Solutions Manual to

PLASTICITY FOR STRUCTURAL ENGINEERS

by

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Springer-Verlag, New York
Berlin, Heidelberg, London,
Paris, Tokyo

1988

CHAPTER 1. INTRODUCTION

1.1 The $\sigma-\epsilon$ response in simple tension for a material is approximated by the following form of Ramberg-Osgood formula

$$\epsilon = \epsilon^e + \epsilon^p = \frac{\sigma}{E} + (\frac{\sigma}{b})^n$$

a. Find tangent modulus E_t and plastic modulus E_p as functions of stress σ, and of plastic strain ϵ^p.

b. Find plastic work W_p as a function of stress σ, and of plastic strain ϵ^p.

c. Express the stress σ and plastic modulus E_p in terms of the plastic work W_p.

d. What is the initial yield stress?

e. Assume $n = 1$, sketch the $\sigma-\epsilon$ curve for loading followed by a complete unloading.

f. Assume $n = 5$, find the offset tensile stresses for the permanent offsets $\epsilon^p = 0.1\%$ and $\epsilon^p = 0.2\%$, respectively.

SOLUTION

a) Using the relationship between the stress increment and the strain increment

$$d\sigma = E_t\, d\epsilon = E_p\, d\epsilon^p$$

and the simple relationship given in this problem,

$$\epsilon^p = (\frac{\sigma}{b})^n, \quad \epsilon^e = \frac{\sigma}{E}$$

we obtain

$$d\epsilon = [\frac{1}{E} + \frac{n}{b}(\frac{\sigma}{b})^{n-1}]\, d\sigma = \frac{1}{E_t}\, d\sigma \qquad (1)$$

$$d\epsilon^p = [\frac{n}{b}(\frac{\sigma}{b})^{n-1}]\, d\sigma = \frac{1}{E_p}\, d\sigma \qquad (2)$$

$$\text{or } E_t = \frac{1}{\frac{1}{E} + \frac{n}{b}(\frac{\sigma}{b})^{n-1}} = \frac{1}{\frac{1}{E} + \frac{n}{b}(\epsilon^p)^{(n-1)/n}} \qquad (3)$$

$$E_p = \frac{b}{n}(\frac{b}{\sigma})^{n-1} = \frac{b}{n}(\epsilon^p)^{-(n-1)/n} \qquad (4)$$

b) Using Eq. (2), the plastic work W_p is obtained as

$$W_p = \int \sigma\, d\epsilon^p = \int n (\frac{\sigma}{b})^n\, d\sigma = \frac{nb}{n+1}(\frac{\sigma}{b})^{n+1} \qquad (5)$$

$$\text{or } W_p = \frac{nb}{n+1}(\epsilon^p)^{(n+1)/n} \qquad (6)$$

c) Solve σ using Eq. (5), and obtain

$$\sigma = b\,(\frac{n+1}{nb} W_p)^{1/(n+1)} = (\frac{n+1}{n} b^n W_p)^{1/(n+1)} \qquad (7)$$

Substituting the above equation to Eq. (4), we obtain

$$E_p = \frac{b}{n}(\frac{b}{\sigma})^{n-1} = \frac{b^n}{n}[\frac{n+1}{n} b^n W_p]^{-(n-1)/(n+1)}$$

d) Noting that the plastic strain takes place for any non-zero σ as specified by the Ramberg-Osgood formula, the initial yield stress is zero, i.e. $\sigma_0 = 0$.

e) For the case of n=1, the Ramberg-Osgood formula becomes

$$\epsilon = \frac{\sigma}{E} + \frac{\sigma}{b},$$

$$\text{or } \sigma = \frac{1}{\frac{1}{E} + \frac{1}{b}}\epsilon = E_t\, \epsilon$$

The σ vs. ϵ curve is plotted in Fig. S1.1.

f) For the case of n = 5, we have

$$\epsilon^p = (\frac{\sigma}{b})^5 \quad \text{or} \quad \sigma = b\,(\epsilon^p)^{1/5}$$

therefore, for $\epsilon^p = 0.001$, we obtain $\sigma_0 = 0.251$ b, and for $\epsilon^p = 0.002$, we obtain $\sigma_0 = 0.289$ b.

