

## Solutions to 2<sup>nd</sup> Edition

### Chapter 1

1. A. Find the cheapest conductor based on the cost to meet a specified maximum resistance per length of conductor. The cost for a fixed resistance per length is proportional to  $\rho_{\text{res}}\rho_{\text{dens}}C$  where  $\rho_{\text{res}}$  is the resistivity,  $\rho_{\text{dens}}$  is the density and  $C$  is the cost per weight.

B. At what price per pound would sodium make an even cheaper conductor? Sodium has a density of  $0.967 \text{ Mg/m}^3$  and a resistivity of  $49.6 \text{ nohm-m}$ .

Metal	density	resistivity	price
		$\text{Mg/m}^3$	$\text{nohm-m}$
iron	7.88	98	0.39
magnesium	1.738	45	0.98
copper	8.90	16.73	1.44
aluminum	2.699	26.55	0.69

Solution:

A. The parameter  $P = C\rho_{\text{res}}\rho_{\text{dens}}$  ranks the different metals. There is no need to change units as long as the data for all metals are in the same units)

For iron,  $P = (0.39)(98)(7.88) = 301$

For magnesium,  $P = (0.98)(45)(1.738) = 60.9$

For copper,  $P = (1.44)(16.73)(8.90) = 214$

For aluminum,  $P = (1.69)(26.55)(2.70) = 49$

Aluminum is the cheapest.

B. For sodium to be competitive,  $(C_{\text{Na}})(49.6)(0.967) = 1.03$ ,  $C_{\text{Na}} = 0.021 \text{ \$/lb}$

2. Vanadium is often added to steels in minor amounts to form carbo-nitrides.

A. If vanadium were not available, what element would you suggest as a substitute to form similar carbo-nitrides?

Assuming that each atom of the substitute is equally effective as a vanadium atom, what weight percent of the substitute would you recommend to replace 0.05 wt % V?

Solution:

A. Examining the periodic table, I would suggest Nb. (If that were not available, I would suggest Ta).

B. On a one mole of Nb for one mole of V basis, the required amount of Nb =  $0.05\% \text{ V}(92.91 \text{ g Nb/mole})/(50.9 \text{ g V/mole}) = 0.09\%$

3. Prior to World War II, most high speed steels contained about 18% W. A typical composition

was 1%C, 4%Cr, 2%V, 18%W, bal. Fe. Because of the shortage of tungsten, a substitute had to be found for some or all of the tungsten.

A. What element would you suggest as a substitute for tungsten?

B. Assuming an equal effectiveness of the substitute and tungsten on an atom-for-atom basis, what % of the substitute would be needed to substitute for each % W?

Solution:

- A. Examining the periodic table, I would suggest Mo.  
 B. On a one mole of Mo for one mole of W basis,  $(92.95 \text{ g Mo/mole}) / (183.86 \text{ g W/mole}) = 0.505 \text{ wt \% Mo}$  would substitute for 1 wt % W.

4. Using the prices in Table II, by what factor would the cost of material in a part increase if magnesium were substituted for aluminum without making any changes in dimensions?

$$\begin{aligned} \text{The relative cost/volume} &= (\text{cost/lb})(\text{Mg/m}^3) \\ (\text{Mg cost/vol})/(\text{Al cost/vol}) &= (0.98)(1.74)/[(0.69)(2.7)] = 2.2 \end{aligned}$$

## Chapter 2

1. The rate at which metals freeze is controlled by the rate at which heat can be extracted. Consider the freezing of aluminum. If the liquid-solid interface advances at 1 mm/s, what is the thermal gradient ( $^{\circ}\text{C/mm}$ ) in the solid?

Data for aluminum: melting point =  $660^{\circ}\text{C}$ , specific heat =  $0.215 \text{ cal/(g-}^{\circ}\text{C)}$ , heat of fusion =  $94.5 \text{ cal/g}$ , atomic wt = 27, density =  $2.7 \text{ Mg/m}^3$ , thermal conductivity =  $0.22 \text{ (watts/mm}^2\text{)/(}^{\circ}\text{C/mm)}$  and coefficient of linear expansion =  $22.5 \times 10^{-6}/^{\circ}\text{C}$ .

