1.1. a. $x(t)=e^{.5t}\quad t\geq 0 \Rightarrow$

$$E=\lim_{T\to\infty}\int_0^T\left(e^{.5t}\right)^2dt=\lim_{T\to\infty}\int_0^T e^{t}dt \Rightarrow E=\lim_{T\to\infty}\left(e^{T}-1\right)$$

$\Rightarrow E=\infty \Rightarrow$ not an energy signal

$$P=\lim_{T\to\infty}\frac{1}{T}\int_0^{T/2}e^{t}dt=\lim_{T\to\infty}\frac{1}{T}\left(e^{T/2}-1\right)=\infty \Rightarrow \text{not a power signal}$$

$\Rightarrow x(t)=e^{.5t}$ is not stable

b. $x(t)=e^{-.5t}\quad t\geq 0 \Rightarrow E=\lim_{T\to\infty}\int_0^T e^{-T}dt=\lim_{T\to\infty}\left(e^{-T}+1\right)=1$ Joule $\Rightarrow$ Energy signal

c. $x(t)=\cos t+\cos 2t \Rightarrow P=\frac{1}{T}\int_{-T/2}^{T/2}\left(\cos t+\cos 2t\right)^2dt=0.5+0.5=1^{W} \Rightarrow$ power signal

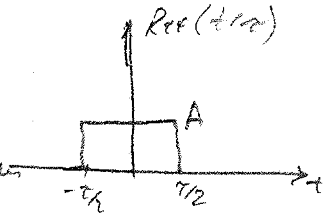
d. $x(t)=e^{-a|t|} \Rightarrow$

$$E=\int_{-\infty}^{\infty}\left(e^{-a|t|}\right)^2dt=2\int_0^{\infty}e^{-2at}dt=\frac{1}{a}J \Rightarrow \text{energy signal.}$$

1.2 $x(t)=A\,Rect(t/\tau)$

$\Rightarrow$

$$E=\int_{-\infty}^{\infty}x^2(t)\,dt=\int_{-\tau/2}^{\tau/2}A^2dt=A^2\left(\tau/2+\tau/2\right)=A^2\tau \text{ Joules}$$



1.4. a. $\int_{-2}^{2}\varphi_1(t)\varphi_1^*(t)dt=\int_0^2(1)dt=2$

$$\int_{-2}^{2}\varphi_1(t)\varphi_2^*(t)\,dt=\int_0^{.5}1\,dt+\int_{.5}^{1}(-1)dt=0$$

Finally

$$\int_{-2}^{2}\varphi_2(t)\varphi_2^*(t)dt=\int_{-1}^{1}(1)dt=2 \Rightarrow \text{Then } \varphi_1 \,\&\, \varphi_2 \text{ are orthogonal}$$

b. $x(t)=\sum_{n=1}^{2}X_n\varphi_n(t)=X_1\varphi_1(t)+X_2\varphi_2(t)$, where

$$X_1=\frac{1}{2}\int_{-2}^{2}(t)\,\varphi_1^*(t)\,dt=\frac{1}{2}\int_0^1 t\,dt+\frac{1}{2}\int_1^2(-t)dt=-1/2$$

$$X_2=\frac{1}{2}\int_{-2}^{2}t\;\varphi_2^*(t)=\frac{1}{2}\left[2\int_0^{.5}t\,dt+2\int_{.5}^{1}(-t)\,dt=-1/4\right.$$

1.3. a. From Fourier transform tables

$$\mathcal{F}\left[2\Delta\left(\frac{t-3}{5}\right)\right] = 5e^{-j3\omega}\,\mathrm{Sinc}^2\left(\frac{5\omega}{4}\right)$$

b. $x_p(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2n\pi t/T}$ where $X_n = \frac{5}{10}\,\mathrm{Sinc}^2\left(\frac{n\pi}{4}\right)$

1.5.

