Solutions to 2nd Edition

Chapter 1

1. A. Find the cheapest conductor based on the cost to meet a specified maximum resistance per length of conductor. The cost for a fixed resistance per length is proportional to $\rho_{res}\rho_{dens}C$ where ρ_{res} is the resistivity, ρ_{dens} is the density and C is the cost per weight.

B. At what price per pound would sodium make an even cheaper conductor? Sodium has a density of 0.967 Mg/m^3 and a resistivity of 49.6 nohm-m.

Metal	density	resistivity	price
		Mg/m^3	nohm-m
iron	7.88	98	0.39
magnesium	1.738	45	0.98
copper	8.90	16.73	1.44
aluminum	2.699	26.55	0.69

Solution:

A. The parameter $P = C\rho_{res}\rho_{dens}$ ranks the different metals. There is no need to change units as long as the data for all metals are in the same units)

For iron, $P = (0.39)(98)(7.88) = 301$

For magnesium, $P = (0.98)(45)(1.738) = 60.9$

For copper, $P = (1.44)(16.73)(8.90) = 214$

For aluminum, $P = (169)(26.55)(2.70) = 49$

Aluminum is the cheapest.

B. For sodium to be competitive, $(C_{Na})(49.6)(0.967) = 1.03$, $C_{Na} = 0.021$ \$/lb

2. Vanadium is often added to steels in minor amounts to form carbo-nitrides.

A. If vanadium were not available, what element would you suggest as a substitute to form similar carbo-nitrides?

Assuming that each atom of the substitute is equally effective as a vanadium atom, what weight percent of the substitute would you recommend to replace 0.05 wt % V?

Solution:

A. Examining the periodic table, I would suggest Nb. (If that were not available, I would suggest Ta).

B. On a one mole of Nb for one mole of V basis, the required amount of Nb = 0.05% V(92.91 g Nb/mole)/(50.9 g V/mole) = 0.09%

3. Prior to World War II, most high speed steels contained about 18% W. A typical composition was 1%C, 4%Cr, 2%V, 18%W, bal. Fe. Because of the shortage of tungsten, a substitute had to be found for some or all of the tungsten.

A. What element would you suggest as a substitute for tungsten?

B. Assuming an equal effectiveness of the substitute and tungsten on an atom-for-atom basis, what % of the substitute would be needed to substitute for each % W?

Solution:

A. Examining the periodic table, I would suggest Mo.
B. On a one mole of Mo for one mole of W basis, (92.95g Mo/mole)/(183.86 g W/mole) = 0.505wt % Mo would substitute for 1 wt % W.

4. Using the prices in Table II, by what factor would the cost of material in a part increase if magnesium were substituted for aluminum without making any changes in dimensions?
The relative cost/volume = (cost/lb)(Mg/m^3)
(Mg cost/vol)/(Al cost/vol) = (0.98)(1.74)/[(0.69)(2.7)] = 2.2

Chapter 2

1. The rate at which metals freeze is controlled by the rate at which heat can be extracted. Consider the freezing of aluminum. If the liquid-solid interface advances at 1 mm/s, what is the thermal gradient (°C/mm) in the solid?
Data for aluminum: melting point = 660°C, specific heat = 0.215 cal/(g-°C), heat of fusion = 94.5 cal/g, atomic wt = 27, density = 2.7 Mg/m^3, thermal conductivity = 0.22 (watts/mm^2)/(°C/mm) and coefficient of linear expansion = 22.5x10^{-6}/°C.
Solution. J = K(ΔT/dx); J = (1mm/s)(2.7x10^6 g/m^3)(10^{-9} m^3/mm^3)(94.5cal/g)(4.2 J/cal) = 1.072 J/(smm^2). ΔT/dx = 1.072 W/mm^2/0.22 (watts/mm^2)/(°C/mm) = 4.87 °C/mm.

2. An ingot of Al-5% Cu is directionally solidified. Assume that there is no diffusion in the solid and that there is perfect mixing in the liquid. Pure aluminum melts at 660°C. At the eutectic temperature of 548°C, the liquid composition is 33.2% Cu and the solid is composition is 5.35 %Cu. Assume that the liquidus and solidus are straight lines.
A. Find the distribution coefficient expressed as k = C_S/ C_L where C_S and C_L are expressed as % Cu.
B. Calculate the composition of the liquid when the solidification is 40% complete.
C. What is the average composition of the solid, $C_{Sav.}$, at this point. (Make sure that 0.4C_{Sav} + 0.6 C_L = 5%.)
D.What is the liquid-solid interface temperature at this point?
E. How much eutectic will be formed?
Solution:
A. k = 5.35/33.2 = 0.161
B. $C_L = C_0 (1-f_S)^{-(1-k)} = 5(0.6)^{-(1-.161)} =$ 7.67%Cu
C. 0.4C_{Sav} + 0.6 C_L = 5%. C_{Sav} = [5 – (.06)(7.67)] /0.4 = 0.995% Cu