1.2 For the material of Problem 1.1 above, assume n = 4, E = 73,000 MPa, b = 800 MPa. A material element is prestrained in tension up to a state with $\epsilon^p = 0.015$ and is subsequently unloaded and then reversed loaded until plastic

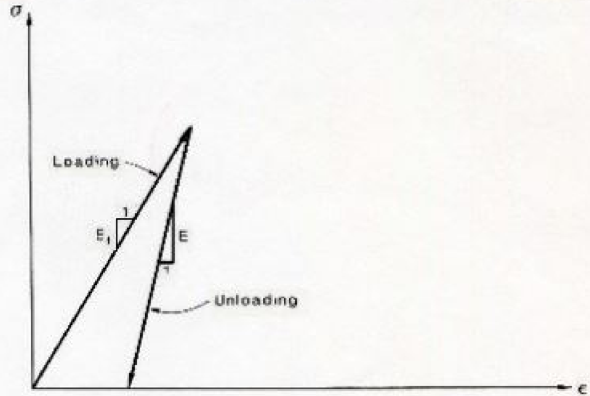

Fig. S1.1 σ vs. ε curve for loading followed by a complete unloading.

flow in compression commences; further compressive yielding continues until $\epsilon^p = -0.015$. The material is assumed to follow (i) isotropic hardening rule; (ii) independent hardening rule, both with the plastic modulus E_p taken to depend on a single hardening modulus κ defined as $\kappa = \int (d\epsilon^p d\epsilon^p)^{1/2}$.

a. Find the stress at the initiation of compressive yielding.

b. Sketch the $\sigma - \epsilon^p$ curve.

SOLUTION

The stress-plastic strain relation according to the Ramberg-Osgood formula is

$$\epsilon^p = (\frac{\sigma}{b})^n, \quad \text{or} \quad \sigma = b\,(\epsilon^p)^{1/n}$$

The generalized relation for any loading path has the form

$$\sigma = b\,(\kappa)^{1/n}, \quad d\sigma = \frac{b}{n}\,(\kappa)^{(1/n)-1}\,d\kappa = E_p\,d\kappa$$

where

$$\kappa = \int (d\epsilon^p d\epsilon^p)^{1/2}, \quad E_p = \frac{b}{n}\,(\kappa)^{(1/n)-1}$$

At the initial tension yield point, $\kappa = \epsilon^p = 0$, we obtain $\sigma_0 = 0$, Point O in Fig. S1.2. In the subsequent tension loading until $\epsilon^p = 0.015$, Path O-A in Fig. S1.2, we have

$$\kappa = \epsilon^p$$

$$\sigma = \sigma_0 + \int_0^{\epsilon^p} d\sigma = \sigma(\epsilon^p) = b\,(\epsilon^p)^{1/n} = 800\,(\epsilon^p)^{1/4}$$

and at Point A,

$$\kappa_A = \epsilon_A^p = 0.015$$

$$\sigma_A = 800 \times (0.015)^{1/4} = 280 \text{ MPa}$$

Case (i): Isotropic Hardening

In the process of unloading and reversed compressive loading, the material begins to yield in compression at Point B in Fig. S1.2. According to the isotropic hardening rule, we have,

$$\sigma_B = -\sigma_A = -280 \text{ MPa}$$

and

$$\kappa_B = \kappa_A = 0.015$$

For the subsequent compression loading along path BC, Fig. S1.2, we have

$$\kappa = \kappa_B + (\epsilon_B^p - \epsilon^p) = 0.03 - \epsilon^p$$

then, the stress in this path is

$$\sigma = \sigma_B + \int_{\kappa_B}^{\kappa} E_p\,d\kappa = \sigma_B - \int_{\epsilon_B^p}^{\epsilon^p} E_p\,d\epsilon^p$$

$$= \sigma_B - \sigma\Big|_{\epsilon_B^p}^{\epsilon^p} = -280 - (800)\,(0.03 - \epsilon^p)^{1/4}\Big|_{\epsilon_B^p}^{\epsilon^p}$$

