

**Photovoltaic Systems Engineering, 4<sup>th</sup> Ed.  
Chapter 1 Problem Solutions**

**1.1** Prove equation 1.5.

Using (1.5) with  $N(n) = 2N_0$ , results in  $2N_0 = N_0 e^{n \ln(1+i)}$ . Solving for  $n$  yields the result

$$n = \frac{\ln 2}{\ln(1+i)} = D \cong \frac{.693}{i} \quad \text{since } \ln(1+i) \cong i \text{ for small } i.$$

**1.2** Calculate the approximate and exact doubling times for annual percentage increases of 5%, 10%, 15% and 20%.

% Increase	5	10	15	20
Exact D	14.2 yr	7.27 yr	4.96 yr	3.80 yr
Approx D	14 yr	7.00 yr	4.67 yr	3.50 yr
Approx D/Exact D	0.99	0.96	0.94	0.92

**1.3** The human population of the earth is approximately 7 billion and is increasing at approximately 2% per year. The diameter of the earth is approximately 8000 miles and the surface of the earth is approximately two-thirds water. Calculate the population doubling time, then set up a spreadsheet that will show a) the population, b) the number of square feet of land area per person, and c) the length of the side of a square that will produce the required area per person. Carry out the spreadsheet for 15 doubling times, assuming that the rate of population increase remains constant. What conclusions can you draw from this exercise?

For 2% annual growth rate, the doubling time is 35 years.

The total area of the earth is  $4\pi \times (4000 \text{ mi} \times 5280 \text{ ft/mi})^2 = 5.61 \times 10^{15} \text{ ft}^2$ .

Hence, the land area of the earth is approximately  $1.87 \times 10^{15} \text{ ft}^2$ . The following table results.

# Doubling Times	Year	Population	Land Area/ Person (ft <sup>2</sup> )	Length of side of square/Person (ft)
0	2017	7.00E+09	2.67E+05	516.9
1	2052	1.40E+10	1.34E+05	365.5
2	2087	2.80E+10	6.68E+04	258.4
3	2122	5.60E+10	3.34E+04	182.7
4	2157	1.12E+11	1.67E+04	129.2
5	2192	2.24E+11	8.35E+03	91.4
6	2227	4.48E+11	4.17E+03	64.6
7	2262	8.96E+11	2.09E+03	45.7
8	2297	1.79E+12	1.04E+03	32.3
9	2332	3.58E+12	5.22E+02	22.8
10	2367	7.17E+12	2.61E+02	16.2
11	2402	1.43E+13	1.30E+02	11.4
12	2437	2.87E+13	6.52E+01	8.1
13	2472	5.73E+13	3.26E+01	5.7
14	2507	1.15E+14	1.63E+01	4.0
15	2542	2.29E+14	8.15E+00	2.9

Conclusions: These are left up to the creative thought processes of the problem solver.

**1.4** Show that in an exponential growth scenario, the amount accumulated in a doubling time equals the amount accumulated in all previous history.

Let  $T$  represent the doubling time,  $r$  the rate,  $P_0$  the initial population, and let  $t > 0$  represent any arbitrary time. The problem is to show that  $P_0(1+r)^{T+t} - P_0(1+r)^t = P_0(1+r)^t$ . This is equivalent to showing  $P_0(1+r)^T = 2P_0$ , which is true by assumption (this equation is what it means for  $T$  to be the doubling time).

- 1.5 Assume there is enough coal left to last for another 300 years at current consumption rates.
- Determine how long the coal will last if its use is increased at a rate of 5% per year.
  - If there is enough coal to last for 10,000 years at current consumption rates, then how long will it last if its use increases by 5% per year?
  - Can you predict any other possible consequences if coal burning increases at 5% per year for the short or long term?
  - Determine the annual percent reduction in coal consumption to ensure that coal will last forever, assuming the 300-year lifetime at present consumption rates.

