SOLUTIONS FOR QUESTIONS AT END OF CHAPTER 1

1.1 Discussion expected to start with units used in barter trade (explain barter trade, example units – e.g. the hand, the foot and the cubit, and limitations due to variation in size of hands etc). Moving on to standard units and improvements in these over time – e.g. platinum bar for length initially, then wavelengths of radiation, then distance travelled by light in an interval of 1/299792458 seconds. Give examples of standard units for other physical quantities. Mention Imperial, metric and SI units.

1.2 **Discussion should cover primary sensor, secondary sensors, conversion elements, signal processing elements, signal transmission element, signal presentation units and signal recording units.**

Sample answer: Discussion should cover the **primary sensor**, explaining its function and giving examples of primary sensors. Mention that some primary sensors like a liquid-in-glass thermometer are a complete measurement system in themselves. Intelligent instruments also have one or more **secondary sensors.** These measure the environmental conditions, particularly temperature and pressure, surrounding a measurement system in order to correct the output of primary sensors affected by the environment conditions.

Conversion elements are needed in a measurement system where the output variable of a primary transducer is in an inconvenient form and has to be converted to a more convenient form. For instance, the displacement-measuring strain gauge has an output in the form of a varying resistance. The resistance change cannot be easily measured and so it is converted to a change in voltage by a bridge circuit, which is a typical example of a conversion element. In some cases, the primary sensor and variable conversion element are combined, and the combination is known as a **transducer. Signal processing elements** exist to improve the quality of the output of a measurement system in some way. A very common type of signal processing element is the electronic amplifier, which amplifies the output of the primary transducer or variable conversion element, thus improving the sensitivity and resolution of measurement. This element of a measuring system is particularly important where the primary transducer has a low output. For example, thermocouples have a typical output of only a few millivolts. Other types of signal processing element are those that filter out induced noise and remove mean levels etc. In some devices, signal processing is incorporated into a transducer, which is then known as a **transmitter**. In addition to a primary sensor, variable conversion element and signal processing element, some measurement systems have one or two other components, firstly to transmit the signal to some remote point and secondly to display or record the signal if it is not fed automatically into a feedback control system. **Signal transmission** is needed when the observation or application point of the output of a measurement system is some distance away from the site of the primary transducer. Sometimes, this separation is made solely for purposes of convenience, but more often, it follows from the physical inaccessibility or environmental unsuitability of the site of the primary transducer for mounting the signal presentation/recording unit. The signal transmission element has traditionally consisted of single or multi-cored cable, which is often screened to minimize signal corruption by induced electrical noise. However, fiber-optic cables are being used in ever increasing numbers in modern installations, in part because of their low transmission loss and imperviousness to the effects of electrical and magnetic fields. The final optional element in a measurement system is the point where the measured signal is utilized. In some cases, this element is omitted altogether because the measurement is used as part of an automatic control scheme, and the transmitted signal is fed directly into the control system. In other cases, this element in the measurement system takes the form either of a **signal presentation unit** or of a **signal-recording unit**. Give examples of signal presentation and signal recording units.

1.3 **Mention should be made of instrument characteristics required, environmental conditions, instrument cost, durability, maintainability and constancy of performance.**

Sample answer: The starting point in choosing the most suitable instrument to use for measurement of a particular quantity in a manufacturing plant or other system is the specification of **the instrument characteristics required**, especially parameters like the desired measurement accuracy, resolution, sensitivity and dynamic performance. **It is also essential to know the environmental conditions** that the instrument will be subjected to, as some conditions will immediately either eliminate the possibility of using certain types of instrument or else will create a requirement for expensive protection of the instrument. It should also be noted that protection reduces the performance of some instruments, especially in terms of their dynamic characteristics (for example, sheaths protecting thermocouples and resistance thermometers reduce their speed of response). Provision of this type of information usually requires the expert knowledge of personnel who are intimately acquainted with the operation of the manufacturing plant or system in question. Then, a skilled instrument engineer, having knowledge of all the instruments that are available for measuring the quantity in question, will be able to evaluate the possible list of instruments in terms of their accuracy, cost and suitability for the environmental conditions and thus choose the most appropriate instrument. As far as possible, measurement systems and instruments should be chosen that are as insensitive as possible to the operating environment, although this requirement is often difficult to meet because of cost and other performance considerations. The extent to which the measured system will be disturbed during the measuring process is another important factor in instrument choice. For example, significant pressure loss can be caused to the measured system in some techniques of flow measurement.

Instrument cost is an important consideration. Generally, the better the characteristics, the higher the cost. However, in comparing the cost and relative suitability of different instruments for a particular measurement situation, considerations of **durability, maintainability and constancy of performance are also very important** because the instrument chosen will often have to be capable of operating for long periods without performance degradation and a requirement for costly maintenance. In consequence of this, the initial cost of an instrument often has a low weighting in the evaluation exercise.

Cost is very strongly correlated with the performance of an instrument, as measured by its static characteristics. Increasing the accuracy or resolution of an instrument, for example, can only be done at a penalty of increasing its manufacturing cost. Instrument choice therefore proceeds by specifying the minimum characteristics required by a measurement situation and then searching manufacturers' catalogues to find an instrument whose characteristics match those required. To select an instrument with characteristics superior to those required would only mean paying more than necessary for a level of performance greater than that needed.

As well as purchase cost, other important factors in the assessment exercise are instrument durability and the maintenance requirements. Assuming that one had $20,000 to spend, one would not spend $15,000 on a new motor car whose projected life was five years if a car of equivalent specification with a projected life of ten years was available for $20,000. Likewise, durability is an important consideration in the choice of instruments. The projected life of instruments often depends on the conditions in that the instrument will have to operate. Maintenance requirements must also be taken into account, as they also have cost implications.