$$= -800\,(0.03 - \epsilon^p)^{1/4}$$

At Point C, we have

$$\kappa_C = 0.03 - (-0.015) = 0.045$$

and

$$\sigma_C = -(800)\,(0.03 + 0.015)^{1/4} = -368.5 \text{ MPa}$$

Case (ii): Independent Hardening

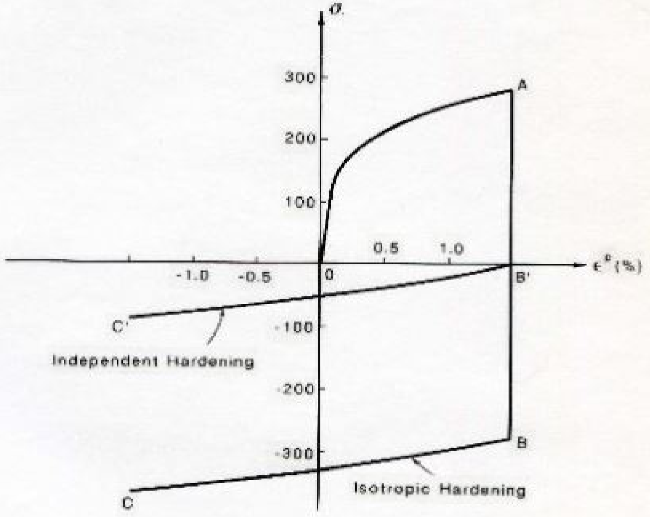

Fig. S1.2 σ vs. ϵ^p curves.

According to the independent hardening rule, at the compressive yield point B′, Fig. S1.2, we have

$$\sigma_B = \sigma_0 = 0,$$

and

$$\kappa_B = \kappa_A = 0.015$$

For further compressive loading, along path B′C′, Fig. S1.2, we have

$$\kappa = \kappa_B + (\epsilon_B^p - \epsilon^p) = 0.03 - \epsilon^p$$

and then the stress is

$$\sigma = \sigma_B + \int_{\kappa_B}^{\kappa} E_p \, d\kappa = \sigma_B - \int_{\epsilon_B^p}^{\epsilon^p} E_p \, d\epsilon^p$$

$$= \sigma_B - \sigma \Big|_{\epsilon_B^p}^{\epsilon^p} = 0 - 800 \times (0.03 - \epsilon^p)^{1/4} \Big|_{\epsilon_B^p}^{\epsilon^p}$$

$$= -800 \times (0.03 - \epsilon^p)^{1/4} + 280 \text{ (MPa)}$$

At Point C$'$, we have

$$\kappa_C = 0.045$$

and

$$\sigma_C = -(800)\,(0.03 + 0.015) + 280 = -88.5 \text{ MPa}$$

1.3 For the material of Problem 1.1, assume $n = 3$, $E = 69{,}000$ MPa, $b = 690$ MPa. A material element is firstly strained in tension up to State 1 with $W_p = 113.85$ kN.m/m^3 and is subsequently unloaded and reversed loaded until plastic flow in compression commences at State 2. Further, it is loaded with a stress increment $d\sigma = -2.07$ MPa up to State 3, and then with another stress increment $d\sigma = -2.07$ MPa up to State 4. After that, the element is unloaded and loaded in tension again until plastic flow occurs at State 5. The material is assumed to follow the isotropic hardening rule with the plastic modulus E_p taken to depend on a hardening parameter κ defined as $\kappa = W_p$.

a. Find the tensile stress σ_1 and plastic strain ϵ_1^p at State 1.

b. Find the stress σ, strain ϵ, plastic strain ϵ^p, plastic work W_p, and plastic modulus E_p at States 2, 3 and 4, respectively.

c. Find the stress σ_5 and plastic modulus E_p at State 5.

SOLUTION

Using the stress-strain relation given in Problem 1.1,

$$\epsilon = \epsilon^e + \epsilon^p, \quad \epsilon^e = \frac{\sigma}{E}, \quad \epsilon^p = \left(\frac{\sigma}{b}\right)^n$$

we obtain

$$\sigma = b\,(\epsilon^p)^{1/n} \quad \text{(MPa)}$$

$$W_p = \frac{nb}{n+1}\,(\epsilon^p)^{(n+1)/n} \quad \text{(MPa)}$$

$$\epsilon^p = \left(\frac{n+1}{nb} W_p\right)^{n/(n+1)}$$

$$E_p = \frac{b}{n}\left(\frac{n+1}{nb} W_p\right)^{(1-n)/(n+1)} \quad \text{(MPa)}$$

