Introduction to Econometrics (4th Edition)

by

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Solutions to End-of-Chapter Exercises: Chapter 2*

*Limited distribution: **For Instructors Only**. Answers to all odd-numbered questions are provided to students on the textbook website.

2.1. (a) Probability distribution function for Y

Outcome (number of heads)	$Y = 0$	$Y = 1$	$Y = 2$
Probability	0.25	0.50	0.25

(b) Cumulative probability distribution function for Y

Outcome (number of heads)	$Y < 0$	$0 \le Y < 1$	$1 \le Y < 2$	$Y \ge 2$
Probability	0	0.25	0.75	1.0

(c) $\mu_Y = E(Y) = (0 \times 0.25) + (1 \times 0.50) + (2 \times 0.25) = 1.00$

Using Key Concept 2.3: $\text{var}(Y) = E(Y^2) - [E(Y)]^2$,

and

$$E(Y^2) = (0^2 \times 0.25) + (1^2 \times 0.50) + (2^2 \times 0.25) = 1.50$$

so that

$$\text{var}(Y) = E(Y^2) - [E(Y)]^2 = 1.50 - (1.00)^2 = 0.50.$$

2.2. We know from Table 2.2 that $\Pr(Y=0)=0.22$, $\Pr(Y=1)=0.78$, $\Pr(X=0)=0.30$, $\Pr(X=1)=0.70$. So

(a)

$$\begin{aligned}\mu_Y &= E(Y) = 0\times\Pr(Y=0)+1\times\Pr(Y=1)\\ &= 0\times 0.22+1\times 0.78 = 0.78,\end{aligned}$$

$$\begin{aligned}\mu_X &= E(X) = 0\times\Pr(X=0)+1\times\Pr(X=1)\\ &= 0\times 0.30+1\times 0.70 = 0.70.\end{aligned}$$

(b)

$$\begin{aligned}\sigma_X^2 &= E[(X-\mu_X)^2]\\ &= (0-0.70)^2\times\Pr(X=0)+(1-0.70)^2\times\Pr(X=1)\\ &= (-0.70)^2\times 0.30+0.30^2\times 0.70 = 0.21,\\ \sigma_Y^2 &= E[(Y-\mu_Y)^2]\\ &= (0-0.78)^2\times\Pr(Y=0)+(1-0.78)^2\times\Pr(Y=1)\\ &= (-0.78)^2\times 0.22+0.22^2\times 0.78 = 0.1716.\end{aligned}$$

(c)

$$\begin{aligned}\sigma_{XY} &= \operatorname{cov}(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]\\ &= (0-0.70)(0-0.78)\Pr(X=0,Y=0)+(0-0.70)(1-0.78)\Pr(X=0,Y=1)\\ &\quad +(1-0.70)(0-0.78)\Pr(X=1,Y=0)+(1-0.70)(1-0.78)\Pr(X=1,Y=1)\\ &= (-0.70)\times(-0.78)\times 0.15+(-0.70)\times 0.22\times 0.15+0.30\times(-0.78)\times 0.07+0.30\times 0.22\times 0.63\\ &= 0.084,\end{aligned}$$

$$\operatorname{corr}(X,Y) = \frac{\sigma_{XY}}{\sigma_X\sigma_Y} = \frac{0.084}{\sqrt{0.21\times 0.1716}} = 0.4425.$$

2.3. For the two new random variables $W = 3 + 6X$ and $V = 20 - 7Y$, we have:

(a)

$$E(V) = E(20 - 7Y) = 20 - 7E(Y) = 20 - 7 \times 0.78 = 14.54,$$
$$E(W) = E(3 + 6X) = 3 + 6E(X) = 3 + 6 \times 0.70 = 7.2.$$

(b)

$$\sigma_W^2 = \text{var}(3 + 6X) = 6^2 \sigma_X^2 = 36 \times 0.21 = 7.56,$$
$$\sigma_V^2 = \text{var}(20 - 7Y) = (-7)^2 \times \sigma_Y^2 = 49 \times 0.1716 = 8.4084.$$

(c)

$$\sigma_{WV} = \text{cov}(3 + 6X, 20 - 7Y) = 6(-7)\text{cov}(X, Y) = -42 \times 0.084 = -3.52$$

$$\text{corr}(W, V) = \frac{\sigma_{WV}}{\sigma_W \sigma_V} = \frac{-3.528}{\sqrt{7.56 \times 8.4084}} = -0.4425.$$

2.4. (a) $E(X^3) = 0^3 \times (1-p) + 1^3 \times p = p$

(b) $E(X^k) = 0^k \times (1-p) + 1^k \times p = p$

(c) $E(X) = 0.3$

$\text{var}(X) = E(X^2) - [E(X)]^2 = 0.3 - 0.09 = 0.21$

Thus, $\sigma = \sqrt{0.21} = 0.46$.