D. For a straight line liquidus, C_L = (33.2)(660 – T)/(660 – 548). When C_L = 7.67%Cu, T – 660 – (7.67/33.2)(660-548) = 634°C

E. Solving $C_L = C_0 (1-f_S)^{-(1-k)}$ for $(1-f_S) = (C_L - C_0)^{-1/(1-k)}$ and substituting C_L = 33.2, $f_{eutectic} = (1-f_S) = (33.2-5))^{-1/(1-.161)}$ = 0.019%

3. Consider the freezing of an aluminum alloy containing 0.005% copper.

A. What would be the composition of the first solid to freeze?

B. What would be the average composition of the first half to freeze?

Solution:

A. $C = kC_0$ = 0.161(.005) = .0008 % Cu

B. $C_L = .005(0.5)^{-(1-.161)}$ = 0.00894; C_{Sav} = [.005 - 0.00894(.5)]/0.5 = 0.0011% Cu

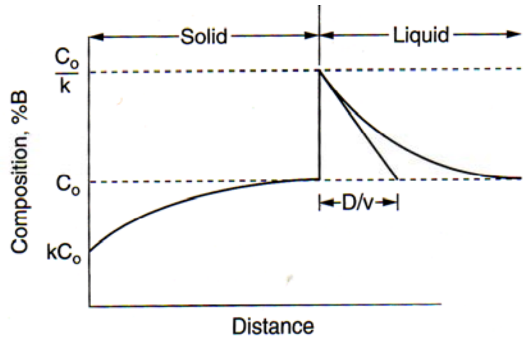
4 . Consider the steady-state freezing of an aluminum alloy containing 0.55% Cu. In steady state freezing the boundary layer is such that the solid freezing has the same composition as the alloy. Assume that the liquid-solid interface moves at a rate of 80μm/s. The diffusion coefficient of copper in liquid aluminum is $3\text{x}10^{-9}\,\text{m}^2$/s.

A. What is the interface temperature?

B. What is the thickness of the boundary layer?

C. What temperature gradient would be required to maintain plane-front growth?

Solution:



A. At steady state, the liquid composition is C_0/k = 0.55/0.161 = 3.41% Cu so the interface temperature is 660 – 3.41(660 – 548)/33.2 = 648.5

B.

Using the results of Problem 2, 0.55% Cu, $T_{solidus}$ = 660 – (5.31/33.2)(660 – 548) = 642°C.

$t = 3\text{x}10^{-9}\,\text{m}^2/\text{s}/(80\text{x}10^{-6}\,\text{m/s.}) = 3.75\ \text{x}10^{-5}\,\text{m} = 37.5\ \mu\text{m}$

B.
$(T/dx)_{crit} = (T_1\text{-}T_3)/(D/v)$ where $T_3 = T_{solidus} = 642°C$ and $T_l = T_{liquidus} = 648.5$
So $(T/dx)_{crit} = (648.5 – 642)/(3x10^{-9} m^2/s/80x10^{-9} m/s) = 17x10^3 °C/m = 17°C/mm$

5. The dependence of the dendrite arm spacing, λ, on the cooling rate, r, is given by $\lambda = kr^{-n}$. λ was found to be 100 µm at r = 0.1 K/s and 10 µm at r = 60 K/s .
A. Find k and n.
B. What rate of cooling would be required to produce a spacing of λ = 1.0 mm?

Solution:
A. $\lambda_1/\lambda_2 = (r_1/r_2)^{-n}$; $n = -\ln(\lambda_1/\lambda_2)/\ln(r_1/r_2) = -\ln(100/10)/\ln(0.1/60) = 0.36$
B. $k = \lambda/r^{-n} = \lambda r^{n} = 100(0.1)\ )^{0.36} = 43.65$.

6. At the melting point of aluminum and one atmosphere partial pressure of hydrogen, the equilibrium solubility of hydrogen is $7x10^{-3}$ cm^3/g of Al in the liquid and $4x10^{-4}$ cm^3/g of Al in the solid. The solubilities are expressed as volumes measured at 20°C and one atmosphere (STP) not the volumes at the melting point.
A. Calculate the equilibrium solubilities at 0.1 atmosphere of H_2. Express tour answer in STP
B. What volumes of H_2 would be liberated per volume of aluminum, if the partial pressure of H_2 were 0.1 atmospheres? (The H_2 is liberated at a total pressure of one atmosphere and the melting point of aluminum) Assume the perfect gas law. Your answer should be the percent gas porosity if the gas is trapped interdendritically.
Solution:
A. The solubility $c_{p2}/c_{p1} = \sqrt{(p_2/p_1)} = \sqrt{0.10} = 0.316$. The solubility in the liquid is
$0.316x7x10^{-3}$ cm^3/g Al = $2.212x10^{-3}$ cm^3/g Al. The solubility in the solid is $0.316x4x10^{-4}$ cm^3/g Al = $1.26x10^{-4}$ cm^3/g Al.
B. Volume liberated per gram of aluminum =
$(2.212x10^{-3}\text{- } 1.26x10^{-4})(660+273)/(20+273) = 6.64x10^{-3} cm^3/g$ Al. Per cm^3 of Al this is $(6.64x10^{-3} cm^3/g\ Al)(2.7g/cm^3) = 1.79x10^{-2}$ or 1.8%