$$y(t) = \sum_{n=-\infty}^{\infty} Y_n e^{j2n\pi t/T} \quad \text{where } Y_n = \frac{1}{T}\int_0^T y(t)e^{-j2n\pi t/T}dt = \frac{1}{T}\int_0^T x(t-t_0)e^{-j2n\pi t/T}dt$$

let $u = t - t_0 \Rightarrow du = dt$

$$\Rightarrow Y_n = \frac{1}{T}\int_{t_0}^{T+t_0} x(u)e^{j2n\frac{\pi}{T}(u-t_0)}du = e^{-j2n\pi\frac{t_0}{T}} \cdot \frac{1}{T}\int_{t_0}^{T-t_0} x(u)e^{-j2n\frac{\pi u}{T}}du = X_n e^{-j2n\pi t_0/T}$$

1.6. a. $R_u = \frac{c}{2f_r}$; $f_r = 200\,Hz \Rightarrow R_u = \frac{3\times10^8}{2\times200} = 750\,Km$

for $f_r = 750\,Hz \Rightarrow R_u = \frac{3\times10^8}{2\times750} = 200\,Km$

b. $PRI = \frac{1}{f_r} \Rightarrow PRI_1 = \frac{1}{200} = 5.0\,msec$

$PRI_2 = \frac{1}{750} = 1.33\,msec$

1.7. a. for $f_r = 200\,Hz$; $d_t = \frac{\tau}{T}$; $T = 5.0\,msec \Rightarrow \tau = 0.3\times5.0\times10^{-3} = 1.5\,msec$

The average transmitted power is $P_{av} = d_t \cdot P_t = .3\times5\times10^3 = 1.5\,KW$

In 20 msec we have 4-periods $\Rightarrow$ The average transmitted power in 20 msec is

$P_{av_{20ms}} = 4\times1.5\,KW = 6\,KW$

The single pulse energy is $E_p = P_t\,\tau = 5\,KW\times1.5\,msec = 7.5$ joules

and in 20 msec $\Rightarrow E_{20msec} = 4\times7.5 = 30$ joules

b. for $f_r = 750\,Hz \Rightarrow \tau = \frac{.3}{750} = 0.4\,msec$

As in 20 msec $\Rightarrow \frac{20}{1.33} = 15$ periods $\Rightarrow P_{av} = 15\times1.5 = 22.5\,KW$

& $E_{20} = 15 * E_p = 15\times5\,KW\times.4 = 30$ joules.

1.8.

$$\Delta R = \frac{c\tau}{2} \Rightarrow \Delta R = \frac{3\times10^{8}\times1\times10^{-6}}{2} = 150\,m$$

1.9.

$$R_u = \frac{c}{2f_r} = \frac{3\times10^{8}}{2\times3\times10^{3}} = 50\,km$$

$$\Delta R = \frac{c}{2B} \Rightarrow B = \frac{c}{2\Delta R} = \frac{3\times10^{8}}{2\times30} = 5\,MHz$$

$$d_t = \frac{\tau}{T} = \tau f_r = \frac{f_r}{B} = \frac{3\times10^{3}}{5\times10^{6}} = 0.006$$

1.10.

$$f_d = \frac{2v_r}{\lambda} \Rightarrow \text{for } v_r = 100\frac{m}{s} \Rightarrow f_{d_1} = \frac{2\times100}{0.3} = 666.67\,kHz$$

$$\text{for } v_r = 200\frac{m}{s} \Rightarrow f_{d_2} = \frac{2\times200}{.3} = 1.33\,kHz$$

$$\text{for } v_r = 350\frac{m}{s} \Rightarrow f_{d_3} = \frac{2\times350}{.3} = 2.33\,kHz$$

1.11.

$$\Delta t = \frac{2R}{c} \Rightarrow \text{for } R_1 = 30\,km \Rightarrow \Delta t_1 = \frac{2\times30\times10^{3}}{3\times10^{8}} = 200\,\mu sec$$

$$\text{for } R_2 = 80\,km \Rightarrow \Delta t_2 = \frac{2\times80\times10^{3}}{3\times10^{8}} = 533.33\,\mu sec$$

$$\text{for } R_3 = 150\,km \Rightarrow \Delta t_3 = \frac{2\times150\times10^{3}}{3\times10^{8}} = 1.0\,msec$$

