# Chapter 2

1. For a particle Newton’s second law says ![vector(F) = m vector(a) = m[(d^2 x)/(d t^2) i-hat + (d^2 y)/(d t^2) j-hat + (d^2 z)/(d t^2)k-hat]]().

Take the second derivative of each of the expressions in Equation (2.1): . Substitution into the previous equation gives ![vector(F) = m vector(a) = m[(d^2 x^(prime))/(d t^2) i-hat + (d^2 y^(prime))/(d t^2) j-hat + (d^2 z^(prime))/(d t^2) k-hat] = vector(F^prime) .]()

1. From Equation (2.1) ![vector(p) = m[(d x)/(d t) i-hat + (d y)/(d t) j-hat + (d z)/(d t) k-hat]]().

In a Galilean transformation .

Substitution into Equation (2.1) gives ![vector(p) = m[(d x^(prime))/(d t) + v] i-hat + (d y^(prime))/(d t) j-hat + (d z^(prime))/(d t) k-hat != vector(p ^prime)]().

However, because ![vector(p ^prime) = m[(d x^(prime))/(d t) i-hat + (d y^(prime))/(d t) j-hat + (d z^(prime))/(d t) k-hat]]() the same form is clearly retained, given the velocity transformation.

1. Using the vector triangle shown, the speed of light coming toward the mirror is  and the same on the return trip. Therefore the total time is . Notice that , so .



1. As in Problem 3, , so ![theta = sin^(negative 1) (v_1 / v_2) = sin^(negative 1) [(0.350 m / s)/(1.25 m / s)] = 16.3 degrees]() and .
2. When the apparatus is rotated by 90°, the situation is equivalent, except that we have effectively interchanged  and . Interchanging  and  in Equation (2.3) leads to Equation (2.4).
3. Let *n* = the number of fringes shifted; then . Because , we have . Solving for *v* and noting that  +  = 22 m, 
4. Letting (where ) the text equation (not currently numbered) for  becomes



which is identical to when  so as required.

1. Since the Lorentz transformations depend on *c* (and the fact that *c* is the same constant for all inertial frames), different values of *c* would necessarily lead two observers to different conclusions about the order or positions of two spacetime events, in violation of postulate 1.
2. Let an observer in K send a light signal along the + *x*-axis with speed *c*. According to the Galilean transformations, an observer in  measures the speed of the signal to be . Therefore the speed of light cannot be constant under the Galilean transformations.
3. From the Principle of Relativity, we know the correct transformation must be of the form (assuming and ):

.

The spherical wave front equations (2.9a) and (2.9b) give us:

 .

Solve the second wave front equation for and substitute into the first:

![c t = [((a c + b) (a c minus b) t)/c]]() or 

Now *v* is the speed of the origin of the -axis. We can find that speed by setting which gives , or , or equivalently *b* = *av*. Substituting this into the equation above for  yields . Solving for *a*: 

This expression, along with *b = av*, can be substituted into the original expressions for *x* and to obtain:



which in turn can be solved for *t* and  to complete the transformation.

1. When we find , so:

 ;

;

;

.

1. (a) First we convert to SI units: 95 km/h = 26.39 m/s, so 

(b) 

(c)  so 

(d) Converting to SI units, 27,000 km/h = 7500 m/s, so 

(e) (25 cm)/(2 ns) = so 

(f) , so 

1. From the Lorentz transformations![Delta t^(prime) = gamma [Delta t minus v Delta x / (c^2)]](). But in this case, so solving for *v* we find . Inserting the values and , we find . We conclude that the frame travels at a speed *c*/2 in the -direction. Note that there is no motion in the transverse direction.
2. Try setting. Thus . Solving for *v* we find , which is impossible. There is no such frame .
3. For the smaller values of β we use the binomial expansion .
4. 
5. 
6. 
7. 
8. 
9. 
10. There is no motion in the transverse direction, so m.

 

 

 

1. (a) 

(b) With we find . Then m, m,![(x^prime) = gamma (x minus v t) = (5/3) [3 m minus (2.40 * (10^8) m / s) (3.86 * 10^(negative 8) s)] = negative 10.4 m]()

![(t^prime) = gamma (t minus v x / c^2) = (5/3) [(3.86 * 10^(negative 8) s) minus (2.40 * (10^8) m / s) (3 m) / (3.00 * (10^8) m / s)^2] = 51.0 n s]()

(c)  which equals *c* to within rounding errors.