$$\begin{aligned} \text{Solution. } J &= K(\Delta T/dx); J = (1 \text{ mm/s})(2.7 \times 10^6 \text{ g/m}^3)(10^{-9} \text{ m}^3/\text{mm}^3)(94.5 \text{ cal/g})(4.2 \text{ J/cal}) = 1.072 \text{ J/(s mm}^2\text{)}. \Delta T/dx = 1.072 \\ \text{W/mm}^2 / 0.22 \text{ (watts/mm}^2\text{)/(}^{\circ}\text{C/mm)} &= 4.87^{\circ}\text{C/mm}. \end{aligned}$$

2. An ingot of Al-5% Cu is directionally solidified. Assume that there is no diffusion in the solid and that there is perfect mixing in the liquid. Pure aluminum melts at  $660^{\circ}\text{C}$ . At the eutectic temperature of  $548^{\circ}\text{C}$ , the liquid composition is 33.2% Cu and the solid composition is 5.35 %Cu.

Assume that the liquidus and solidus are straight lines.

A. Find the distribution coefficient expressed as  $k = C_S / C_L$  where  $C_S$  and  $C_L$  are expressed as % Cu.

B. Calculate the composition of the liquid when the solidification is 40% complete.

C. What is the average composition of the solid,  $C_{Sav.}$ , at this point. (Make sure that  $0.4C_{Sav} + 0.6 C_L = 5\%$ .)

D. What is the liquid-solid interface temperature at this point?

E. How much eutectic will be formed?

Solution:

$$\text{A. } k = 5.35/33.2 = 0.161$$

$$\text{B. } C_L = C_0 (1-f_S)^{-(1-k)} = 5(0.6)^{-(1-0.161)} = 7.67\% \text{ Cu}$$

$$\text{C. } 0.4C_{Sav} + 0.6 C_L = 5\%. C_{Sav} = [5 - (0.6)(7.67)] / 0.4 = 0.995\% \text{ Cu}$$

D. For a straight line liquidus,  $C_L = (33.2)(660 - T)/(660 - 548)$ . When

$$C_L = 7.67\% \text{Cu}, T = 660 - (7.67/33.2)(660 - 548) = 634^\circ\text{C}$$

E. Solving  $C_L = C_0 (1 - f_S)^{-(1-k)}$  for  $(1 - f_S) = (C_L - C_0)^{-1/(1-k)}$  and substituting  $C_L = 33.2$ ,  $f_{\text{eutectic}} = (1 - f_S) = (33.2 - 5)^{-1/(1-0.161)} = 0.019\%$

3. Consider the freezing of an aluminum alloy containing 0.005% copper.

A. What would be the composition of the first solid to freeze?

B. What would be the average composition of the first half to freeze?

Solution:

A.  $C = kC_0 = 0.161(0.005) = 0.0008\% \text{Cu}$

B.  $C_L = 0.005(0.5)^{-(1-0.161)} = 0.00894$ ;  $C_{\text{Sav}} = [0.005 - 0.00894(0.5)]/0.5 = 0.0011\% \text{Cu}$

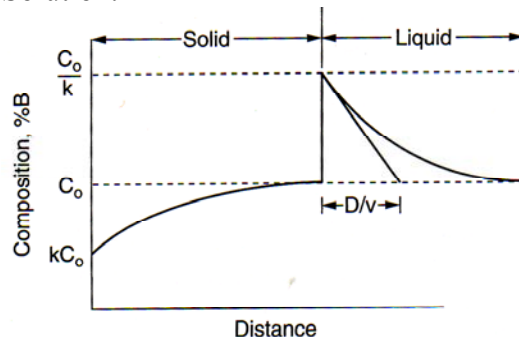
4. Consider the steady-state freezing of an aluminum alloy containing 0.55% Cu. In steady state freezing the boundary layer is such that the solid freezing has the same composition as the alloy. Assume that the liquid-solid interface moves at a rate of  $80 \mu\text{m/s}$ . The diffusion coefficient of copper in liquid aluminum is  $3 \times 10^{-9} \text{ m}^2/\text{s}$ .

A. What is the interface temperature?

B. What is the thickness of the boundary layer?

C. What temperature gradient would be required to maintain plane-front growth?

Solution:



A. At steady state, the liquid composition is  $C_0/k = 0.55/0.161 = 3.41\% \text{Cu}$  so the interface temperature is  $660 - 3.41(660 - 548)/33.2 = 648.5$

B.