a) At State 1, $W_p = 113.85$ kN.m/m^3 = 0.113850 MPa, we obtain

$$\epsilon_1^p = \left(\frac{n+1}{nb} W_p\right)^{n/(n+1)} = \left(\frac{4}{3 \times 690} \times 0.11385\right)^{3/4} = 0.001806$$

$$\sigma_1 = b\,(\epsilon^p)^{1/n} = 690 \times (0.001806)^{1/3} = 84.028 \text{ MPa}$$

$$E_p = \frac{b}{n}\left(\frac{n+1}{nb} W_p\right)^{(1-n)/(n+1)} = \frac{690}{3} \times \left(\frac{4}{3 \times 690} \times 0.11385\right)^{-2/4} = 15{,}507 \text{ MPa}$$

$$\epsilon_1 = \frac{\sigma_1}{E} + \epsilon_1^p = \frac{84.028}{69000} + 0.001806 = 0.003024$$

b) According to the isotropic hardening rule, the stress increment from State 1 to State 2 is

$$d\sigma = -2\,\sigma_1 = -168.056 \text{ MPa}$$

and noting that there is no new plastic deformation from State 1 to State 2 therefore, we obtain

$$\epsilon_2 = \epsilon_1 + \frac{d\sigma}{E} = 0.003024 - \frac{168.056}{69000} = 0.000588$$

$$\sigma_2 = \sigma_1 + d\sigma = 84.028 - 168.056 = -84.028 \text{ MPa}$$

$$\epsilon_2^p = \epsilon_1^p = 0.001806$$

$$W_{p2} = W_{p1} = 0.113850 \text{ MPa}$$

$$E_{p2} = E_{p1} = 15{,}507 \text{ MPa}$$

From State 2 to State 3, the stress increment is

$$d\sigma = -2.07 \text{ MPa}$$

and using the plastic modulus at State 2, we obtain at State 3

$$\sigma_3 = \sigma_2 + d\sigma = -84.028 - 2.07 = -86.098 \text{ MPa}$$

$$\epsilon_3^p = \epsilon_2^p + \frac{d\sigma}{E_{p2}} = 0.001806 - \frac{2.07}{15{,}507} = 0.001673$$

$$\epsilon_3 = \epsilon_2 + d\epsilon^e + d\epsilon^p = 0.000588 - \frac{2.07}{69{,}000} - \frac{2.07}{15{,}507} = 0.0004247$$

$W_{p3} = W_{p2} + \sigma_3\, d\epsilon^p = 0.11385 + 86.098 \times 0.0001335 = 0.12534$ MPa

$$E_{p3} = \frac{b}{n}\left(\frac{n+1}{nb}W_{p3}\right)^{(1-n)/(n+1)} = \frac{690}{3} \times \left(\frac{4}{3x690} \times 0.12534\right)^{-2/4} = 14{,}778 \text{ MPa}$$

From State 3 to State 4, the stress increment is

$$d\sigma = -2.07 \text{ MPa}$$

and using the plastic modulus at State 3, we obtain at State 4

$$\sigma_4 = \sigma_3 + d\sigma = -88.168 \text{ MPa}$$

$$\epsilon_4^p = \epsilon_3^p + \frac{d\sigma}{E_{p3}} = 0.001533$$

$$\epsilon_4 = \epsilon_3 + \frac{d\sigma}{E} + \frac{d\sigma}{E_{p3}} = 0.0002546$$

$$W_{p4} = W_{p3} + \sigma_4\, d\epsilon^p = 0.13769 \text{ MPa}$$

$$E_{p4} = \frac{b}{n}\left(\frac{n+1}{nb}W_{p4}\right)^{(1-n)/(n+1)} = 14{,}100 \text{ MPa}$$

c) According to the isotropic hardening rule, the stress at State 5 is

$$\sigma_5 = -\sigma_4 = 88.168 \text{ MPa}$$

and because no new plastic deformation occurs from State 4 to State 5,

$$E_{p5} = E_{p4} = 14{,}100 \text{ MPa}$$

1.4 For the overlay material model of Example 1.1 (see Fig. 1.7), assume that the material parameters are selected as $A_1 = \frac{2}{3}$, $A_2 = \frac{1}{3}$, $\sigma_{01} = 138$ MPa, $\sigma_{02} = 345$ MPa and $E = 69{,}000$ MPa. The strains at Points c and f in Fig. 1.8 are taken to be $\epsilon_c = 0.013$ and $\epsilon_f = 0.011$, and State h is assumed to correspond to a compressive stress in Bars 2 of the value $\frac{\sigma_{02}}{2}$.