As a general rule, a good assessment criterion is obtained if the total purchase cost and estimated maintenance costs of an instrument over its life are divided by the period of its expected life. The figure obtained is thus a cost per year. However, this rule becomes modified where instruments are being installed on a process whose life is expected to be limited, perhaps in the manufacture of a particular model of car. Then, the total costs can only be divided by the period of time that an instrument is expected to be used for, unless an alternative use for the instrument is envisaged at the end of this period.

To summarize therefore, instrument choice is a compromise between performance characteristics, ruggedness and durability, maintenance requirements and purchase cost. To carry out such an evaluation properly, the instrument engineer must have a wide knowledge of the range of instruments available for measuring particular physical quantities, and he/she must also have a deep understanding of how instrument characteristics are affected by particular measurement situations and operating conditions.

1.4 **Discussion should cover regulating trade, monitoring functions and control functions.**

Sample answer: Present-day applications of measuring instruments can be classified into three major areas.

The first of these is their use in regulating trade, applying instruments that measure physical quantities such as length, volume and mass in terms of standard units. Give examples.

The second application area of measuring instruments is in monitoring functions. These provide information that enables human beings to take some prescribed action accordingly. The gardener uses a thermometer to determine whether he should turn the heat on in his greenhouse or open the windows if it is too hot. Regular study of a barometer allows us to decide whether we should take our umbrellas if we are planning to go out for a few hours. Whilst there are thus many uses of instrumentation in our normal domestic lives, the majority of monitoring functions exist to provide the information necessary to allow a human being to control some industrial operation or process. In a chemical process for instance, the progress of chemical reactions is indicated by the measurement of temperatures and pressures at various points, and such measurements allow the operator to take correct decisions regarding the electrical supply to heaters, cooling water flows, valve positions etc. One other important use of monitoring instruments is in calibrating the instruments used in the automatic process control systems described below.

Use as part of automatic feedback control systems forms the third application area of measurement systems. Give an example – e.g. a heating control system. Explain that the characteristics of the measuring instruments used in any feedback control system are of fundamental importance to the quality of control achieved. The accuracy and resolution with which an output variable of a process is controlled can never be better than the accuracy and resolution of the measuring instruments used.

1.5 **Mention and explanation is expected about specifying characteristics required in sensor, environmental conditions around measurement system, choice of primary and possibly secondary sensors, durability (lifetime) and cost, choice of conversion element in case of non-voltage sensor output, signal processing, signal transmission, signal display/recording.**

Sample answer:

Specifying characteristics required: mention of parameters like the desired measurement accuracy, resolution, sensitivity and dynamic performance.

Environmental conditions that the instrument will be subjected to: mention of temperature, humidity, dirt, gases, flames or explosive environment. Some conditions will eliminate the possibility of using certain types of instrument or else create a requirement for expensive protection of the instrument. Mention should be noted that protection reduces the performance of some instruments, especially in terms of their dynamic characteristics (for example, sheaths protecting thermocouples and resistance thermometers reduce their speed of response).

Primary sensor: need to choose a sensor that satisfies characteristics required (accuracy etc) and will work in expected environmental conditions (with protection if necessary). Also need to balance cost of sensor against maintenance requirements plus expected durability and lifetime.

Secondary sensor: if characteristics of primary sensor will be affected by operating conditions, secondary sensors need to be specified that will measure environmental conditions and allow output of primary sensor to be corrected.

Conversion element: this will be needed if output variable of primary transducer is not in a voltage form and has to be converted. Give example, e.g. displacement-measuring strain gauge has an output in the form of a varying resistance that is usually converted to a varying voltage by a bridge circuit.

Signal processing: This improves the quality of the output of a measurement system in some way. Give examples, e.g. electronic amplifier that amplifies the output of the primary transducer or conversion element, thus improving the sensitivity and resolution of measurement. Mention that this is particularly important where the primary transducer has a low output like a thermocouple with a typical output of only a few millivolts. Other types of signal processing element are those that filter out induced noise and remove mean levels etc.

Signal transmission: some measurement systems need a component to transmit the signal to some remote point when the observation or application point of the output of a measurement system is some distance away from the site of the primary transducer. Mention that this is needed where the environmental conditions (e.g. heat) are not suitable for signal display/ signal recording equipment or a controller that the measurement system feeds into. Briefly mention electrical (via copper wires), fiber optic, open air path, radio and pneumatic transmission.

Signal display/recording: Some measurement systems include a component to display or record the measured signal if it is not fed automatically into a feedback control system. Briefly discuss chart recorders, oscilloscopes, digital data recorders etc.

1.6 Explanation and examples are expected as follows:

(a) Imperial units: An early system of measurement units, characterized by having varying and cumbersome multiplication factors relating fundamental units to subdivisions. Examples: length measurement units and conversion factors (1760 for miles to yards, 3 for yards to feet and 12 for feet to inches). Other examples are volume (cubic inch, cubic feet, gallon), mass (pound, hundredweight, ton), pressure (pounds per square inch, atmosphere) etc.

(b) SI units: an internationally-agreed system of metric units where all multiples and sub-divisions of basic metric units are related to the base unit by factors of ten, and such units are therefore much easier to use. Examples are meter for length, cubic meter for volume, gram for mass, bar for pressure etc.

(c) Primary sensor: A sensor that measures the value of some measured quantity. Some examples are thermocouple to measure temperature, load cell to measure mass etc.

(d) Secondary sensor: In some measurement systems, the output of the primary sensor is affected by the environmental conditions (especially the temperature) of the environment that the sensor is operating in. One or more secondary sensors are used to measure each aspect of the environment that is affecting the measurement reading of the primary sensor). For example, if a pressure sensor is affected by the temperature of the environment it is working in, a temperature sensor is used as a secondary sensor to measure the temperature of the environment and allow the reading from the pressure sensor to be corrected.