1.12.

for S-Band radar where $f = 3\,GHz \Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^{8}}{3\times10^{9}} = 0.1\,m$

$$\Rightarrow \text{for } v_r = 50\,m/sec \Rightarrow f_{d_1} = \frac{2\times50}{0.1} = 1\,KHz$$

$$\text{for } v_r = 200\frac{m}{sec} \Rightarrow f_{d_2} = \frac{2\times200}{0.1} = 4\,KHz$$

$$\text{for } v_r = 250\frac{m}{sec} \Rightarrow f_{d_3} = \frac{2\times250}{0.1} = 5\,KHz$$

1.13.

for $f = 10\,GHz \Rightarrow \lambda = \frac{3\times10^{8}}{10\times10^{9}} = 0.03\,m = 30\,mm$

$$\Rightarrow \text{for } v_r = 50\frac{m}{sec} \Rightarrow f_{d_1} = \frac{2\times50}{.03} = 3.33\,KHz$$

for $v_r = 200\ \frac{m}{sec} \Rightarrow f_{d_2} = \frac{2\times 200}{.03} = 13.33\ KHz$

for $v_r = 250\ \frac{m}{sec} \Rightarrow f_{d_3} = \frac{2\times 250}{.03} = 22.0\ KHz$

1.14.

$f_{d_{max}} = \frac{2 v_{r_{max}}}{\lambda}$; we know that $f_{r_{min}} \geq 2\frac{2 v_{r_{max}}}{\lambda}$

$\Rightarrow v_{r_{max}} = \frac{f_{r_{min}}}{4}\lambda$; & $\lambda = \frac{c}{f} = \frac{3\times 10^8}{1.5\times 10^9} = 0.2m$

$\Rightarrow v_{r_{max}} = 10\times 10^3 \cdot \frac{.2}{4} = 500\ \frac{m}{sec} \Rightarrow f_{d_{max}} = \frac{2\times 500}{.2} = 5\ KHz$

1.15. Start with

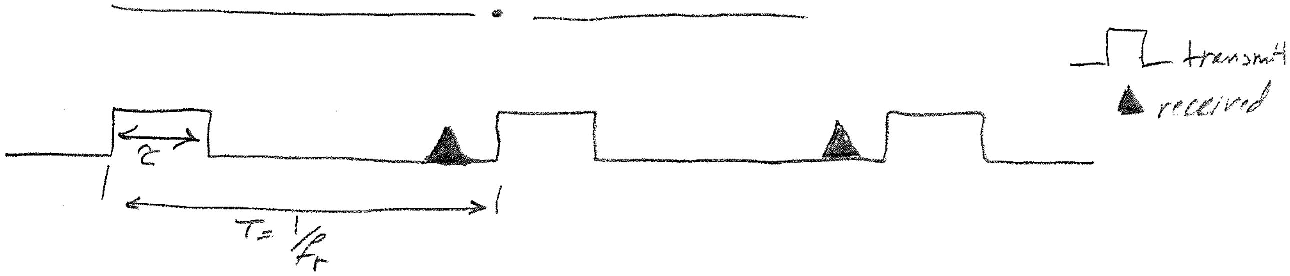
$R(t) = R_0 + v(t - t_0)$ & follow same steps outlined in text.