a. Find residual stresses in Bars 1 and 2 when $\sigma = 0$ along the unloading Paths c-d and f-g, and the reloading Path h-i.

b. Determine the stress in Bars 2, corresponding to States g and i.

c. What are the values of the stress in Bars 2, σ_2, and strain ϵ when stress in Bars 1 is completely relieved (i.e. $\sigma_1 = 0$) during unloading along Path f-g and during reloading along Path h-i.

d. For $\sigma - \epsilon$ paths in Fig. 1.8, plot bar stresses σ_1 vs σ_2. Showing the line of equivalent stress $\sigma = 0$.

SOLUTION

a) At Point c, $\sigma_1 = \sigma_{01}$, $\sigma_2 = \sigma_{02}$, and

$$\sigma_c = \frac{\sigma_{01}A_1 + \sigma_{02}A_2}{A_1 + A_2} = 207 \text{ MPa}$$

Along the unloading Path c-d, from Point c to the point at which $\sigma = 0$, the strain increment is obtained as

$$\Delta\epsilon = -\frac{\sigma_c}{E}$$

and the stresses at this point are given by

$$\sigma_1 = \sigma_{01} + E\,\Delta\epsilon = \sigma_{01} - \sigma_c = -69 \text{ MPa}$$

$$\sigma_2 = \sigma_{02} + E\,\Delta\epsilon = \sigma_{02} - \sigma_c = 138 \text{ MPa}$$

Because the stress state at Point f is the same as at Point c, the stress at the point at which $\sigma = 0$ in the unloading path f-g is the same as in the unloading path c-d, i.e.,

$$\sigma_1 = -69 \text{ MPA}$$

$$\sigma_2 = 138 \text{ MPa}$$

At Point g, Bar 1 yields, the strain increment from Point f to Point g is found to be

$$\Delta\epsilon = \frac{1}{E}(\sigma_{1g} - \sigma_{1f}) = \frac{1}{E}(-\sigma_{01} - \sigma_{01}) = -\frac{2\sigma_{01}}{E}$$

and the stress in Bar 2 and the strain are obtained as

$$\sigma_{2g} = \sigma_{2f} + E\,\Delta\epsilon = \sigma_{02} - 2\,\sigma_{01} = 69 \text{ MPa}$$

$$\epsilon_g = \epsilon_f + \Delta\epsilon = 0.011 - 0.004 = 0.007$$

At Point h, we know $\sigma_1 = -\sigma_{01}$, $\sigma_2 = -\sigma_{02}/2$, then, the stress increment in Bar 2 is

$$\Delta\sigma_2 = \sigma_{2h} - \sigma_{2g} = -241.5 \text{ MPa}$$

We can find the stress and strain at Point h,

$$\sigma_h = \frac{\sigma_{1h}A_1 + \sigma_{2h}A_2}{A_1 + A_2} = -149.5 \text{ MPa}$$

$$\epsilon_h = \epsilon_g + \frac{\Delta\sigma_2}{E} = 0.007 - 0.0035 = 0.0035$$

Along the unloading Path h-i, the strain increment from Point h to the point at which $\sigma = 0$ is given by

$$\Delta\epsilon = -\frac{\sigma_h}{E}$$

then, we can find the residual stresses in Bars 1 and 2 when $\sigma = 0$ along the reloading Path h-i:

$$\sigma_1 = \sigma_{1h} + E\,\Delta\epsilon = -\sigma_{01} - \sigma_h = 11.5 \text{ MPa}$$

$$\sigma_2 = \sigma_{2h} + E\,\Delta\epsilon = -\frac{\sigma_{02}}{2} - \sigma_h = -23 \text{ MPa}$$

b) We have already obtained $\sigma_{2g} = 69$ MPa in (a). At Point i, Bar 1 yields in tension at $\sigma_{1i} = \sigma_{01}$, therefore, the strain increment from Point h to i is $\Delta\epsilon = 2\sigma_0/E$, thus the stress in Bar 2 at Point i can be found as