(e) Signal processing: Signal processing elements exist to improve the quality of the output of a measurement system in some way. A very common type of signal processing element is the electronic amplifier, which amplifies the output of the primary transducer or conversion element, thus improving the sensitivity and resolution of measurement. Other types of signal processing element are those that filter out induced noise and remove mean levels etc.

QUESTIONS AND SOLUTIONS AT END OF CHAPTER 2

2.1 Describe active and passive, null-type and deflection type, analog and digital, indicating and signal-output types. Explain difference between these. Give examples of each. (See section 2.2 in book for fuller details).

2.2 **Sample answer:**

(a) quantity being measured modulates the magnitude of some external power source.

(b) instrument output is entirely produced by the quantity being measured.

Give examples (see section 2.2 in book).

Relative merits:

Active: better measurement resolution (but limited by heating effect due to power source and also by safety considerations – small voltage necessary)

Passive: simpler construction, no power supply needed, limited measurement resolution.

2.3 Null type more accurate but tedious to use.

Deflection-type less accurate but easier to use.

Explain difference by means of an example, e.g. null and deflection types of pressure gauge. (See section 2.2 of book for full details).

Null-type normally reserved for calibration duties where best accuracy is needed.

2.4 Analog instrument gives an output that varies continuously as the quantity being measured changes. The output can have an infinite number of values within the range that the instrument is designed to measure but the number of different positions that the eye can discriminate between is strictly limited, according to how large the scale is and how finely it is divided.

A digital instrument has an output that varies in discrete steps and so can only have a finite number of values.

Give examples of each.

Digital instrument is best for computer control systems since analog to digital conversion is needed for an analog instrument. Explain problems with A-D conversion – cost, conversion time (see section 2.2 in book).

2.5 **Static characteristics:** These are their steady-state attributes (when the output measurement value has settled to a constant reading after any initial varying output) such as accuracy, measurement sensitivity and resistance to errors caused by variations in their operating environment.

Define and explain the various static characteristics (see section 2.3 in book).

Dynamic characteristics: describe behavior of a measuring instrument between the time a measured quantity changes value and the time when the instrument output attains a steady value in response. Depending on time allowed, student may be expected to sketch the main types of dynamic response (zero, first and second order) – see section 2.4 in book.

2.6 Definition and explanation expected for accuracy (or inaccuracy or measurement uncertainty), precision (or repeatability or reproducibility), tolerance, range (or span), linearity (or non-linearity), sensitivity of measurement, threshold, resolution, sensitivity to disturbance – explaining zero drift (or bias) and sensitivity drift, hysteresis, dead space. Student should draw sketches to illustrate as appropriate.

2.7 **Accuracy** more usually expressed as inaccuracy or measurement uncertainty. The latter quantify the extent to which a measurement may be incorrect and are usually expressed as a percentage of the full scale instrument output reading.

Precision is a term that describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high precision instrument, then the spread of readings will be very small.

Precision is often, though incorrectly, confused with accuracy. High precision does not imply anything about measurement accuracy. A high precision instrument may have a low accuracy. Low accuracy measurements from a high precision instrument are normally caused by a bias in the measurements, which is removable by re-calibration.

Depending on time allowed, student should be expected to illustrate with an example of high precision but low accuracy (e.g. figure 2.5 in book).

2.8 See section 2.4 in book for appropriate sketches.

2.9 Dynamic characteristics describe the behavior of an instrument following the time that the measured quantity changes value up until the time when the output reading attains a steady value. Various kinds of dynamic behavior can be observed in different instruments ranging from an output that varies slowly until it reaches a final constant value to an output that oscillates about the final value until a steady reading is obtained. The dynamic characteristics are a very important factor in deciding on the suitability of an instrument for a particular measurement application.

A zero-order instrument responds instantaneously (or effective so) to a change in measured quantity and so is suitable for all measurement situations.

A large number of instruments have a first order characteristic. This limits their use in control systems because it is necessary to take account of the time lag that occurs between a measured quantity changing in value and the measuring instrument indicating the change. Fortunately, the time constant of many first order instruments is small relative to the dynamics of the process being measured, and so no serious problems are created in such cases. However, if there is a need to sample the output of a measurement system at a high frequency, the time lag before the instrument responds to a change in the value of the measured quantity may preclude the use of instruments with a first-order characteristic.

A second-order instrument has an oscillatory output unless damping is applied. When damped, the output response resembles that of a first-order instrument and the above arguments apply.

(See discussion on first and second order instruments in section 2.4 of book for fuller explanation).

2.10 The ***accuracy*** of an instrument is a measure of how close the output reading of the instrument is to the correct value. In practice, it is more usual to quote the ***inaccuracy*** or ***measurement uncertainty*** value rather than the accuracy value for an instrument. Inaccuracy or measurement uncertainty is the extent to which a reading might be wrong, and is often quoted as a percentage of the full-scale reading of an instrument.

Likely error is 1.5% of 1100 ºC (the full scale reading), i.e. 16.5 ºC.

2.11 Tolerance is a term that is closely related to accuracy. It describes the maximum deviation of a manufactured component from some specified value. For instance, crankshafts are machined with a diameter tolerance quoted as so many microns (10^{-6} m), and electric circuit components such as resistors have tolerances of perhaps 5%.

Expected shortest rod is 5000 mm – 2% = 4900 mm. Expected longest rod is 5000 mm + 2% = 5100 mm.

2.12 Maximum error is 1.5% of the full scale reading, i.e. 1.5% × 20,000 = 100 bar.

2.13 Tolerance is a term that is closely related to accuracy. It describes the maximum deviation of a manufactured component from some specified value. Examples might be expected, such as crankshafts are machined with a diameter tolerance quoted as so many microns (10^{-6} m), and electric circuit components such as resistors have tolerances of perhaps 5%.