1. At the point of reflection the light has traveled a distance . On the return trip it travels . Then the total time is . But from time dilation we know (with ) that . Comparing these two results for  we get which reduces to . This is Equation (2.21).
2. (a) With a contraction of 1%, . Thus  Solving for , we find or 

(b) The time for the trip in the Earth-based frame is . With the relativistic factor (corresponding to a 1% shortening of the ship’s length), the elapsed time on the rocket ship is 1% less than the Earth-based time, or a difference of 

1. The round-trip distance is *d* = 40 ly. Assume the same constant speed for the entire round trip. In the rocket’s reference frame the distance is only  Then in the rocket’s frame of reference  Rearranging . Solving for we find , or . To find the elapsed time *t* on Earth, we know  y, so 
2. In the muon’s frame  In the lab frame the time is longer; see Equation (2.19): . In the lab the distance traveled is , since . Therefore , so . Now all quantities are known except β. Solving for β we find  or .
3. Converting the speed to m/s we find 25,000 mi/h = 11,176 m/s. From tables the distance is . In the earth’s frame of reference the time is the distance divided by speed, or . In the astronauts’ frame the time elapsed is. The time difference is ![Delta t = t minus t^(prime) = t minus t sqrt(1 minus beta^2) = t[1 minus sqrt(1 minus beta^2)]](). Evaluating numerically ![Delta t = 34,359 s [1 minus sqrt(1 minus ((11,176 m / s)/(3.00 * (10^8) m / s))^2)] = 2.4 * 10^(negative 5) s .]()
4. , so we know that . Solving for *v* we find 
5. so clearly in this case. Thus  and solving for *v* we find 
6. The clocks’ rates differ by a factor of. Because  is very small we will use the binomial theorem approximation . Then the time difference is . Using and the fact that the time for the trip equals distance divided by speed,





1. (a) 

(b) Earth’s frame: 

 Golf ball’s frame: 

1. Spacetime invariant (see Section 2.9): . We know  km, , and km. Thus and  s.
2. (a) Converting *v* = 120 km/h = 33.3 m/s. Now with *c* = 100 m/s, we have  and . We conclude that the moving person ages 6.1% slower.

(b) 

1. Converting *v* = 300 km/h = 83.3 m/s. Now with *c* =100 m/s, we have  and  So the length is  m.
2. Let subscript 1 refer to firing and subscript 2 to striking the target. Therefore we can see that m, m, and ns. 

To find the four primed quantities we can use the Lorentz transformations with the known values of , , , and . Note that with , .

 

1. Start from the formula for velocity addition, Equation (2.23a): .

(a) 

(b) 

1. Velocity addition, Equation (2.24):  with and 



1. Conversion: 110 km/h = 30.556 m/s and 140 km/h = 38.889 m/s. Let  m/s and m/s. Our premise is that m/s. Then by velocity addition,  By symmetry each observer sees the other one traveling at the same speed.
2. From Example 2.5 we have ![u = (c/n) [(1 + n v / c)/(1 + v / n c)]](). For light traveling in opposite directions ![Delta u = (c/n) [(1 + n v / c)/(1 + v / n c) minus (1 minus n v / c)/(1 minus v / n c)]](). Because  is very small, use the binomial expansion: , where we have dropped terms of order  Similarly  Thus ![Delta u approximately (c/n) [(1 + n v / c minus v / n c) minus (1 minus n v / c + v / n c)] = (2 v)/n(1 minus 1 / n) = 2 v(1 minus 1 / n^2) .]() Evaluating numerically we find 
3. Clearly the speed of B is just . To find the speed of C use and : 
4. We can ignore the 400 km, which is small compared with the Earth-to-moon distance m. The rotation rate is  rad/s. Then the speed across the moon’s surface is 
5. Classical: . Then ![N = N_0 exp[(negative (ln 2) t)/(t_(1 / 2))] = 14.6]() or about 15 muons.

Relativistic:  so ![N = N_0 exp[(negative (ln 2) t)/(t_(1 / 2))] = 2710 muons .]() Because of the exponential nature of the decay curve, a factor of five (shorter) in time results in many more muons surviving.

1. The circumference of the fixed point’s rotational path is , where  Earth’s radius = 6378 km. Thus the circumference of the path is 31,143 km. The rotational speed of that point is . The observatory clock runs slow by a factor of . In 41.2 h the observatory clock is slow by  h = 107 ns. In 48.6 h it is slow by  h = 126 ns. The Eastward-moving clock has a ground speed of 31,143 km/41.2 h = 755.9 km/h = 210.0 m/s and thus has a net speed of 210.0 m/s + 360.5 m/s = 570.5 m/s. For this clock  and in 41.2 hours it runs slow by  h = 268 ns. The Westward-moving clock has a ground speed of 31,143 km/48.6 h = 640.8 km/h = 178.0 m/s and thus has a net speed of 360.5 m/s − 178.0 m/s = 182.5 m/s. For this clock  and in 48.6 hours it runs slow by  h = 32 ns. So our prediction is that the Eastward-moving clock is off by 107 ns – 269 ns ns, while the Westward-moving clock is off by 126 ns − 32 ns = 94 ns. These results are correct for special relativity but do not reconcile with those in the table in the text, because general relativistic effects are of the same order of magnitude.
2. The derivations of Equations (2.31) and (2.32) in the beginning of Section 2.10 will suffice. Mary receives signals at a rate  for and a rate for . Frank receives signals at a rate  for  and a rate for .
3. ; Frank sends signals at rate *f*, so Mary receives  signals.