$$\sigma_{2i} = \sigma_{2h} + E\,\Delta\epsilon = -\frac{\sigma_{02}}{2} + 2\sigma_{01} = 103.5 \text{ MPa}$$

c) In Path f-g, from Point f to the point where $\sigma_1 = 0$, we have

$$\Delta\sigma_1 = -\sigma_{01}, \quad \Delta\epsilon = -\frac{\sigma_{01}}{E}$$

therefore at that point, we have

$$\sigma_2 = \sigma_f + E\,\Delta\epsilon = \sigma_{02} - \sigma_{01} = 207 \text{ MPa}$$

$$\epsilon = \epsilon_f + \Delta\epsilon = 0.011 - 0.002 = 0.009$$

In Path h-i, from Point h to the point where $\sigma_1 = 0$, we have

$$\Delta\sigma_1 = \sigma_{01}, \quad \Delta\epsilon = \frac{\sigma_{01}}{E}$$

therefore, at that point, we have

$$\sigma_2 = \sigma_h + E\,\Delta\epsilon = -\frac{\sigma_{02}}{2} + \sigma_{01} = -34.5 \text{ MPa}$$

$$\epsilon = \epsilon_h + \Delta\epsilon = 0.0035 + 0.002 = 0.0055$$

d) The stresses σ_1 and σ_2 at each stage as shown in Fig. 1.8 are listed as follows, in MPa.

	a	b	c	d	e	f	g	h	i
σ_1	138	138	138	− 138	− 138	138	− 138	− 138	138
σ_2	138	345	345	69	− 345	345	69	− 172.5	103.5

Let $\sigma = 0$, we obtain

$$\sigma_1 A_1 + \sigma_2 A_2 = 0$$

or

$$2\sigma_1 + \sigma_2 = 0$$

The σ_1 vs σ_2 curve is plotted in Fig. S1.4

1.5 An initially unstressed and unstrained element of the same linear strain-hardening material as in Example 1.2 is subjected to different loading histories which produce the stress paths given below. For each of the three hardening rules considered in Example 1.2, find the final strain state, ϵ, and the corresponding ϵ^p attained at the end of each loading path. In the following, stress σ is in MPa.

i. $\sigma = 0 \rightarrow 414 \rightarrow -414 \rightarrow 0 \rightarrow 414$

ii. $\sigma = 0 \rightarrow 621 \rightarrow 0$

For each case, show schematic representations of the stress-strain paths followed in the $\sigma - \epsilon$ and $\sigma - \epsilon^p$ spaces.

SOLUTION

In Example 1.2, the stress-strain relation is given as

$$\sigma = \sigma_0 + m\,\epsilon^p, \quad \text{for } \sigma \geq \sigma_0$$

$$\epsilon^e = \frac{\sigma}{E}$$

and $\sigma_0 = 207$ MPa, $E = 207{,}000$ MPa, $m = 25{,}900$ MPa, $E_p = 25{,}900$ MPa, $E_t = 23{,}020$ MPa.

<u>Isotropic Hardening.</u>

Case (i): (Figs. S1.5.1a, 1b)

At Point A, the element initially yields in tension, we obtain

$$\sigma_A = \sigma_0 = 207 \text{ MPa}, \quad \epsilon_A^p = 0, \quad \epsilon_A^e = \frac{\sigma_A}{E} = 0.001$$