Maximum deviation in length (given by the tolerance) is 1.5% of the nominal length, i.e. 1.5% × 250 mm = 3.75 mm. Thus shortest and longest bricks likely are 250 – 3.75 and 250 + 3.75, i.e. 246.25 and 253.75

2.14 Range is 7.5 – 5.0 = 2.5 cm.

2.15 (a) Range is 50 bar

(b) Possible measurement error is 0.5% × 50 bar = 0.25 bar

(Error is quoted as % of full scale reading, not % of actual reading)

2.16

The minimum likely value is 5000Ω – 3% = 5000 – 150 = 4850Ω.

The maximum likely value is 5000Ω + 3% = 5000 + 150 = 5150Ω.

2.17 (a) The maximum error expected in any measurement reading is 0.5% of the full scale reading, which is 30 bar for this particular instrument. Hence, the maximum likely error is 0.5% x 30 bar = 0.15 bar [30%]

(b) The maximum measurement error is a constant value related to the full scale reading of the instrument, irrespective of the magnitude of the quantity that the instrument is actually measuring. In this case, as worked out above, the magnitude of the error is 0.15 bar. Thus, when measuring a pressure of 5 bar, the maximum possible error of 0.15 bar is 3% of the measurement value. [30%]

(c) If the measurement error is deemed to be too high, you could either use a higher quality instrument with better accuracy (usually more expensive) or use a pressure sensor with a similar quality (and price) but that has a smaller range, for example 0-10 bar. The recommended option is the latter (because it is cheaper). [40%]

2.18

(a) The mean (average) value of the ten measurements made with the ultrasonic rule is 4.293 meters.

The maximum deviation below this mean value is -0.003 meters and the maximum deviation above the mean value is +0.003 meters. Thus the precision of the ultrasonic rule can be expressed as ±0.003 meters (±3 mm).

[50%]

(b) The correct value of the room width has been measured as 4.276 meters by the calibrated steel rule. All ultrasonic rule measurements are above this, with the largest value being 4.296 meters. This last measurement is the one that exhibits the largest measurement error. This maximum measurement error can be calculated as : 4.296 – 4.276 = 0.020 meters (20 mm). Thus the maximum measurement inaccuracy can be expressed as +20 mm.
[50%]

2.19

(a)

Precision is a term that describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high precision instrument, then the spread of readings will be very small.

Accuracy more usually expressed as inaccuracy or measurement uncertainty. The latter quantify the extent to which a measurement may be incorrect and are usually expressed as a percentage of the full scale instrument output reading.

Precision is often, though incorrectly, confused with accuracy. High precision does not imply anything about measurement accuracy. A high precision instrument may have a low accuracy. Low accuracy measurements from a high precision instrument are normally caused by a bias in the measurements, which is removable by re-calibration.

An illustration showing high precision but low accuracy should be given (such as figure 2.5 in book).

[60%]

(b)

Precision expressed as the spread of values from minimum to maximum = 70.7 - 69.4 = 1.3 Kg

Mean reading = 69.9 Kg; correct reading = 70.5 Kg; Inaccuracy = 0.6 Kg = 0.85% of correct value

[40%]

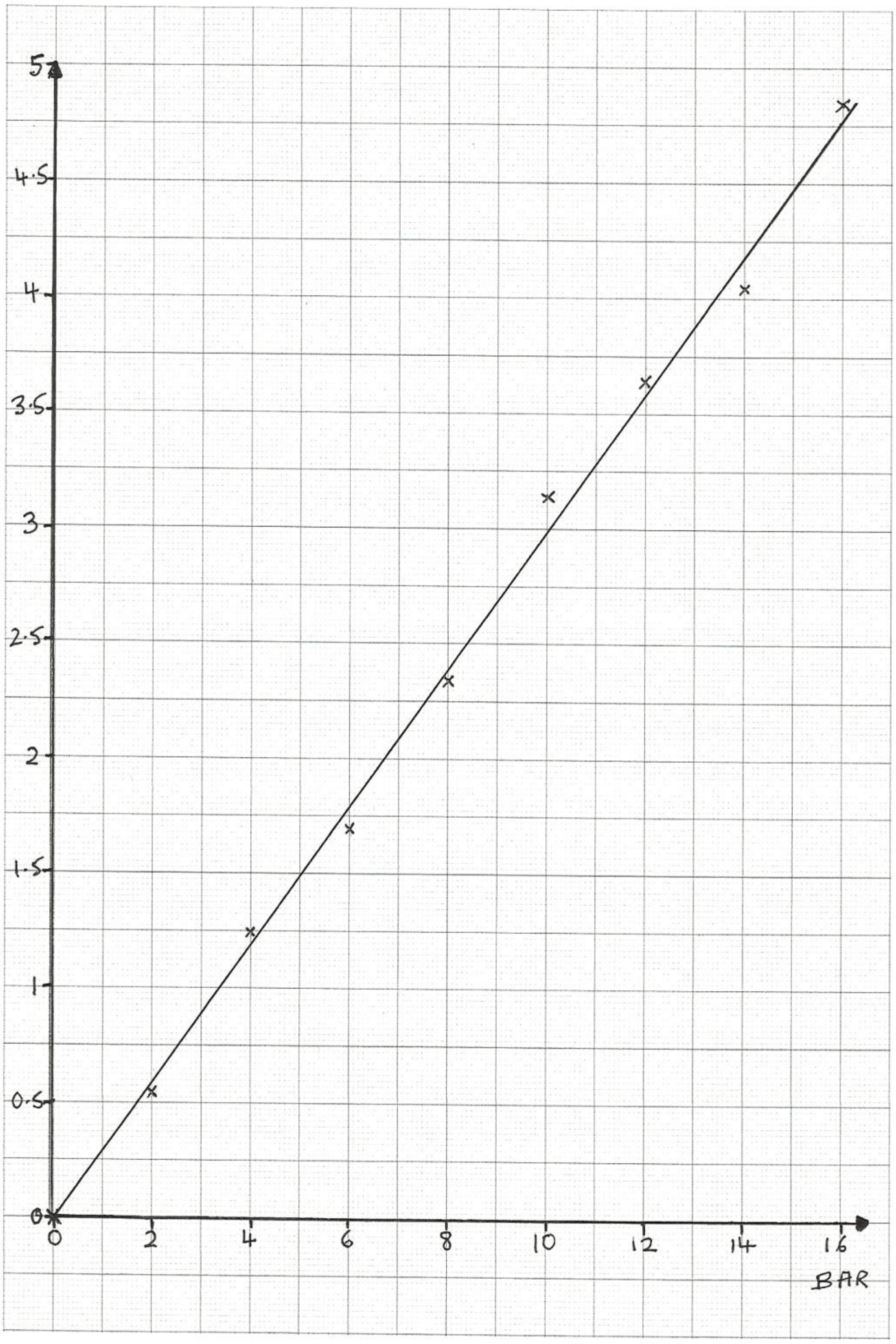
2.20 Given table of data values is:

mV	4.37	8.74	13.11	17.48
°C	250	500	750	1000

Students may make a graph of these data points and fit a straight line in order to determine the sensitivity. This is acceptable. However, there is a quicker analytical solution in this case since the output e.m.f. increments by exactly 4.37 mV for each 250 °C rise in temperature. The sensitivity can therefore be calculated as 4.37/250 = 0.0175 mV/°C. (N.B. The sensitivity is expressed in units of mV/°C since this is what the question asked for, but it would be more appropriate in practice to express the sensitivity as 17.5 μV/°C).

2.21 Sensitivity is 47 mV/bar

2.22 **(a)**



(b) From graph, output is 4.75 Volts for input of 16 bar. Thus, sensitivity is 4.75/16 = 0.297 V/bar

(c) Maximum non-linearity: On graph drawn, this is apparently the data point for input of 10 bar.

Measured output for data point at 10 bar input is 3.15V (taken from table of measured input-output data).

Output read from graph for input of 10 bar is 3.00V.

Thus non-linearity is 3.15 – 3.00 = 0.15V

Full scale deflection is 4.85V.

Hence, maximum non-linearity expressed as a percentage $= \frac{0.15}{4.85} \times 100 = 3.09\ \%$

[Marking: 40% for part (a) , 30% for part (b) and 30% for part (c)]

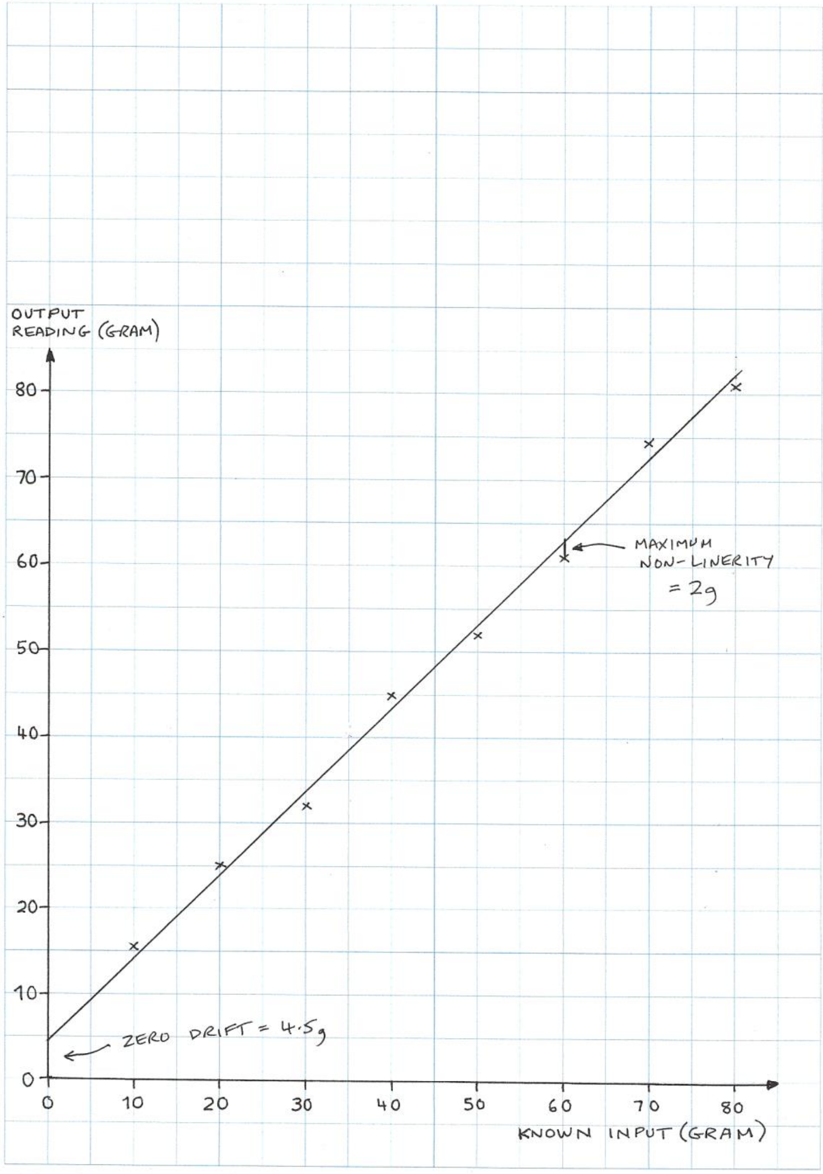
2.23 **Sensitivity Drift** (also known as scale factor drift) defines the amount by which an instrument's sensitivity of measurement varies as environmental conditions (e.g. temperature and pressure) change.

Zero drift (also known as bias) describes the effect where the zero reading of an instrument is modified by a change in environmental conditions. This causes a constant error that exists over the full range of measurement of the instrument.