1.16. $f_d = \frac{2 v_r}{\lambda} \cos\theta_e \cos\theta_a$

$\Rightarrow f_d = \frac{2\times 150}{0.1} \cos 30^\circ \cos 15^\circ = 2.51\ KHz$

$\Delta R = \frac{c\tau}{2} \Rightarrow 0.3 = \frac{3\times 10^8 \tau}{2} \Rightarrow \tau = 2\ nsec \Rightarrow B = \frac{1}{\tau} = 500\ MHz$

1.18



The minimum PRF must be chosen so that the target range is unambigous $\Rightarrow$

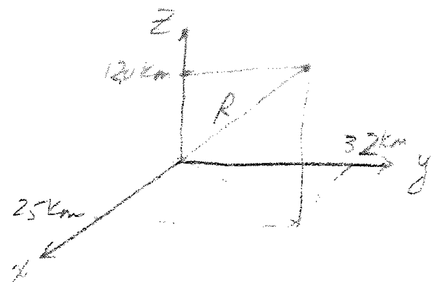
$f_{r_{min}} \geq 2\frac{2 v_{r_{max}}}{\lambda} \geq 2 f_{d_{max}} \Rightarrow$

$f_{r_{min}} = 4\frac{400}{.2} = 8\ KHz$

& $R_u = \frac{c}{2 f_r} = \frac{3\times 10^8}{2\times 8\times 10^3} = 18.75\ KHz$

1.19.

$$R = \sqrt{x^2 + y^2 + z^2}$$



$\Rightarrow$

a.
$$R = \sqrt{120^2 + 25^2 + 32^2}$$
$$= \sqrt{179.3} \qquad = 42.344\ Km.$$

b. $v_x = -250\ m/sec \quad ; v_z = v_y = 0.$

$$\dot{R} = \text{Range rate} = \frac{dR}{dt} = \frac{\Delta R}{\Delta t} = \frac{\Delta v . R}{\Delta R} = \frac{\Delta v_x . R_x + \Delta v_y . R_y + \Delta v_z . R_z}{\Delta R}$$

$$= \frac{-250 * 25 + 32 * 0 + 12 * 0}{42.344} \Rightarrow \dot{R} = -147\ m/sec.$$

c. Round trip delay is $\Delta t = \frac{2 * 42.344 \times 10^3}{3 \times 10^8} = 282.29\ \mu sec$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^9} = 33.33 \times 10^{-3}\ m$$

$\Rightarrow$

$$f_d = \frac{2 \dot{R}}{\lambda} = \frac{2 * -147}{33.33 \times 10^{-3}} = -8.82\ KHz \quad (\text{openning target}).$$

1.20

$$\Delta R = \frac{c \tau}{2} \Rightarrow \tau = \frac{2 \times 10^2}{3 \times 10^8} = 66.67\ \mu sec$$

$$R_u = \frac{c}{2 f_r} \Rightarrow f_r = \frac{2 \times 100 \times 10^3}{3 \times 10^8} = 1.5\ KHz$$

$$P_{av} = P_t d_t = P_t \tau f_r = 500 \times 66.667 \times 10^{-6} \times 1.5 \times 10^3 = 0.5\ KW \Leftrightarrow 27 dB$$

2.1. $x(t) = Rect\left(\frac{t}{\tau}\right) \cos\left(\omega_0 t + \frac{B t^2}{2\tau}\right)$

$$x_I(t) = Rect(t/\tau) \cos \frac{B}{2\tau} t^2 \quad \& \quad x_Q(t) = Rect\left(\frac{t}{\tau}\right) \sin\left(\frac{B t^2}{2\tau}\right)$$

2.2. for $\tau = 15\,\mu sec$ & $B = 10\,MHz$ $\Rightarrow$

$$x_I(t) = Rect\left(\frac{t}{15\mu}\right) \cos(333.336\, t^2) \quad \& \quad x_Q(t) = Rect\left(\frac{t}{15\mu}\right) \sin(333.336\, t^2)$$

2.3.