Using (1.9)

$$\text{a) } m = \frac{\ln[300 \ln(1+0.05)+1]}{\ln(1+0.05)} = 56.36 \text{ yr} \qquad \text{b) } m = \frac{\ln[10000 \ln(1+0.05)+1]}{\ln(1+0.05)} = 126.91 \text{ yr}$$

c) To mention a few: significant increases in CO, CO<sub>2</sub>, NO<sub>2</sub>, SO<sub>2</sub> and particulates in the air, affecting acid rain, global warming and respiratory ailments.

$$\text{d) } \text{Set } m = \frac{\ln[300 \ln(1+n)+1]}{\ln(1+n)} = \infty \text{ and solve for } n. \text{ Note that if } n < 0, \text{ then } \ln(1+n) < 0.$$

Also, if  $[300 \ln(1+n) + 1] = 0$ , then  $\ln[300 \ln(1+n) + 1] = -\infty$ . So  $\ln(1+n) = -1/300$  and, hence,

$$n = e^{\frac{-1}{300}} - 1 = -3.328 \times 10^{-3}, \text{ or, approximately } 0.333\% \text{ per year decrease in consumption to ensure the supply will last forever.}$$

- 1.6 The half-life is the time it takes to decay exponentially to half the original amount. If the half-life of a radioactive isotope is 500 years, how many years will it take for an amount of the isotope to decay to 1% of its original value? Assume the isotope decays exponentially.

The time constant is given by  $\tau = \frac{t_{1/2}}{\ln 2} = \frac{500}{.693} = 721.5 \text{ yr}$ . Hence, setting  $e^{-\frac{t}{\tau}} = 0.01$  and solving for  $t$  gives the result  $t = \tau \ln 100 = 721.5 \ln 100 = 3323 \text{ yr}$ .

- 1.7 If a colony of bacteria lives in a jar and doubles in number every day, and it takes 30 days to fill the jar with bacteria,
- How long does it take for the jar to be half full?
  - How long before the bacteria notice they have a problem? (You may want to pretend you are a bacterium.)
  - If on the 30th day, 3 more jars are found, how much longer will the colony be able to continue to multiply at its present rate?
- a) 29 days                      b) Probably about when the jar is about 1/8 full. This is 3 days before the jar will be full. It sort of depends whether bacteria are as smart as humans.
- c) 2 more days
- 1.8 An enterprising young engineer enters an interesting salary agreement with an employer. She agrees to work for a penny the first day, 2 cents the second, 4 cents the third, and so on, each day doubling the amount of the previous day. Set up a spreadsheet that will show her daily and cumulative earnings for her first 30 days of employment.

Day	Daily Earnings	Cumulative Earnings	Day	Daily Earnings	Cumulative Earnings
1	\$0.01	\$0.01	16	327.68	655.35
2	\$0.02	\$0.03	17	\$655.36	\$1,310.71
3	\$0.04	\$0.07	18	\$1,310.72	\$2,621.43
4	\$0.08	\$0.15	19	\$2,621.44	\$5,242.87
5	\$0.16	\$0.31	20	\$5,242.88	\$10,485.75
6	\$0.32	\$0.63	21	\$10,485.76	\$20,971.51
7	\$0.64	\$1.27	22	\$20,971.52	\$41,943.03
8	\$1.28	\$2.55	23	\$41,943.04	\$83,886.07
9	\$2.56	\$5.11	24	\$83,886.08	\$167,772.15
10	\$5.12	\$10.23	25	\$167,772.16	\$335,544.31
11	\$10.24	\$20.47	26	\$335,544.32	\$671,088.63
12	\$20.48	\$40.95	27	\$671,088.64	\$1,342,177.27
13	\$40.96	\$81.91	28	\$1,342,177.28	\$2,684,354.55
14	\$81.92	\$163.83	29	\$2,684,354.56	\$5,368,709.11
15	\$163.84	\$327.67	30	\$5,368,709.12	\$10,737,418.23

**1.9** Burning a gallon of petroleum produces approximately 25 pounds of carbon dioxide and burning a ton of coal produces approximately 7000 pounds of carbon dioxide.

- If a barrel of petroleum contains 42 gallons, if the world consumes 80 million barrels of petroleum per day and if the atmosphere weighs 14.7 pounds per square inch of earth surface area, calculate the weight of carbon dioxide generated each year from burning petroleum and compare this amount with the weight of the atmosphere.
- If a total of 16 million tons of coal are burned every day on the earth, calculate the weight of carbon dioxide generated each year from coal burning and compare it with the weight of the atmosphere.

a)  $(25 \text{ lb CO}_2/\text{gal}) \times (42 \text{ gal/bbl}) \times (80 \times 10^6 \text{ bbl/day}) \times (365 \text{ days/yr}) = 3.066 \times 10^{13} \text{ lb/yr.}$

The atmosphere weighs  $(14.7 \text{ lb/in}^2) \times (5.61 \times 10^{15} \text{ ft}^2) \times (144 \text{ in}^2/\text{ft}^2) = 1.188 \times 10^{19} \text{ lb.}$

Hence, annual CO<sub>2</sub> production is  $2.584 \times 10^{-4}\%$  of the total weight of the atmosphere under the assumptions given.

b)  $(16 \times 10^6 \text{ ton/day}) \times (7000 \text{ lb CO}_2/\text{ton}) \times (365 \text{ days/yr}) = 4.088 \times 10^{13} \text{ lb/yr, or } 3.445 \times 10^{-4}\%$  of the weight of the atmosphere under the assumptions given.