Sensitivity drift and Zero drift are collectively known as the Sensitivity to Disturbance of an instrument. As variations occur in the temperature, pressure etc. in the environment surrounding a measurement system, certain static instrument characteristics change, and the sensitivity to disturbance is a measure of the magnitude of this change. This change can cause either zero drift or sensitivity drift, or sometimes both of these.

See section 2.3 in the book for a fuller explanation.

2.24 **(a)**



(b) The three measurement characteristics evident in the data plotted on the graph are non-linearity, zero drift and sensitivity drift.

(c) Maximum non-linearity: On graph drawn, this is apparently the data point for input of 60 gram.

Measured output for data point at 60g input is 61g (taken from table of measured input-output data).

Output read from graph for input of 60g is 63g.

Thus non-linearity is 63 - 61 = 2g

Zero drift: This is the output when the input is zero. From the graph, zero drift = 4.5g.

Sensitivity drift: Correct measurement sensitivity is 1 (output should equal input)

Sensitivity calculated from graph (gradient of straight line fitted to the data $\frac{(82.0-4.5)}{80} = \frac{77.5}{80} = 0.97$) is

Hence, sensitivity drift $= 0.97 - 1.00 = -0.03$

[Marking: 40% for part (a) , 20% for part (b) and 40% for part (c)]

2.25 **Zero drift** (also known as bias) describes the effect where the zero reading of an instrument is modified by a change in environmental conditions (e.g. environmental temperature and pressure). This causes a constant error that exists over the full range of measurement of the instrument.

The zero drift coefficient is the amount of change in the instrument output for a given change in an environmental parameter such as temperature or pressure. If the characteristic of an instrument is sensitive to several environmental parameters, then it will have several zero drift coefficients, one for each environmental parameter.

If an instrument has a voltage output, the zero drift due to environmental temperature change would be expressed in units of volts/°C. [40%]

The zero drift at the temperature of 30ºC is the constant difference between the pairs of output readings, i.e. 0.19 pascal. [20%]

The zero drift coefficient is the magnitude of drift (0.19 pascal) divided by the magnitude of the temperature change causing the drift (10ºC). Thus the zero drift coefficient is 0.19/10 = 0.019 pascal/ºC.

[30%]

2.26 **(a)** Given table of data values at a temperature of 20°C is:

y	13.1	26.2	39.3	52.4	65.5	78.6
x	5	10	15	20	25	30

Students may make a graph of these data points and fit a straight line in order to determine the sensitivity. This is acceptable. However, there is a quicker analytical solution in this case since the value of y increments by exactly 13.1 for each increment of 5 in the value of x.

The sensitivity, expressed as the ratio y/x can therefore be calculated as 13.1/5 = 2.62

[40%]

(b) When the instrument is subsequently used in an environment at a temperature of 50°C, the table of data values changes to the following:

y	14.7	29.4	44.1	58.8	73.5	88.2
x	5	10	15	20	25	30

As before, graphical analysis of the data is unnecessary since this time value of y increments by exactly 14.7 for each increment of 5 in the value of x. The new sensitivity, expressed as the ratio y/x can therefore be calculated as 14.7/5 = 2.94

The sensitivity of 2.62 at 20°C has changed to a sensitivity of 2.94 at 50°C

Thus, the sensitivity changes by 0.32 as the temperature increases by 30°C

Hence, the sensitivity drift (change in sensitivity per °C) can be expressed as 0.32/30 = 0.01067/°C

[60%]

2.27 At 20°C, deflection/load characteristic is a straight line. Sensitivity = 90 degrees/Kg.

At 27°C, deflection/load characteristic is still a straight line. Sensitivity $= \frac{(191-6)}{2} = 92.5$ degrees/Kg.

Sensitivity drift = 2.5 degrees/Kg [20%]

Zero drift (bias) = 6 degrees (the no-load deflection) [20%]

Sensitivity drift/°C = 2.5/7 = 0.357 (degrees per Kg)/°C [30%]

Zero drift/°C = 6/7 = 0.857 degrees/°C [30%]

2.28 The table of measurements given is:

Values measured by uncalibrated instrument (°C)	**Correct value of temperature (°C)**
20	21.5
35	36.5
50	51.5
65	66.5

The bias is the difference between the correct value and the measured value. This difference is 1.5°C for each pair of measurements.

Hence, the bias due to the instrument being out of calibration is 1.5°C

2.29 Bias is + 3 bar.

2.30

Zero drift at 40°C = value of y for x = 0 at 40°C = 0.5

Subtract the zero drift from all values of y at 40°C:

x:	0	20	40	60	80	100
y:	0	31.5	63.0	94.5	126.0	157.5

Sensitivity at 20°C = y/x = 1.55

Sensitivity at 40°C = y/x = 1.575

Hence, total sensitivity drift = 1.575 - 1.55 = 0.025

2.31 The measurements in an environment at a temperature of 21°C are:

Load(Kg)	0	50	100	150	200
Deflection(mm)	0.0	1.0	2.0	3.0	4.0

At 35°C, the measurements change to:

Load(Kg)	0	50	100	150	200
Deflection(mm)	0.2	1.3	2.4	3.5	4.6

(a) At 21°C, the deflection increases by 1.0 mm for each 50 Kg increase in load. Therefore, the sensitivity is 1.0/50 = 0.020 mm/Kg = 20 µm/Kg

At 35°C, the deflection increases by 1.1 mm for each 50 Kg increase in load. Therefore, the sensitivity is 1.1/50 = 0.022 mm/Kg = 22 µm/Kg

[40%]

(b) The total zero drift due to the increase in temperature is the change in deflection when the load is zero, i.e. 0.2 mm.

The total sensitivity drift due to the increase in temperature is the change in sensitivity, i.e. (22 – 20) = 2.0 µm/Kg.

