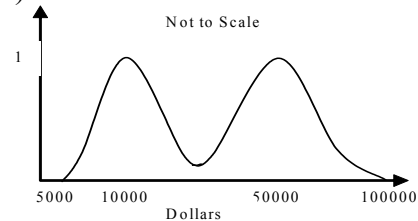
e) $\widetilde{D_1 / D_2} = \widetilde{D_1} \cap \widetilde{D_2} =$

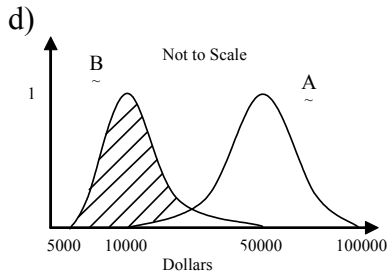
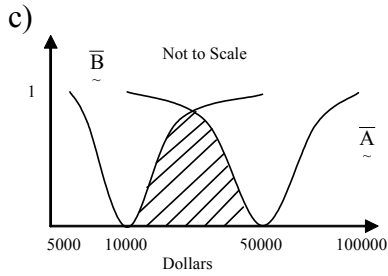
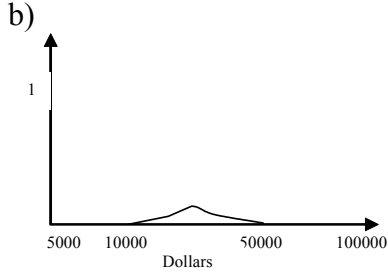
$$\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}$$

f) $\widetilde{\widetilde{D_1 \cup D_2}} = \widetilde{\widetilde{D_1} \cap \widetilde{D_2}} =$

$$\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}$$

2.5 a)





2.6 a) $\tilde{A} \cup \tilde{B} = \frac{0.2}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.2}{4}$

b) $\tilde{A} \cap \tilde{B} = \frac{0.1}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.1}{4}$

c) $\overline{\tilde{A}} = \frac{0.9}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.8}{4}$

d) $\overline{\tilde{B}} = \frac{0.8}{0} + \frac{0.5}{1} + \frac{0}{2} + \frac{0.3}{3} + \frac{0.9}{4}$

2.7 a)

$$\tilde{M} \cup \tilde{F} = \frac{1}{0} + \frac{1}{10} + \frac{1}{20} + \frac{0.7}{40} + \frac{0.9}{80} + \frac{0}{100}$$

b)

$$\tilde{M} \cap \tilde{F} = \frac{0.3}{1} + \frac{0.7}{10} + \frac{0.4}{20} + \frac{0}{40} + \frac{0}{80} + \frac{0}{100}$$

c)

$$\overline{\tilde{M}} = \frac{0}{1} + \frac{0.3}{10} + \frac{0.6}{20} + \frac{1}{40} + \frac{1}{80} + \frac{1}{100}$$

d) $\overline{\tilde{F}} = \frac{0.7}{0} + \frac{0}{10} + \frac{0}{20} + \frac{0.5}{40} + \frac{0.8}{80} + \frac{1}{100}$

e) $\tilde{C} \cap \overline{\tilde{F}} = \frac{0}{1} + \frac{0}{10} + \frac{0}{20} + \frac{0.5}{40} + \frac{0.8}{80} + \frac{1}{100}$

f) $\overline{\tilde{M} \cap \tilde{C}} = \overline{\tilde{M}} \cup \overline{\tilde{C}}$
 $= \frac{0.3}{1} + \frac{0.7}{10} + \frac{0.4}{20} + \frac{0}{40} + \frac{0}{80} + \frac{0}{100}$

2.8

$\tilde{Flow}_1 \cap \tilde{Flow}_2$

$$= \frac{0}{0} + \frac{0.45}{20} + \frac{0.6}{40} + \frac{0.45}{60} + \frac{0.3}{80} + \frac{0.1}{100}$$