**1.10** Assume a world population of 7.3 billion and a U.S. population of 323 million.

- Look up the present total annual U.S. primary energy consumption. Then determine the total world energy consumption in quads if the rest of the world were to use the same per capita energy as in the U.S.
- If the energy source mix were to remain the same as the present mix in achieving the scenario of part a, what would be the percentage increase in CO<sub>2</sub> emissions?

a) The US EIA shows 97 quads of U.S. Primary Energy Consumption in 2013 and total world primary energy consumption of 543 quads. The U. S. fraction of total world primary consumption thus was 18% in 2013.

Thus, on a per person basis, U.S. energy consumption was  $97 \times 10^{15} \text{ Btu} / 323 \times 10^6 \text{ people} = 3 \times 10^8 \text{ Btu/pers}$  and worldwide energy consumption is  $543 \times 10^{15} \text{ Btu} / 7.3 \times 10^9 \text{ people} = 7.44 \times 10^7 \text{ Btu/pers.}$

Thus, if everyone used  $3 \times 10^8 \text{ Btu/yr}$ , total world consumption would have been  $7.3 \times 10^9 \times 3 \times 10^8 = 21.9 \times 10^{17} \text{ Btu} = 2,190 \text{ quads}$ , which is 4.03 times the actual consumption.

- For the same energy mix, the CO<sub>2</sub> production rate would be 4.03 times as much, or 403% of the actual 2013 rate.

**1.11** Obtain data on worldwide energy consumption by sector from the United States Department of Energy, Energy Information Administration website. Plot the data and estimate annual percentage growth rates for the seven regions reported and then for the world.

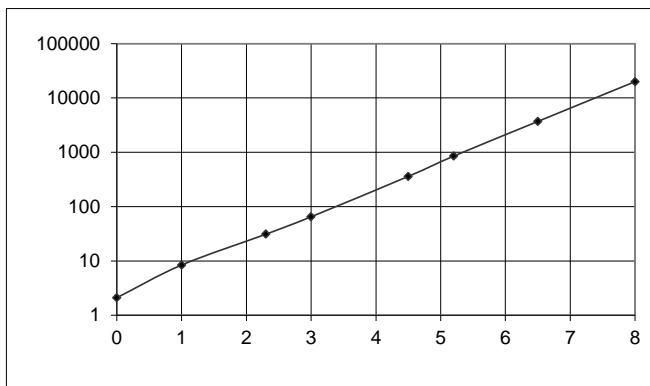
Note that the sectors listed on the EIA website include residential, industrial, commercial and transportation. The seven regions of the world, as called for in the problem, are: North America, Central and South America, Europe, Eurasia, Middle East, Africa, Asia and Oceania. New data becomes available frequently, so the reader is invited to access this data and plot historic trends. The end product might be a set of graphs with energy use by sector for each region, or vice versa. There is a lot of data that can be massaged in the various databases, with projections of energy use. These projections can then be compared with projections made by the EIA.

**1.12** The following measurements of  $x(t)$  are made:

t	0	1	2.3	3.0	4.5	5.2	6.5	8.0
x	2.1	8.4	31	65	360	850	3700	20,000

Construct a semilog plot of  $x(t)$  either manually or with a computer, and determine whether the function appears to have an exponential dependence. If so, determine  $x(t)$ .

The semi-log plot as an Excel plot is as follows:



This is clearly a pretty good straight line on semilogarithmic coordinates. The function can thus be represented fairly closely as

$$x = A \times (10)^{bt}, \text{ where } A = 2.1 \text{ and } b = \frac{\log x_2 - \log x_1}{t_2 - t_1} = \frac{\log 20000 - \log 2.1}{8 - 0} = \frac{3.979}{8} = 0.497.$$

Alternatively, using the Excel trendline option yields  $x = 2.2995e^{1.1334t}$  with an  $R^2$  value of 0.9994. Since  $e^t = 10^{\log e}$ ,  $x$  can also be expressed as  $x = 2.2995 \times (10)^{0.4922t}$ , which is reasonably close to the graphical estimate.