[30%]

(c) The zero drift of 0.2 mm is caused by a temperature increase of 14°C. Thus, the zero drift coefficient can be expressed as 0.2/14 = 0.0143 mm/°C = 14.3 µm/°C

The sensitivity drift caused by the temperature increase of 14°C is 2.0 µm/Kg. Thus, the sensitivity drift coefficient can be expressed as 2/14 = 0.143 (µm per Kg)/(°C).

[30%]

2.32

Let the temperature reported by the balloon at some general time t be T_r. Then T_h is related to T_r by the relation: $T_r = \frac{T_h}{1+\tau D} = \frac{T_o - 0.012h}{1+\tau D} = \frac{20-0.012h}{1+10D}$

It is given that h = 6t (velocity is 6 m/s), thus: $T_r = \frac{20-0.072t}{1+10D}$

The transient or complementary function part of the solution (T_h = 0) is given by: $T_{r_{cf}} = Ce^{-t/10}$

The particular integral part of the solution is given by: $T_{r_{pi}} = 20 - 0.072(t-10)$

Thus, the whole solution is given by: $T_r = T_{r_{cf}} + T_{r_{pi}} = Ce^{-t/10} + 20 - 0.072(t-10)$

Applying initial conditions: At t = 0, T_r = 20, i.e. $20 = Ce^{-0} + 20 - 0.072(-10)$

Thus C = –0.72 and the solution can be written as: $T_r = 20 - 0.72e^{-t/10} - 0.072(t-10)$

Using the above expression to calculate T_r for various values of t, the following table is constructed:

Time	Altitude	Temperature reading	Temperature error
0	0	0.00	0.00
10	60	19.28	0.46
20	120	18.56	0.62
30	180	17.84	0.68
40	240	17.12	0.71
50	300	16.40	0.72
60	360	15.68	0.72
70	420	14.96	0.72
80	480	14.24	0.72
90	540	13.52	0.72
100	600	12.80	0.72

[50%]

(b) At 8000m, t = 1333.3 seconds. Calculating T_r from the above expression:

$$T_r = 20 - 0.72e^{-13333/10} - 0.072(1333.3-10)$$

The exponential term approximates to zero and so T_r can be written as:

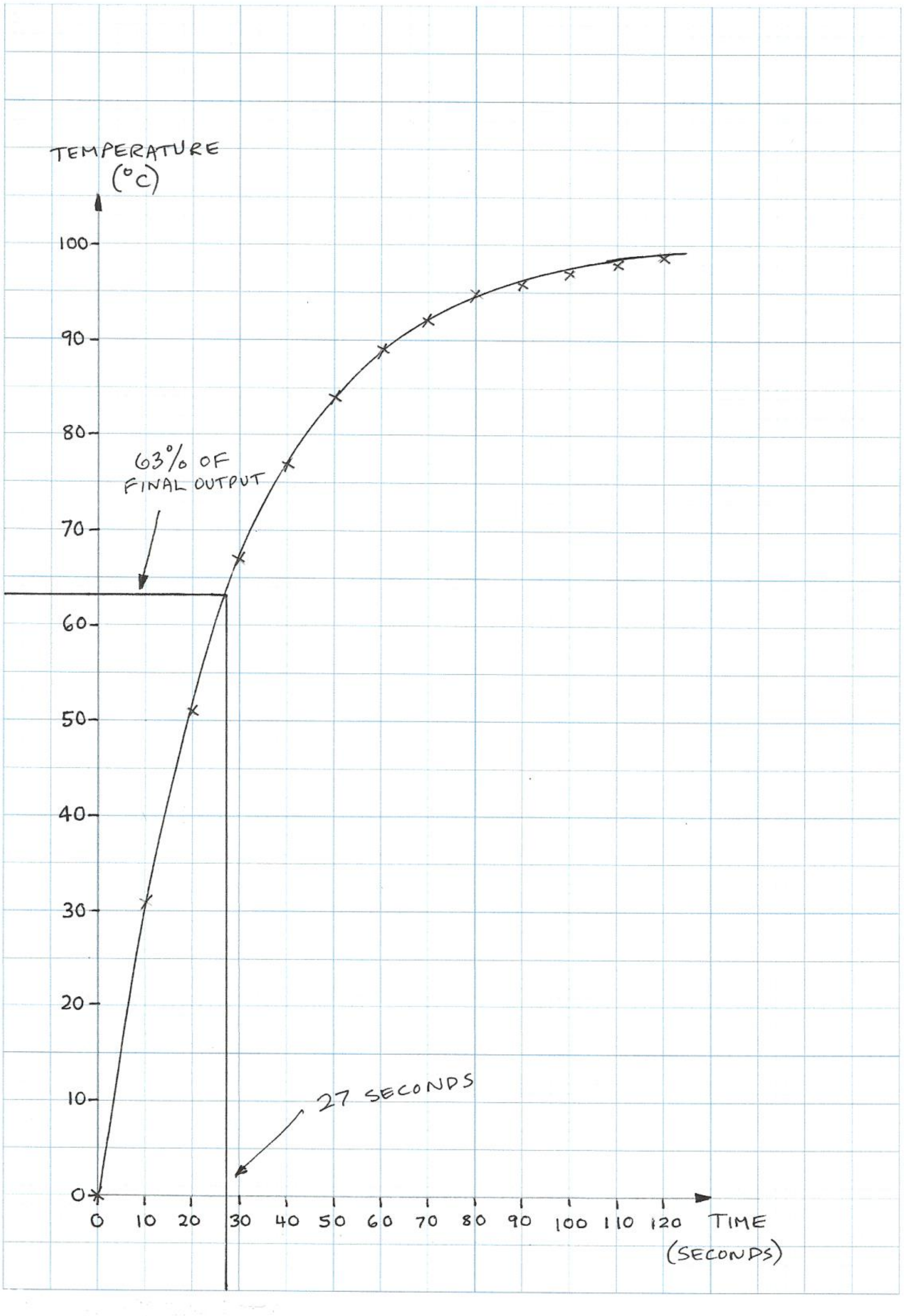
$$T_r \approx 20 - 0.072(1323.3) = -75.28^\circ C$$ [30%]

(c) Since the temperature falls according to the relation: T_h = T_0 - 0.012h, the true temperature at an altitude of 8000m is T_h = T_0 - 0.012h = 20 - (0.012×8000) = 20 – 96 = 76

Thus the temperature error is 0.72 [20%]

This result might have been inferred from the table above where it can be seen that the error has converged to a value of 0.72. For large values of *t*, the transducer reading lags the true temperature value by a period of time equal to the time constant of 10 seconds. In this time, the balloon travels a distance of 60 metres and the temperature falls by 0.72°. Thus for large values of *t*, the output reading is always 0.72° less than it should be.