Using the results of Problem 2,  $0.55\% \text{Cu}$ ,  $T_{\text{solidus}} = 660 - (5.31/33.2)(660 - 548) = 642^\circ\text{C}$ .

$$t = 3 \times 10^{-9} \text{ m}^2/\text{s} / (80 \times 10^{-6} \text{ m/s.}) = 3.75 \times 10^{-5} \text{ m} = 37.5 \mu\text{m}$$

B.

$(T/dx)_{crit} = (T_1 - T_3)/(D/v)$  where  $T_3 = T_{solidus} = 642^\circ\text{C}$  and  $T_1 = T_{liquidus} = 648.5$

So  $(T/dx)_{crit} = (648.5 - 642)/(3 \times 10^{-9} \text{ m}^2/\text{s} / 80 \times 10^{-9} \text{ m/s}) = 17 \times 10^3 \text{ }^\circ\text{C/m} = 17^\circ\text{C/mm}$

5. The dependence of the dendrite arm spacing,  $\lambda$ , on the cooling rate,  $r$ , is given by  $\lambda = kr^{-n}$ .  $\lambda$  was found to be  $100 \text{ } \mu\text{m}$  at  $r = 0.1 \text{ K/s}$  and  $10 \text{ } \mu\text{m}$  at  $r = 60 \text{ K/s}$ .

A. Find  $k$  and  $n$ .

B. What rate of cooling would be required to produce a spacing of  $\lambda = 1.0 \text{ mm}$ ?

Solution:

A.  $\lambda_1/\lambda_2 = (r_1/r_2)^{-n}$ ;  $n = -\ln(\lambda_1/\lambda_2)/\ln(r_1/r_2) = -\ln(100/10)/\ln(0.1/60) = 0.36$

B.  $k = \lambda/r^{-n} = \lambda r^n = 100(0.1)^{0.36} = 43.65$ .

6. At the melting point of aluminum and one atmosphere partial pressure of hydrogen, the equilibrium solubility of hydrogen is  $7 \times 10^{-3} \text{ cm}^3/\text{g}$  of Al in the liquid and  $4 \times 10^{-4} \text{ cm}^3/\text{g}$  of Al in the solid. The solubilities are expressed as volumes measured at  $20^\circ\text{C}$  and one atmosphere (STP) not the volumes at the melting point.

A. Calculate the equilibrium solubilities at 0.1 atmosphere of  $\text{H}_2$ . Express your answer in STP

B. What volumes of  $\text{H}_2$  would be liberated per volume of aluminum, if the partial pressure of  $\text{H}_2$  were 0.1 atmospheres? (The  $\text{H}_2$  is liberated at a total pressure of one atmosphere and the melting point of aluminum) Assume the perfect gas law. Your answer should be the percent gas porosity if the gas is trapped interdendritically.

Solution:

A. The solubility  $c_{p2}/c_{p1} = \sqrt{p_2/p_1} = \sqrt{0.10} = 0.316$ . The solubility in the liquid is

$0.316 \times 7 \times 10^{-3} \text{ cm}^3/\text{g Al} = 2.212 \times 10^{-3} \text{ cm}^3/\text{g Al}$ . The solubility in the solid is  $0.316 \times 4 \times 10^{-4} \text{ cm}^3/\text{g Al} = 1.26 \times 10^{-4} \text{ cm}^3/\text{g Al}$ .

B. Volume liberated per gram of aluminum =

$(2.212 \times 10^{-3} - 1.26 \times 10^{-4})(660+273)/(20+273) = 6.64 \times 10^{-3} \text{ cm}^3/\text{g Al}$ . Per  $\text{cm}^3$  of Al this is  $(6.64 \times 10^{-3} \text{ cm}^3/\text{g Al})(2.7 \text{ g/cm}^3) = 1.79 \times 10^{-2}$  or 1.8%

7. Consider an aluminum-rich binary aluminum-silicon alloy. The melting temperature of aluminum is  $660^\circ\text{C}$ , the eutectic is at  $577^\circ\text{C}$  and 12.6 wt% Si. The maximum solubility of silicon in aluminum is 1.65% Si at  $577^\circ\text{C}$ .

The liquidus and solidus can be approximated by straight lines. The diffusivity of silicon in liquid aluminum is  $8 \times 10^{-8} \text{ m}^2/\text{s}$ . Freezing occurs at a rate of  $10 \mu\text{m/s}$ .