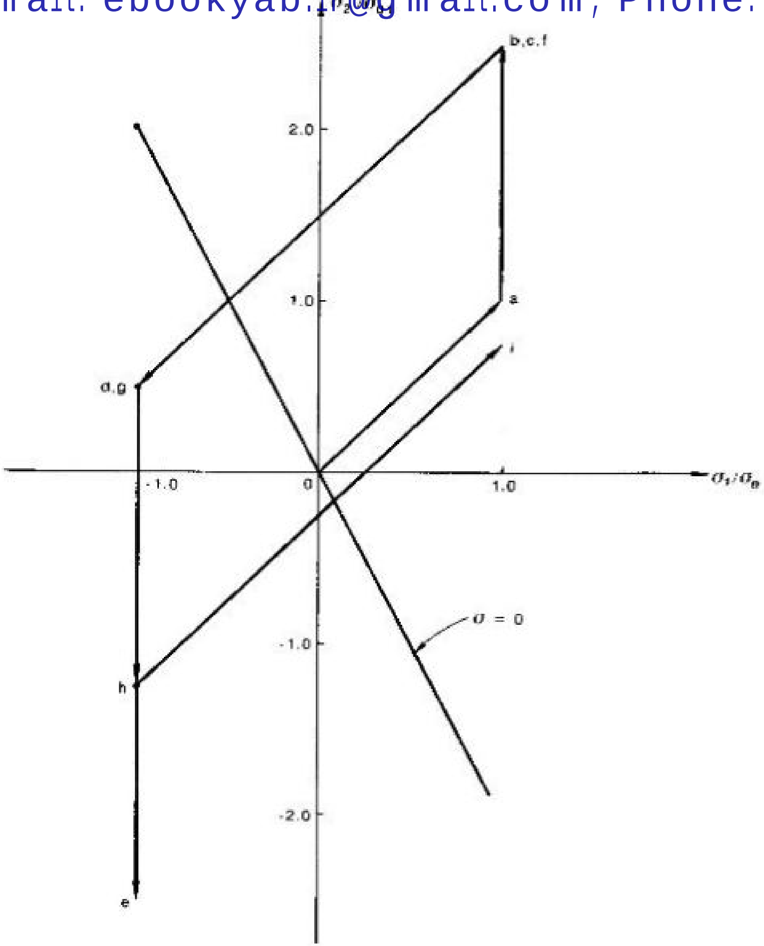

Fig. S1.4 The σ_1 vs. σ_2 curve.

$$\epsilon_A = \epsilon_A^p + \epsilon_A^e = 0.001$$

From Point A to Point B, the element is subjected to plastic loading, we have

$$\sigma_B = 414 \text{ MPa}, \quad d\sigma = 207 \text{ MPa}$$

$$\epsilon_B^p = \epsilon_A^p + \frac{d\sigma}{E_p} = 0.008, \quad \epsilon_B = \epsilon_A + \frac{d\sigma}{E_t} = 0.01$$

Unloading occurs from Point B to Point C, according to the isotropic hardenig rule, we have

$$\sigma_C = -414 \text{ MPa}, \quad d\sigma = -828 \text{ MPa}$$

$$\epsilon_C^p = \epsilon_B^p = 0.008, \quad \epsilon_C = \epsilon_B + \frac{d\sigma}{E} = 0.006$$

From C to D, the material is in elastic loading, we have

$$\sigma_D = 0, \quad d\sigma = 414 \text{ MPa},$$

$$\epsilon_D = \epsilon_C + \frac{d\sigma}{E} = 0.008, \quad \epsilon_D^p = \epsilon_C^p = 0.008$$

From D to E, the material is also in elastic loading, thus, we have

$$\sigma_E = 414 \text{ MPa}, \quad d\sigma = 414 \text{ MPa}$$

$$\epsilon_E^p = \epsilon_D^p = 0.008, \quad \epsilon_E = \epsilon_D + \frac{d\sigma}{E} = 0.01$$

Case (ii): (Figs. S1.5.2a, 2b)

At Point A, the material initially yields in tension, and we have

$$\sigma_A = \sigma_0 = 207 \text{ MPa}, \quad \epsilon_A^p = 0, \quad \epsilon_A = \frac{\sigma_A}{E} = 0.001$$

From A to B, the material is in plastic loading, we obtain

$$\sigma_B = 621 \text{ MPa}, \quad d\sigma = 414 \text{ MPa}$$

$$\epsilon_B^p = \epsilon_A^p + \frac{d\sigma}{E_p} = 0.016, \quad \epsilon_B = \epsilon_A + \frac{d\sigma}{E_t} = 0.019$$