To compute the skewness, use the formula from exercise 2.21:

$$E(X-\mu)^3 = E(X^3) - 3[E(X^2)][E(X)] + 2[E(X)]^3$$
$$= 0.3 - 3 \times 0.3^2 + 2 \times 0.3^3 = 0.084$$

Alternatively, $E(X-\mu)^3 = [(1-0.3)^3 \times 0.3] + [(0-0.3)^3 \times 0.7] = 0.084$

Thus, skewness $= E(X-\mu)^3/\sigma^3 = .084/0.46^3 = 0.87$.

To compute the kurtosis, use the formula from exercise 2.21:

$$E(X-\mu)^4 = E(X^4) - 4[E(X)][E(X^3)] + 6[E(X)]^2[E(X^2)] - 3[E(X)]^4$$
$$= 0.3 - 4 \times 0.3^2 + 6 \times 0.3^3 - 3 \times 0.3^4 = 0.0777$$

Alternatively, $E(X-\mu)^4 = [(1-0.3)^4 \times 0.3] + [(0-0.3)^4 \times 0.7] = 0.0777$

Thus, kurtosis is $E(X-\mu)^4/\sigma^4 = .0777/0.46^4 = 1.76$

2.5. Let X denote temperature in °F and Y denote temperature in °C. Recall that $Y = 0$ when $X = 32$ and $Y = 100$ when $X = 212$.

This implies $Y = (100/180) \times (X - 32)$ or $Y = -17.78 + (5/9) \times X$.

Using Key Concept 2.3, $\mu_X = 70$°F implies that $\mu_Y = -17.78 + (5/9) \times 70 = 21.11$°C, and $\sigma_X = 7$°F implies $\sigma_Y = (5/9) \times 7 = 3.89$°C.

2.6. The table shows that $\Pr(X=0, Y=0)=0.026$, $\Pr(X=0, Y=1)=0.576$, $\Pr(X=1, Y=0)=0.009$, $\Pr(X=1, Y=1)=0.389$, $\Pr(X=0)=0.602$, $\Pr(X=1)=0.398$, $\Pr(Y=0)=0.035$, $\Pr(Y=1)=0.965$.

(a)

$$\begin{aligned} E(Y) = \mu_Y &= 0 \times \Pr(Y=0) + 1 \times \Pr(Y=1) \\ &= 0 \times 0.035 + 1 \times 0.965 = 0.965. \end{aligned}$$

(b)

$$\begin{aligned} \text{Unemployment Rate} &= \frac{\#(\text{unemployed})}{\#(\text{labor force})} \\ &= \Pr(Y=0) = 1 - \Pr(Y=1) = 1 - E(Y) = 1 - 0.965 = 0.035. \end{aligned}$$

(c) Calculate the conditional probabilities first:

$$\Pr(Y=0|X=0) = \frac{\Pr(X=0, Y=0)}{\Pr(X=0)} = \frac{0.026}{0.602} = 0.043,$$

$$\Pr(Y=1|X=0) = \frac{\Pr(X=0, Y=1)}{\Pr(X=0)} = \frac{0.576}{0.602} = 0.957,$$

$$\Pr(Y=0|X=1) = \frac{\Pr(X=1, Y=0)}{\Pr(X=1)} = \frac{0.009}{0.398} = 0.023,$$

$$\Pr(Y=1|X=1) = \frac{\Pr(X=1, Y=1)}{\Pr(X=1)} = \frac{0.389}{0.398} = 0.978.$$

The conditional expectations are

$$\begin{aligned} E(Y|X=1) &= 0 \times \Pr(Y=0|X=1) + 1 \times \Pr(Y=1|X=1) \\ &= 0 \times 0.023 + 1 \times 0.978 = 0.978, \end{aligned}$$

$$\begin{aligned} E(Y|X=0) &= 0 \times \Pr(Y=0|X=0) + 1 \times \Pr(Y=1|X=0) \\ &= 0 \times 0.043 + 1 \times 0.957 = 0.957. \end{aligned}$$

(d) Use the solution to part (b),

Unemployment rate for college graduates = 1 – $E(Y|X=1)$ = 1−0.978 = 0.023.