A. For an alloy of 0.05% Si, what is the interface temperature for steady state freezing?

A. Find the thickness of the boundary layer.

C. What temperature gradient is necessary to maintain plane-front growth?

D. Repeat A, B, and C for an alloy containing 1% silicon.

Solution:

A. The interface temperature is the solidus temperature =  $660 - (0.05/1.65)(660-577) = 657.5^\circ\text{C}$

B.  $t = D/v = (8 \times 10^{-8} \text{ m}^2/\text{s}) / (10 \times 10^{-6} \text{ m}) = 8 \times 10^{-3} \text{ m} = 1 \text{ mm}$

C.  $(dT/dx)_{\text{crit}} = (T_{\text{liquidus}} - T_{\text{solidus}}) / (D/v)$ ;  $T_{\text{liquidus}} = 660 - (0.05/12.6)(660-577) = 659.7^\circ\text{C}$ ;  $(dT/dx)_{\text{crit}} = (659.7-657.5)(8 \text{ mm}) = 17.7^\circ\text{C/mm}$

D. For 1% Si, interface temperature =  $660 - (1/1.65)(660-577) = 609.7^\circ\text{C}$ ;  $t = 1 \text{ mm}$

$T_{\text{liquidus}} = 660 - (1/12.6)(660-577) = 653.4^\circ\text{C}$ ;  $(dT/dx)_{\text{crit}} = (653.4-609.7)(8 \text{ mm}) = 350^\circ\text{C/mm}$

8. Predict the morphology of each of the eutectics listed below. The compositions are from phase diagrams in the *Metals Handbook*, v. 8, 8<sup>th</sup> ed. Some of the densities are estimates.

system	phase	composition	density
Bi/Cd	$\alpha$ Bi	0 % Cd	$9.8 \text{ Mg/m}^3$
	eutectic	39.7% Cd	
	$\beta$ Cd	100 % Cd	
Fe/C	$\gamma$	2.1% C	7.9
	eutectic	4.3% C	7.9
	graphite	100% C	2.25
Cu/Al	$\theta$	47% Al	6.6
	eutectic	66.8% Al	
	$\alpha$	94% Al	2.7
BiPb	BiPb <sub>2</sub>	42% Bi	11.4
	eutectic	56% Bi	
	Bi	100% Bi	9.8

Solution:

Bi/Cd Wt fraction  $\alpha$  Bi =  $(100-39.7)/100 = 0.397$ ;

Vol fraction  $\alpha = (39.7/9.8) / (39.7/9.8 + 60.3/7.9) = 0.35$

Fe/C Wt fraction G =  $(2.1)/100 = 2.1$ ;

Vol fract  $\gamma = (2.1/2.15)/(2.1/2.15 + 97.9/7.9) = 0.073$  Predict isolated particles  
 Cu/Al Wt fraction  $\theta = (94-47)/(94-0) = 0.50$   
 Vol fraction  $\alpha = (50/6.6)/(50/6.6 + 50/2.7) = 0.41$  Predict parallel plates

BiPb Wt fraction BiPb<sub>2</sub> =  $(100-56)/(100-42) = 75.9$   
 Vol fraction BiPb<sub>2</sub> =  $(75.9/11.4)/(75.9/11.4 + 24.1/9.8) = 0.73$   
 Vol fraction Bi = .27 predict rods

9. The melting point of pure aluminum is 660°C and aluminum and silicon form a eutectic, the eutectic temperature is 577°C, the eutectic composition is 12%Si and the maximum solubility of silicon in solid aluminum is 1.65%. Assume the phase diagram consists of straight lines. If aluminum containing 0.15 wt % Si were solidified, what would be the composition of the first solid to form?

Solution:  $k = 1.65/12 = 0.137$ ;  $C = kC_0 = 0.137(.15) = .02\% \text{ Si}$ .

10. Some solutes raise the melting temperature, causing both the liquidus and solidus to increase with additional solute. In this case the distribution coefficient,  $k > 1$ .

A. Is the Scheil equation (2.11) still valid?

B. Describe qualitatively how having  $k > 1$  affects the segregation.

Solution:

A. yes

B. If  $k > 1$ , impurities will segregate to first part to freeze. The composition,  $C$ , of the first solid =  $kC_0$  so  $C > C_0$

### Chapter 3

1. A block of an alloy of Cu-6% Al was welded to a block of Cu-14% Al and heated to 700°C. Sketch the concentration profile after some diffusion occurred. The phase diagram is below.