7.Consider an aluminum-rich binary aluminum-silicon alloy. The melting temperature of aluminum is 660°C, the eutectic is at 577°C and 12.6 wt% Si. The maximum solubility of silicon in aluminum is 1.65% Si at 577°C.

The liquidus and solidus can be approximated by straight lines. The diffusivity of silicon in liquid aluminum is 8x10^{-8} m^{2}/s. Freezing occurs at a rate of 10μm/s.

A. For an alloy of 0.05% Si, what is the interface temperature for steady state freezing?

A. Find the thickness of the boundary layer.

C. What temperature gradient is necessary to maintain plane-front growth?

D. Repeat A, B, and C for an alloy containing 1% silicon.

Solution:

A. The interface temperature is the solidus temperature = 660-(.05/1.65)(660-577) = 657.5°C

B. t = D/v = (8x10^{-8} m^{2}/s)/(10x10^{-6} m) = 8x10^{-3} m = 1 mm

C. $(dT/dx)_{crit}$ = $(T_{liquidus} - T_{solidus})$/(D/v); $T_{liquidus}$ = 660-(.05/12.6)(660-577) = 659.7°C; $(dT/dx)_{crit}$ = (659.7-657.5)(8 mm) = 17.7 °C/mm

D. For 1% Si, interface temperature = 660-(1/1.65)(660-577) = 609.7°C; t = 1 mm

$T_{liquidus}$ = 660-(1/12.6)(660-577) = 653.4°C; $(dT/dx)_{crit}$ = (653.4-609.7)(8 mm) = 350°C/mm

8. Predict the morphology of each of the eutectics listed below. The compositions are from phase diagrams in the *Metals Handbook,* v. 8, 8th ed. Some of the densities are estimates.

system	phase	composition	density
Bi/Cd	α Bi	0 % Cd	9.8 Mg/ m^{3}
	eutectic	39.7% Cd	
	β Cd	100 % Cd	7.9
Fe/C	γ	2.1%C	7.9
	eutectic	4.3% C	
	graphite	100%C	2.25
Cu/Al	θ	47% Al	6.6
	eutectic	66.8% Al	
	α	94% Al	2.7
BiPb	$BiPb_2$	42% Bi	11.4
	eutectic	56% Bi	
	Bi	100% Bi	9.8

Solution:

Bi/Cd Wt fraction α Bi = (100-39.7)/100 = 0.397;

Vol fraction α= (39.7/9.8)/(39.7/9.8 + 60.3/7.9) = 0.35

Fe/C Wt fraction G = (2.1)/100 = 2.1;

Vol fract γ = (2.1/2.15)/(2.1/2.15 +97.9/7.9) = 0.073 Predict isolated particles

Cu/Al Wt fraction θ = (94-47)/(94-0) = 0.50

Vol fraction α = (50/6.6)/(50/6.6 + 50/2.7) = 0.41 Predict parallel plates

BiPb Wt fraction $BiPb_2$ = (100-56)/(100-42) = 75.9

Vol fraction $BiPb_2$ = (75.9/11.4)/(75.9/11.4 + 24.1/9.8) = 0.73

Vol fraction Bi = .27 predict rods

9. The melting point of pure aluminum is 660°C and aluminum and silicon form a eutectic, the eutectic temperature is 577°C, the eutectic composition is 12%Si and the maximum solubility of silicon in solid aluminum is 1.65%. Assume the phase diagram consists of straight lines. If aluminum containing 0.15 wt % Si were solidified, what would be the composition of the first solid to form?

Solution: k = 1.65/12 = 0.137; $C = kC_o$ = 0.137(.15) = .02% Si.

10. Some solutes raise the melting temperature, causing both the liquidus and solidus to increase with additional solute. In this case the distribution coefficient, $k > 1$.

A. Is the Scheil equation (2.11) still valid?

B. Describe qualitatively how having $k>1$ affects the segregation.

Solution:

A. yes

B. If $k>1$, impurities will segregate to first part to freeze. The composition, C, of the first solid = kC_o so $C > C_o$

Chapter 3

1. A block of an alloy of Cu-6% Al was welded to a block of Cu-14% Al and heated to 700°C. Sketch the concentration profile after some diffusion occurred. The phase diagram is below.