Unemployment rate for non-college graduates = 1 – $E(Y|X=0)$ = 1−0.957 = 0.043

(e) The probability that a randomly selected worker who is reported being unemployed is a college graduate is

$$\Pr(X = 1|Y = 0) = \frac{\Pr(X = 1, Y = 0)}{\Pr(Y = 0)} = \frac{0.009}{0.035} = 0.257.$$

The probability that this worker is a non-college graduate is

$$\Pr(X = 0|Y = 0) = 1 - \Pr(X = 1|Y = 0) = 1 - 0.257 = 0.743.$$

(f) Educational achievement and employment status are not independent because they do not satisfy that, for all values of x and y,

$$\Pr(X = x|Y = y) = \Pr(X = x).$$

For example, from part (e) $\Pr(X = 0|Y = 0)$ = 0.743, while from the table $\Pr(X = 0)$ = 0.602.

2.7. Using obvious notation, $C = M + F$; thus $\mu_C = \mu_M + \mu_F$ and $\sigma_C^2 = \sigma_M^2 + \sigma_F^2 + 2\text{cov}(M, F)$. This implies

(a) $\mu_C = 40 + 45 = \$85{,}000$ per year.

(b) $\text{corr}(M, F) = \frac{\text{cov}(M,F)}{\sigma_M \sigma_F}$, so that $\text{cov}(M, F) = \sigma_M \sigma_F \text{corr}(M, F)$. Thus $\text{cov}(M, F) = 12 \times 18 \times 0.80 = 172.80$, where the units are squared thousands of dollars per year.

(c) $\sigma_C^2 = \sigma_M^2 + \sigma_F^2 + 2\text{cov}(M, F)$, so that $\sigma_C^2 = 12^2 + 18^2 + 2 \times 172.80 = 813.60$, and $\sigma_C = \sqrt{813.60} = 28.524$ thousand dollars per year.

(d) First you need to look up the current Euro/dollar exchange rate in the Wall Street Journal, the Federal Reserve web page, or other financial data outlet. Suppose that this exchange rate is e (say e = 0.85 Euros per Dollar or $1/e$ = 1.18 Dollars per Euro); each 1 Eollar is therefore with e Euros. The mean is therefore $e \times \mu_C$ (in units of thousands of euros per year), and the standard deviation is $e \times \sigma_C$ (in units of thousands of euros per year). The correlation is unit-free, and is unchanged.

2.8. $\mu_Y = E(Y) = 1,\ \sigma_Y^2 = \text{var}(Y) = 4.$ With $Z = \frac{1}{2}(Y-1),$

$$\mu_Z = E\left(\frac{1}{2}(Y-1)\right) = \frac{1}{2}(\mu_Y - 1) = \frac{1}{2}(1-1) = 0,$$

$$\sigma_Z^2 = \text{var}\left(\frac{1}{2}(Y-1)\right) = \frac{1}{4}\sigma_Y^2 = \frac{1}{4}\times 4 = 1.$$

2.9.

		Value of Y					Probability Distribution of X
		14	**22**	**30**	**40**	**65**	
Value of X	**1**	0.02	0.05	0.10	0.03	0.01	0.21
	5	0.17	0.15	0.05	0.02	0.01	0.40
	8	0.02	0.03	0.15	0.10	0.09	0.39
Probability distribution of Y		0.21	0.23	0.30	0.15	0.11	1.00

(a) The probability distribution is given in the table above.

$$E(Y) = 14\times0.21 + 22\times0.23 + 30\times0.30 + 40\times0.15 + 65\times0.11 = 30.15$$

$$E(Y^2) = 14^2\times0.21 + 22^2\times0.23 + 30^2\times0.30 + 40^2\times0.15 + 65^2\times0.11 = 1127.23$$

$$\text{var}(Y) = E(Y^2) - [E(Y)]^2 = 218.21$$

$$\sigma_Y = 14.77$$

(b) The conditional probability of $Y|X = 8$ is given in the table below

Value of Y				
14	**22**	**30**	**40**	**65**
0.02/0.39	0.03/0.39	0.15/0.39	0.10/0.39	0.09/0.39

$$E(Y|X=8) = 14\times(0.02/0.39) + 22\times(0.03/0.39) + 30\times(0.15/0.39)$$
$$+ 40\times(0.10/0.39) + 65\times(0.09/0.39) = 39.21$$

$$E(Y^2|X=8) = 14^2\times(0.02/0.39) + 22^2\times(0.03/0.39) + 30^2\times(0.15/0.39)$$
$$+ 40^2\times(0.10/0.39) + 65^2\times(0.09/0.39) = 1778.7$$

$$\text{var}(Y) = 1778.7 - 39.21^2 = 241.65$$

$$\sigma_{Y|X=8} = 15.54$$

(c)

$$E(XY) = (1\times14\times0.02) + (1\times22:0.05) + \cdots(8\times65\times0.09) = 171.7$$

$$\text{cov}(X,Y) = E(XY) - E(X)E(Y) = 171.7 - 5.33\times30.15 = 11.0$$

$$\text{corr}(X,Y) = \text{cov}(X,Y)/(\sigma_X\sigma_Y) = 11.0/(2.60\times14.77) = 0.286$$