$\tilde{Flow}_1 \cup \tilde{Flow}_2$

$$= \frac{1}{0} + \frac{0.8}{20} + \frac{0.65}{40} + \frac{0.8}{60} + \frac{.95}{80} + \frac{1.0}{100}$$

$\overline{\tilde{Flow}_1 | \tilde{Flow}_2} = \overline{\tilde{Flow}_1} \cap \overline{\tilde{Flow}_2}$

$$\overline{\tilde{Flow}_2} = \left\{ \frac{1.0}{0} + \frac{0.55}{20} + \frac{0.4}{40} + \frac{0.2}{60} + \frac{0.05}{80} + \frac{0}{100} \right\}$$

$\tilde{Flow}_1 | \tilde{Flow}_2$

$$= \frac{1.0}{0} + \frac{0.55}{20} + \frac{0.4}{40} + \frac{0.2}{60} + \frac{0.05}{80} + \frac{0}{100}$$

$\overline{\tilde{Flow}_2} | \tilde{Flow}_1 = \overline{\tilde{Flow}_2} \cap \overline{\tilde{Flow}_1}$

$$\overline{\tilde{Flow}_1} = \left\{ \frac{0}{0} + \frac{0.2}{20} + \frac{0.35}{40} + \frac{0.55}{60} + \frac{0.7}{80} + \frac{0.9}{100} \right\}$$

$\tilde{Flow}_2 | \tilde{Flow}_1$

$$= \frac{0}{0} + \frac{0.2}{20} + \frac{0.35}{40} + \frac{0.55}{60} + \frac{0.7}{80} + \frac{0.9}{100}$$

2.9 a) $\tilde{A} \cup \tilde{B} =$

$$\frac{0}{0.73} + \frac{0.8}{0.735} + \frac{1}{0.74} + \frac{1}{0.745} + \frac{0.6}{0.750}$$

b) $\tilde{A} \cap \tilde{B} =$

$$\frac{0}{0.73} + \frac{0.4}{0.735} + \frac{0.8}{0.74} + \frac{0.6}{0.745} + \frac{0}{0.750}$$

c)

$$\bar{A} = \frac{1}{0.73} + \frac{0.2}{0.735} + \frac{0}{0.74} + \frac{0.4}{0.745} + \frac{1}{0.750}$$

d) $A|B = A \cap \bar{B}$

$$\frac{0}{0.73} + \frac{0.6}{0.735} + \frac{0.2}{0.74} + \frac{0}{0.745} + \frac{0}{0.750}$$

f) $\overline{A \cup B} = \bar{A} \cap \bar{B} =$

$$\frac{1}{0.73} + \frac{0.2}{0.735} + \frac{0}{0.74} + \frac{0}{0.745} + \frac{0.4}{0.75}$$

e) $\overline{A \cap B} = \bar{A} \cup \bar{B}$

$$\frac{1}{0.73} + \frac{0.6}{0.735} + \frac{0.2}{0.74} + \frac{0.4}{0.745} + \frac{1}{0.75}$$

2.10

$$A \cup B = \left\{ \frac{0.15}{\text{winter}} + \frac{0.55}{\text{spring}} + \frac{0.9}{\text{summer}} + \frac{0.25}{\text{fall}} \right\}$$

$$A \cap B = \left\{ \frac{0.1}{\text{winter}} + \frac{0.3}{\text{spring}} + \frac{0.52}{\text{summer}} + \frac{0.2}{\text{fall}} \right\}$$

$$A|B = A \cap \bar{B},$$

$$\bar{B} = \left\{ \frac{0.9}{\text{winter}} + \frac{0.45}{\text{spring}} + \frac{0.1}{\text{summer}} + \frac{0.8}{\text{fall}} \right\}$$

$$A|B = \left\{ \frac{0.15}{\text{winter}} + \frac{0.33}{\text{spring}} + \frac{0.1}{\text{summer}} + \frac{0.25}{\text{fall}} \right\}$$

$$B|A = B \cap \bar{A},$$

$$\bar{A} = \left\{ \frac{0.85}{\text{winter}} + \frac{0.67}{\text{spring}} + \frac{0.48}{\text{summer}} + \frac{0.75}{\text{fall}} \right\}$$

$$B|A = \left\{ \frac{0.1}{\text{winter}} + \frac{0.55}{\text{spring}} + \frac{0.48}{\text{summer}} + \frac{0.2}{\text{fall}} \right\}$$