2.33



[50%]

The time constant is the time taken for the output reading to rise to 63% of its final value. Since the output reading is rising from 0°C to 100°C, this means the time when the output has risen to 63 °C. Using the graph of temperature readings, this point is reached after 27 seconds. Thus the time constant of the thermometer is 27 seconds. [50%]

2.34 Note: it is assumed that students are familiar with the solution of a first order differential equation, as commonly covered in mathematics courses.

The water temperature on the sea surface, T_0, is 20°C.

The temperature T_x at a depth of x meters is given by the relation: $T_x = T_0 - 0.01x$

The temperature-measuring instrument characteristic is approximately first order with a time constant (τ) of 50s.

It is also given that the submarine is diving at a rate of 0.5 meters/second, i.e. $x = 0.5t$

Part (a)

Let the temperature reported by the temperature sensor at some general time (t) be T_r .

Thus, the temperature reading T_r is related to T_x by the expression:

$$T_r = \frac{T_x}{1+\tau D} = \frac{T_0 - 0.01x}{1+50D} = \frac{20-(0.01\times[0.5t])}{1+50D} = \frac{20-0.005t}{1+50D}$$

The transient or complementary function part of the solution (T_x = 0) is given by:

$$T_{r_{cf}} = Ce^{-t/50}$$

The particular integral part of the solution is given by:

$$T_{r_{pi}} = 20 - 0.005(t-50)$$

Thus, the whole solution is given by:

$$T_r = T_{r_{cf}} + T_{r_{pi}} = Ce^{-t/50} + 20 - 0.005(t-50)$$

Applying initial conditions: At t = 0, T_r = 20, i.e. $20 = Ce^{-0} + 20 - 0.005(-50)$

Thus, $C = 0.25$ and therefore $T_r = 0.25e^{-t/50} + 20 - 0.005(t-50)$

Using the above expression to calculate T_r , for various values of t, the following table can be constructed:

Time (t)	Depth (x) in meters	Actual temperature (T_x) in °C	Temperature reading (T_r) in °C	Temperature error in °C
0	0	20.0	20.0	0.0
100	50	19.5	19.716	0.216
200	100	19.0	19.245	0.245
300	150	18.5	18.749	0.249
400	200	18.0	18.250	0.250
500	250	17.5	17.750	0.250

[70%]

Part (b) At 1000 meters, t = 2000 seconds.

Calculating T_r from the above expression:

$$T_r = 0.25e^{-t/50} + 20 - 0.005(t-50)$$

The exponential term approximates to zero and so T_r can be written as:

$$T_r \approx 20 - 0.005(1950) = 20 - 9.75 = 10.25\,°\text{C}.$$ [30%]

2.35 For a step input, the general differential equation describing the behavior of a second order system can be written as:

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (1)$$

where q_i is the measured quantity, q_o is the instrument output reading, and a_o , a_1 , a_2 and b_o are constants.

Applying the D operator: $a_2 D^2 q_o + a_1 D q_o + a_0 q_o = b_0 q_i$, and re-arranging:

$$q_o = \frac{b_0 q_i}{a_0 + a_1 D + a_2 D^2} \quad (2)$$

It is convenient to re-express the variables a_0, a_1, a_2 and b_0 in equation (2) in terms of three parameters K (static sensitivity), ω (undamped natural frequency) and ξ (damping ratio), where:

$$K = b_0/a_0 \quad ; \quad \omega = \sqrt{a_0/a_2} \quad ; \quad \xi = a_1/2\sqrt{a_0 a_2}$$

ξ can be written as $\xi = \frac{a_1}{2a_0\sqrt{a_2/a_0}} = \frac{a_1 \omega}{2a_0}$

If equation (2) is now divided through by a_0 we get:

$$q_o = \frac{(b_0/a_0)q_i}{1 + (a_1/a_0)D + (a_2/a_0)D^2} \quad (3)$$

The terms in equation (2.9) can be written in terms of ω and ξ as follows:

$$\frac{b_0}{a_0} = K \quad ; \quad \left(\frac{a_1}{a_0}\right)D = \frac{2\xi D}{\omega} \quad ; \quad \left(\frac{a_2}{a_0}\right)D^2 = \frac{D^2}{\omega^2}$$

Hence, dividing equation (3) through by q_i and substituting for a_0, a_1 and a_2 gives:

$$\frac{q_0}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1} \quad (4)$$

See Figure 2.12 in section 2.4(c) for sketches of instrument response in heavy damped, critically camped and lightly damped cases.

Student should identify the critically damped case (ξ = 0.707) as the usual design target.

2.36

Let T_r be the temperature indicated by the thermocouple at some time t and T_x be the actual temperature.

For a first order system with time constant τ, $T_r = T_x - (T_x - T_0)e^{-t/\tau}$

where T_0 is the temperature at time 0.

Substituting in values: T_x = 100°C, T_0 = 20°C, τ = 4 seconds and t = 8 seconds:

Hence $T_r = 100 - \left[(100 - 20) \times e^{-8/4}\right] = 100 - 80/e^2$ = 100 - 10.83 = 89.17°C.

Thus, it would indicate a temperature of 89.2°C after 8 seconds.