2.10. Using the fact that if $Y \sim N\left(\mu_Y, \sigma_Y^2\right)$ then $\frac{Y-\mu_Y}{\sigma_Y} \sim N(0,1)$ and Appendix Table 1, we have

(a) $\Pr(Y \le 3) = \Pr\left(\frac{Y-1}{2} \le \frac{3-1}{2}\right) = \Phi(1) = 0.8413.$

(b)

$$\Pr(Y > 0) = 1 - \Pr(Y \le 0)$$
$$= 1 - \Pr\left(\frac{Y-3}{3} \le \frac{0-3}{3}\right) = 1 - \Phi(-1) = \Phi(1) = 0.8413.$$

(c)

$$\Pr(40 \le Y \le 52) = \Pr\left(\frac{40-50}{5} \le \frac{Y-50}{5} \le \frac{52-50}{5}\right)$$
$$= \Phi(0.4) - \Phi(-2) = \Phi(0.4) - [1 - \Phi(2)]$$
$$= 0.6554 - 1 + 0.9772 = 0.6326.$$

(d)

$$\Pr(6 \le Y \le 8) = \Pr\left(\frac{6-5}{\sqrt{2}} \le \frac{Y-5}{\sqrt{2}} \le \frac{8-5}{\sqrt{2}}\right)$$
$$= \Phi(2.1213) - \Phi(0.7071)$$
$$= 0.9831 - 0.7602 = 0.2229.$$

2.11. (a) 0.90

(b) 0.05

(c) 0.05

(d) When $Y \sim \chi^2_{10}$, then $Y/10 \sim F_{10,\infty}$.

(e) $Y = Z^2$, where $Z \sim N(0,1)$, thus $\Pr(Y \leq 1) = \Pr(-1 \leq Z \leq 1) = 0.32.$

2.12. (a) 0.05

(b) 0.950

(c) 0.953

(d) The t_{df} distribution and N(0, 1) are approximately the same when *df* is large.

(e) 0.10

(f) 0.01

2.13. (a) $E(Y^2) = \text{Var}(Y) + \mu_Y^2 = 1 + 0 = 1$; $E(W^2) = \text{Var}(W) + \mu_W^2 = 100 + 0 = 100$.

(b) Y and W are symmetric around 0, thus skewness is equal to 0; because their mean is zero, this means that the third moment is zero.

(c) The kurtosis of the normal is 3, so $3 = \frac{E(Y - \mu_Y)^4}{\sigma_Y^4}$; solving yields $\text{E}(Y^4) = 3$; a similar calculation yields the results for W.

(d) First, condition on $X = 0$, so that $S = W$:

$E(S|X = 0) = 0$; $E(S^2|X = 0) = 100$, $E(S^3|X = 0) = 0$, $E(S^4|X = 0) = 3 \times 100^2$.

Similarly,

$E(S|X = 1) = 0$; $E(S^2|X = 1) = 1$, $E(S^3|X = 1) = 0$, $E(S^4|X = 1) = 3$.

From the large of iterated expectations

$$E(S) = E(S|X = 0) \times \Pr(X = 0) + E(S|X = 1) \times \Pr(X = 1) = 0$$

$$E(S^2) = E(S^2|X = 0) \times \Pr(X = 0) + E(S^2|X = 1) \times \Pr(X = 1) = 100 \times 0.01 + 1 \times 0.99 = 1.99$$

$$E(S^3) = E(S^3|X = 0) \times \Pr(X = 0) + E(S^3|X = 1) \times \Pr(X = 1) = 0$$

$$E(S^4) = E(S^4|X = 0) \times \Pr(X = 0) + E(S^4|X = 1) \times \Pr(X = 1)$$
$$= 3 \times 100^2 \times 0.01 + 3 \times 1 \times 0.99 = 302.97$$

(e) $\mu_S = E(S) = 0$, thus $E(S - \mu_S)^3 = E(S^3) = 0$ from part (d). Thus skewness = 0.

Similarly, $\sigma_S^2 = E(S - \mu_S)^2 = E(S^2) = 1.99$, and $E(S - \mu_S)^4 = E(S^4) = 302.97$.
Thus, kurtosis $= 302.97 / (1.99^2) = 76.5$