2.11

$$\bar{MP} = \frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5}$$

$$+ \frac{0.8}{6} + \frac{0.6}{7} + \frac{0.4}{8} + \frac{0.2}{9} + \frac{0}{10}$$

$$\overline{\bar{MP}} = \frac{1}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.4}{3} + \frac{0.2}{4} + \frac{0}{5}$$

$$+ \frac{0.2}{6} + \frac{0.4}{7} + \frac{0.6}{8} + \frac{0.8}{9} + \frac{1}{10}$$

$$\bar{HP} = \frac{0}{0} + \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3} + \frac{0.4}{4} + \frac{0.5}{5}$$

$$+ \frac{0.6}{6} + \frac{0.7}{7} + \frac{0.8}{8} + \frac{0.9}{9} + \frac{1}{10}$$

$$\overline{\bar{HP}} = \frac{1}{0} + \frac{0.9}{1} + \frac{0.8}{2} + \frac{0.7}{3} + \frac{0.6}{4} + \frac{0.5}{5}$$

$$+ \frac{0.4}{6} + \frac{0.3}{7} + \frac{0.2}{8} + \frac{0.1}{9} + \frac{0}{10}$$

$$\bar{MP}/\bar{HP} = \frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.6}{4} + \frac{0.5}{5}$$

$$+ \frac{0.4}{6} + \frac{0.3}{7} + \frac{0.2}{8} + \frac{0.1}{9} + \frac{0}{10}$$

$$\bar{MP} \cup \bar{HP} = \frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5}$$

$$+ \frac{0.8}{6} + \frac{0.7}{7} + \frac{0.8}{8} + \frac{0.9}{9} + \frac{1}{10}$$

$$\overline{\bar{MP} \cap \bar{HP}} = \frac{1}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.4}{3} + \frac{0.2}{4} + \frac{0}{5}$$

$$+ \frac{0.2}{6} + \frac{0.3}{7} + \frac{0.2}{8} + \frac{0.1}{9} + \frac{0}{10}$$

2.12

$$\bar{Q} \cup \bar{C} = \frac{1}{0} + \frac{1}{1} + \frac{0.8}{2} + \frac{0.4}{5} + \frac{0.6}{7} + \frac{0.8}{9} + \frac{1}{10}$$

$$\bar{Q} \cap \bar{C} = \frac{0}{0} + \frac{0}{1} + \frac{0}{2} + \frac{0.3}{5} + \frac{0.1}{7} + \frac{0}{9} + \frac{0}{10}$$

$$\bar{Q} = \frac{0}{0} + \frac{0}{1} + \frac{0.2}{2} + \frac{0.7}{5} + \frac{0.9}{7} + \frac{1}{9} + \frac{1}{10}$$

$$\bar{C} = \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{0.6}{5} + \frac{0.4}{7} + \frac{0.2}{9} + \frac{0}{10}$$

$$\bar{Q}/\bar{C} = \frac{1}{0} + \frac{1}{1} + \frac{0.8}{2} + \frac{0.3}{5} + \frac{0.1}{7} + \frac{0}{9} + \frac{0}{10}$$

$$\overline{\bar{Q} \cup \bar{C}} = \frac{0}{0} + \frac{0}{1} + \frac{0.2}{2} + \frac{0.6}{5} + \frac{0.4}{7} + \frac{0.2}{9} + \frac{0}{10}$$

$$\overline{\bar{Q} \cap \bar{C}} = \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{0.7}{5} + \frac{0.9}{7} + \frac{1}{9} + \frac{1}{10}$$

$$\overline{\bar{Q} \cup \bar{Q}} = \frac{1}{0} + \frac{1}{1} + \frac{0.8}{2} + \frac{0.7}{5} + \frac{0.9}{7} + \frac{1}{9} + \frac{1}{10}$$