- 1.13**
- What does the area under the Gaussian curve represent?
  - Show that 68% of the area under the Gaussian curve lies within one standard deviation,  $s$ , of maximum value of the function.
  - What percentage of the area lies within  $2s$ ?
    - The area under the Gaussian curve represents the total sample, or total supply.
    - This is probably most easily proven by a numerical integration or else by looking up the function in a book of tables, such as the *CRC Standard Mathematical Tables*. The 20th Edition of the *CRC Standard Mathematical Tables* tabulates the Normal probability function and related functions, beginning on p. 582.

For one standard deviation from the mean,  $x = 1.00$ .  $F(1)$  represents the area under the curve between  $x = -\infty$  and  $x = 1.00$ , which is tabulated as 0.8413. Note that  $f(x)$  is normalized so that  $F(\infty) = 1.00$ . Since  $f(x)$  is symmetrical about  $x = 0$ , this means that the area under the curve between  $x = 0$  and  $x = 1$  is  $0.8413 - 0.500 = 0.3413$ . Thus, the area between  $x = -1$  and  $x = 0$  is also 0.3413, **so the area between  $x = -1$  and  $x = 1$  is 0.6826**. Hence, 68.26% of the distribution lies within 1 standard deviation of the mean.

c) In a similar fashion,  $F(2) = 0.9772$ . Hence,  $2 \times (0.9772 - 0.500) = 0.954$ , or, 95.4% of the distribution lies within 2 standard deviations of the mean.

**1.14** Determine  $R_m$ ,  $t_0$  and  $s$  for the worldwide graph of Figure 1.8.

The worldwide graph shows  $R_m \cong 36.5 \times 10^9$  bbl/yr consumption rate occurring somewhere near the year 2005. The Gaussian function has the property that it has decreased to 60.7% of its peak value at one standard deviation from the peak (mean). Thus,  $.607 \times R_m = 22.2 \times 10^9$  bbl/yr, occurs at the years 1980 and 2030. So  $s \cong 25$  years and hence this implies that 68% of the world's petroleum will be used between the years 1980 and 2030.

**1.15** Look up actual U. S. and World petroleum production figures and plot them on Hubbert's curves to compare the actual production with the theoretical production.

Data is available from the U.S. Energy Information Agency. 2016 World production 96 million barrels/day. 2015 US production 9.25 million barrels/day. These figures change monthly, but the idea is to see where they lie on the Hubbert curve, so they need to be converted to barrels/yr. The results: U.S. =  $3.4 \times 10^9$  and World =  $35 \times 10^9$ . 2016 Hubbert curves show U.S.  $\approx 2 \times 10^9$  barrels/yr and World  $\approx 35 \times 10^9$  barrels/yr. Is this interesting, or what?

**1.16** Based on the data of Figure 1.9,

- Estimate the year when PV shipments will reach 100 GW.
- Estimate the year when PV shipments will reach 1,000 GW.
- Estimate the year when PV shipments will reach 2,700 GW.

The annual percent increase needs to be estimated from Figure 1.9. Since there is some uncertainty involved in predicting the future, one needs to decide upon how much of the curve of Figure 1.9 to use to estimate the annual growth rate, since the slope of the curve is time dependent. Noting the recent trend, since about 2000, estimates can be made of shipments between 2000 and 2015 from the data on Figure 1.9 as follows:

Year	2000	2003	2006	2009	2012	2015
Normalized year (t)	0	3	6	9	12	15
GW shipped (GW)	0.22	0.55	1.7	7.5	30.0	50.0

Plotting this data in Excel and obtaining the best exponential curve fit yields  $GW = 0.200e^{0.3868t}$  with  $R^2 = 0.9875$ , a reasonable fit.

Thus, to determine  $t$  to reach a particular value of GW, solve the equation for  $t$  to get

$$t = \frac{1}{0.3868} \ln \frac{MW(t)}{200}$$

Thus

GW(t) = 100 GW when t = 16 yr	(i.e., in 2016)
GW(t) = 1,000 GW when t = 22 yr	(i.e., in 2022)
GW(t) = 2,700 GW when t = 24.6 yr	(i.e., in 2025)

**1.17** The Earth Policy Institute [31] reports the following worldwide PV production figures. Plot the data on an Excel graph, establish an equation to represent the data, and then answer the three questions posed in Problem 1.16. Compare the results of the two problems.