From B to C, the material is in elastic unloading, we have

$$\sigma_C = 0, \quad d\sigma = -621 \text{ MPa}$$

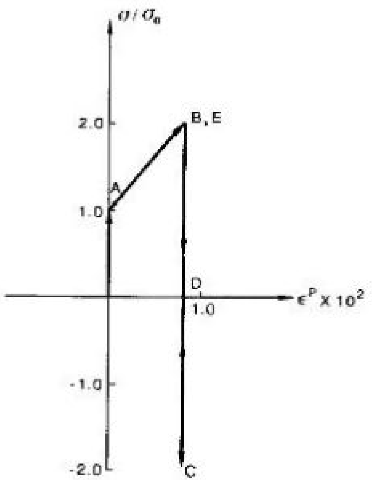

Fig. S1.5.1a

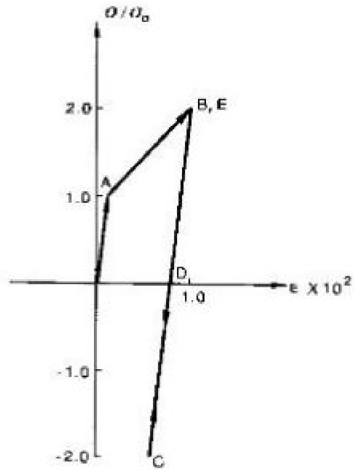

Fig. S1.5.1b

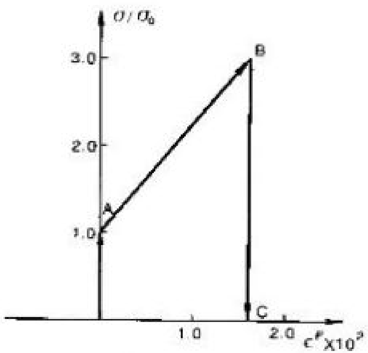

Fig. S1.5.2a

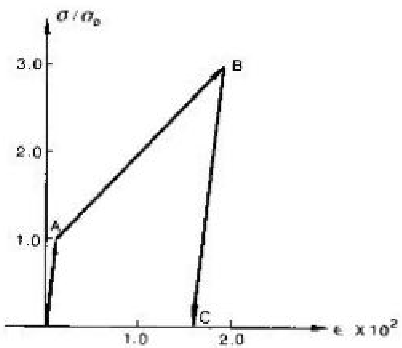

Fig. S1.5.2b

$$\epsilon_C^p = \epsilon_B^p = 0.016, \quad \epsilon_C = \epsilon_B + \frac{d\sigma}{E} = 0.016$$

Kinematic Hardening

Case (i): (Figs. S1.5.3a, 3b)

At Point A, the material initially yields in tension and we have

$$\sigma_A = \sigma_0 = 207 \text{ MPa}, \quad \epsilon_A^p = 0, \quad \epsilon_A = \frac{\sigma_A}{E} = 0.001$$

From A to B, the material is in plastic loading, and we have

$$\sigma_B = 414 \text{ MPa}, \quad d\sigma = 207 \text{ MPa},$$

$$\epsilon_B^p = \epsilon_A^p + \frac{d\sigma}{E_p} = 0.008, \quad \epsilon_B = \epsilon_A + \frac{d\sigma}{E_t} = 0.01$$

From B to C, the material is in elastic unloading, and at Point C, the material starts yielding in compression, we have

$$\sigma_C = \sigma_B - 2\sigma_0 = 0, \quad d\sigma = -2\sigma_0 = -414 \text{ MPa}$$

$$\epsilon_C^p = \epsilon_B^p = 0.008, \quad \epsilon_C = \epsilon_B + \frac{d\sigma}{E} = 0.008$$

From C to D, the material is in plastic loading, we have

$$\sigma_D = -414 \text{ MPa}, \quad d\sigma = -414 \text{ MPa}$$

$$\epsilon_D^p = \epsilon_C^p + \frac{d\sigma}{E_p} = -0.008, \quad \epsilon_D = \epsilon_C + \frac{d\sigma}{E_t} = -0.01$$

From D to E, the material is in elastic unloading, and at Point E, the material yields in tension again, we have

$$\sigma_E = 0, \quad d\sigma = 414 \text{ MPa}, \quad \epsilon_E^p = \epsilon_D^p = -0.008, \quad \epsilon_E = \epsilon_D + \frac{d\sigma}{E} = -0.008$$

From E to F, the material is in plastic loading, and we have

$$\sigma_F = 414 \text{ MPa}, \quad d\sigma = 414 \text{ MPa}$$

$$\epsilon_F^p = \epsilon_E^p + \frac{d\sigma}{E_p} = 0.008, \quad \epsilon_F = \epsilon_E + \frac{d\sigma}{E_t} = 0.01$$

Case (ii): (Figs. S1.5.4a, 4b)

At Point A, the material initially yields in tension, and we have