$$\begin{aligned} Q \cap \bar{Q} &= \frac{0}{0} + \frac{0}{1} + \frac{0.2}{2} + \frac{0.3}{5} + \frac{0.1}{7} + \frac{0}{9} + \frac{0}{10} \\ C \cup \bar{C} &= \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{0.6}{5} + \frac{0.6}{7} + \frac{0.8}{9} + \frac{1}{10} \\ C \cap \bar{C} &= \frac{0}{0} + \frac{0}{1} + \frac{0}{2} + \frac{0.4}{5} + \frac{0.4}{7} + \frac{0.2}{9} + \frac{0}{10} \end{aligned}$$

2.13

$$\begin{aligned} A &= \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.62}{2} + \frac{0.88}{3} + \frac{1}{4} + \frac{0.88}{5} \\ &\quad + \frac{0.61}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0}{10} \\ B &= \frac{0.003}{0} + \frac{0.0022}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} \\ &\quad + \frac{0.32}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10} \\ \bar{A} &= \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.38}{2} + \frac{0.12}{3} + \frac{0}{4} + \frac{0.12}{5} \\ &\quad + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{1}{10} \\ \bar{B} &= \frac{0.997}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} \\ &\quad + \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10} \\ A \cup B &= \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{1}{4} \\ &\quad + \frac{0.88}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10} \\ A \cap B &= \frac{0.00}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} \\ &\quad + \frac{0.32}{5} + \frac{0.61}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0.01}{10} \\ \bar{A} \cup \bar{B} &= \frac{0.99}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} \\ &\quad + \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{0.99}{10} \\ \bar{A} \cap \bar{B} &= \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.39}{2} + \frac{0.12}{3} + \frac{0}{4} + \frac{0.12}{5} \\ &\quad + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10} \\ A/B &= \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{0.86}{4} \end{aligned}$$

$$\begin{aligned} &+ \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.04}{9} + \frac{0.01}{10} \\ \bar{A} \cap B &= \frac{0.00}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0}{4} \\ &\quad + \frac{0.12}{5} + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.88}{9} + \frac{0.61}{10} \\ A \cup \bar{A} &= \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{1}{4} \\ &\quad + \frac{0.88}{5} + \frac{0.61}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{0.99}{10} \\ A \cap \bar{A} &= \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.39}{2} + \frac{0.12}{3} + \frac{0}{4} \\ &\quad + \frac{0.12}{5} + \frac{0.39}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0.01}{10} \\ B \cup \bar{B} &= \frac{0.9997}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} \\ &\quad + \frac{0.68}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10} \\ B \cap \bar{B} &= \frac{0.0003}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} \\ &\quad + \frac{0.32}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10} \end{aligned}$$

2.14 $A =$ "fast" chips

$$\begin{aligned} A &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.2}{5} + \frac{0.6}{6} + \frac{1}{7} + \frac{1}{8} \\ D &= \text{"hot" chips} \\ D &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0.5}{4} + \frac{0.1}{5} + \frac{1}{6} + \frac{0.5}{7} + \frac{1}{8} \\ A \cup D &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0.5}{4} + \frac{0.2}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \\ A \cap D &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.1}{5} + \frac{0.6}{6} + \frac{0.5}{7} + \frac{1}{8} \\ \bar{A} &= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{0.8}{5} + \frac{0.4}{6} + \frac{0}{7} + \frac{0}{8} \\ A \cap \bar{D} &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.2}{5} + \frac{0}{6} + \frac{0.5}{7} + \frac{0}{8} \\ \bar{A} \cap \bar{D} &= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{0.5}{4} + \frac{0.8}{5} + \frac{0}{6} + \frac{0}{7} + \frac{0}{8} \\ \bar{A} \cup \bar{D} &= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{0.9}{5} + \frac{0.4}{6} + \frac{0.5}{7} + \frac{0}{8} \end{aligned}$$