; Mary sends signals at rate *f* , so Frank receives  signals.

1. ; Using the Lorentz transformation



1. For a timelike interval so . We will prove by contradiction. Suppose that there is a frame is which the two events were simultaneous, so that . Then by the spacetime invariant . But because , this implies  which is impossible because is real.
2. As in Problem 42, we know that for a spacelike interval so . Then in a frame  in which the two events occur in the same place, and . But because we have , which is impossible because is real.
3. In order for two events to be simultaneous in , the two events must lie along the axis, or along a line parallel to the axis. The slope of the axis is , so . Solving for *v*, we find. Since the slope of the axis must be less than one, we see that  so  is required.



1. parts (a) and (b) To find the equation of the line use the Lorentz transformation. With we have or, rearranging, . Thus the graph of *ct* vs. *x* is a straight line with a slope . (c) Now with constant, the Lorentz transformation gives . Again we solve for *ct* : . This line is parallel to the  line we found earlier but shifted by the constant. (d) Here both the  and  axes are shifted from their normal (*x*, *ct*) orientation and they are not perpendicular. The angle between the axes decreases as *β* increases.



1. The diagram is shown here. Note that there is only one worldline for light, and it bisects both the *x*, *ct* axes and the axes. The and axes are not perpendicular. This can be seen as a result of the Lorentz transformations, since defines the axis and defines the axis.

 

1. The diagram shows that the events A and B that occur at the same time in K occur at different times in .



1. The Doppler shift gives . With numerical values  nm and nm, solving this equation for gives . The astronaut’s speed is m/s. In addition to a red light violation, the astronaut gets a speeding ticket.
2. According to the fixed source (K) the signal and receiver move at speeds *c* and *v*, respectively, in opposite directions, so their relative speed is . The time interval between receipt of signals is . By time dilation . Using  and  we find and .
3. For a fixed source and moving receiver, the length of the wave train is  Since *n* waves are emitted during time *T*, and the frequency is. As in the text  and  Therefore .
4. 



1. The Doppler shift function 

is the rate at which #1 and #2 receive signals from each other and the rate at which #2 and #3 receive signals from each other. But for signals between #1 and #3 the rate is .

1. The Doppler shift function  is the rate at which #1 and #2 receive signals

from each other and the rate at which #2 and #3 receive signals from each other. As for #1 and #3 we will assume that these plumbing vans are non-relativistic . Otherwise it would be necessary to use the velocity addition law and apply the transverse Doppler shift. From the figure we see that . Now  and . With an angle of ,  and .



1. The Doppler shift to higher wavelengths is (with  nm) . Solving for  we find . Then which is 20.25 days. One problem with this analysis is that we have only computed the time as measured by Earth. We are not prepared to handle the non-inertial frame of the spaceship.
2. Let the instantaneous momentum be in the *x*-direction and the force be in the *y*-direction. Then  and  is also in the *y*-direction. So we have .
3. The magnitude of the centripetal force is for circular motion. For a charged particle , so or, rearranging . Therefore



When the speed increases the momentum increases, and thus for a given value of *B* the radius must increase.

1.  and . The momentum is the product of two factors that contain the velocity, so we apply the product rule for derivatives:

 ![vector(F) = m d/(d t) [(m vector(v))/(sqrt(1 minus v^2 / c^2))] = m[(d vector(v) / d t)/(sqrt(1 minus v^2 / c^2)) + vector(v) d/(d t) (1/(sqrt(1 minus v^2 / c^2)))] = gamma m vector(a) + m vector(v) (negative 1/2) (negative (2 v)/(c^2)) (gamma^3) (d v)/(d t) = gamma m vector(a) + (gamma^3) m vector(a) ((v^2)/(c^2)) = (gamma^3) m vector(a) [1 minus (v^2)/(c^2) + (v^2)/(c^2)] = (gamma^3) m vector(a)]()

1. From the preceding problem . We have m/s2 and  kg.
2. 
3. As in (a) and  N
4. As in (a) and N
5. As in (a) and N
6.  with ; 
7. The initial momentum is.

(a) 

 

Substituting forand solving for *v*, ![v = [1/((.58312 c)^2) + 1/(c^2)]^(negative 1 / 2) = 0.504 c .]()

 (b) Similarly ![v = [1/((.63509 c)^2) + 1/(c^2)]^(negative 1 / 2) = 0.536 c]()

 (c) Similarly ![v = [1/((1.1547 c)^2) + 1/(c^2)]^(negative 1 / 2) = 0.756 c]()

1. 6.3 GeV protons have MeV and MeV. Then . Converting to SI units 

 From Problem 56 we have .

1. Initially Mary throws her ball with velocity (primes showing the measurements are in Mary’s frame):  After the elastic collision, the signs on the above expressions are reversed, so the change in momentum as measured by Mary is .

Now for Frank’s ball, we know and . The velocity transformations give for Frank’s ball as measured by Mary: .