$f_i(t) = \frac{1}{2\pi} Im\left\{\frac{d}{dt} \ln \psi(t)\right\}$, $\psi(t)$ is the analytic signal

$\Rightarrow$

$f_i(t) = \frac{1}{2\pi} Im\left\{\frac{\psi'(t)}{|\psi(t)|^2}\right\}$ where ψ' indicates derivative wrt t

but $\psi(t) = \tilde{x}(t)\, e^{j2\pi f_0 t}$ $\quad$ $\tilde{x}(t)$ is the complex envelope for $x(t)$

$\Rightarrow$

$f_i(t) = \frac{1}{2\pi |\psi(t)|^2} \left[\hat{x}'(t)\, x(t) - x'(t)\, \hat{x}(t)\right]$ where $\hat{x}$ is the Hilbert transform for x

Recall $\tilde{x} = x_I + j x_Q$ & $x = x_I \cos 2\pi f_0 t - x_Q \sin 2\pi f_0 t$

So when $\psi(t) = Rect\left(\frac{t}{\tau}\right) \cos\left(2\pi f_0 t + \frac{B}{2\tau} t^2\right)$ we get

$$\ln[\psi(t)] = C + j\left(2\pi f_0 t + \frac{B}{2\tau} t^2\right)$$

$$\Rightarrow f_i(t) = \frac{1}{2\pi} Im\left\{\frac{d}{dt} \ln \psi(t)\right\} = f_0 + \frac{1}{2\pi} \frac{B}{\tau} t$$

2.4.

$h(t) = \delta(t) - \frac{\omega_0}{\omega_d} e^{-2t} \sin \omega_0 t \qquad t \geq 0$

In general $h(t) = h_I(t) \cos \omega_0 t - h_Q(t) \sin \omega_0 t$

$\Rightarrow$ by inspection

$$h_I(t) = \delta(t) \quad \& \quad h_Q(t) = \frac{\omega_0}{\omega_d} e^{-2t}$$

2.8

a) REPLACE $D(y)$ BY THE CONSTANT d_0 $\Rightarrow$

$$E(\xi) = d_0 \int_{-r/2}^{r/2} \exp\left(2\pi y j \frac{\sin\xi}{\lambda}\right) dy$$

$$= d_0 \left[\frac{\exp\left(2\pi y j \frac{\sin\xi}{\lambda}\right)}{j 2\pi \frac{\sin\xi}{\lambda}} \right] \Bigg|_{-r/2}^{r/2}$$

$\Rightarrow$

$$E[\xi] = \frac{d_0}{j 2\pi \frac{\sin\xi}{\lambda}} \left[\exp\left(\pi j \frac{\sin\xi}{\lambda} r\right) - \exp\left(-\pi j \frac{\sin\xi}{\lambda} r\right) \right]$$

USING THE RELATION $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ AND $Sa(\theta) = \frac{\sin\theta}{\theta}$ $\Rightarrow$

$$E(\xi) = d_0 r\, Sa\left(\pi \frac{\sin\xi}{\lambda} r\right) \Rightarrow |E(\xi)|^2 = d_0^2 a^2 Sa^2\left(\pi \frac{\sin\xi}{\lambda} r\right)$$

b) $P(\xi) = \frac{E(\xi)}{E(0)} = Sa^2\left(\frac{\pi \sin\xi\, r}{\lambda}\right) = Sa^2(x)$, $x = \frac{\pi \sin\xi\, r}{\lambda}$

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$6\pi/8$	$7\pi/8$	π
$P(x)$	1	0.949	0.8105	0.61499	0.40528	0.2214	0.090	0.019	0
$P(x)_{dB}$	0	-0.2244	-0.9121	-2.111	-3.9223	-6.548	-10.45	-17.12	$-\infty$

2.9 a.

$$P_{rad} = \iint |F(\theta,\varphi)|^2 d\Omega = \int_0^{2\pi}\int_0^{\pi} |\cos^n\theta| \sin\theta\, d\theta\, d\varphi = 2\pi(2)\int_0^{\pi/2} \cos^n\theta \sin\theta\, d\theta$$

$\Rightarrow$ for $n=1$

$$P_{rad} = 4\pi \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta = 4\pi \left(\frac{1}{2}\sin^2\theta \Big|_0^{\pi/2}\right) = 2\pi$$

for $n=2$

$$P_{rad} = 4\pi \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta = 4\pi \left(-\frac{\cos^3\theta}{3}\Big|_0^{\pi/2}\right) = \frac{4\pi}{3}$$