

Chapter 1

1.1. a. $x(t) = e^{st} \quad t \geq 0 \Rightarrow E = \lim_{T \rightarrow \infty} \int_0^T (e^{st})^2 dt = \lim_{T \rightarrow \infty} \int_0^T e^{2t} dt \Rightarrow E = \lim_{T \rightarrow \infty} (e^{2T} - 1)$
 $\Rightarrow E = \infty \Rightarrow$ not an energy signal

$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} e^{2t} dt = \lim_{T \rightarrow \infty} \frac{1}{T} (e^{T/2} - 1) = \infty \Rightarrow$ not a power signal
 $\Rightarrow x(t) = e^{st}$ is not stable

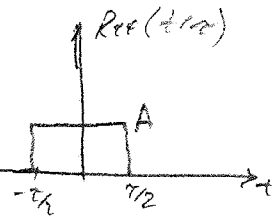
b. $x(t) = e^{-st} \quad t \geq 0 \Rightarrow E = \lim_{T \rightarrow \infty} \int_0^T e^{-2t} dt = \lim_{T \rightarrow \infty} (-e^{-2t} + 1) = 1 \text{ joule} \Rightarrow$ Energy signal

c. $x(t) = \cos t + \cos 2t \Rightarrow P = \frac{1}{T} \int_{-T/2}^{T/2} (\cos t + \cos 2t)^2 dt = 0.5 + 0.5 = 1^W \Rightarrow$ power signal

d. $x(t) = e^{-at} \Rightarrow E = \int_{-\infty}^{\infty} (e^{-at})^2 dt = 2 \int_0^{\infty} e^{-2at} dt = \frac{1}{a} \text{ J} \Rightarrow$ energy signal.

1.2. $x(t) = A \text{rect}(t/\tau)$

$\Rightarrow E = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-\tau/2}^{\tau/2} A^2 dt = A^2 (\tau/2 + \tau/2) = A^2 \tau \text{ joules}$



1.4. a. $\int_{-2}^2 \phi_1(t) \phi_1^*(t) dt = \int_0^2 (1) dt = 2$

$\int_{-2}^2 \phi_1(t) \phi_2^*(t) dt = \int_0^{0.5} 1 dt + \int_{0.5}^1 (-1) dt = 0$

Finally $\int_{-2}^2 \phi_2(t) \phi_2^*(t) dt = \int_{-1}^1 (1) dt = 2 \Rightarrow$ Then ϕ_1 & ϕ_2 are orthogonal

b. $x(t) = \sum_{n=1}^2 X_n \phi_n(t) = X_1 \phi_1(t) + X_2 \phi_2(t)$, where

$X_1 = \frac{1}{2} \int_{-2}^2 (t) \phi_1^*(t) dt = \frac{1}{2} \int_0^1 t dt + \frac{1}{2} \int_1^2 (-t) dt = -1/2$

$X_2 = \frac{1}{2} \int_{-2}^2 t \phi_2^*(t) dt = \frac{1}{2} \left[2 \int_0^{0.5} t dt + 2 \int_{0.5}^1 (-t) dt \right] = -1/4$

1.3. a. From Fourier transform tables

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$$\mathcal{F}\left[2\Delta\left(t-\frac{3}{5}\right)\right] = 5e^{-j3\omega} \text{Sinc}^2\left(\frac{5\omega}{4}\right)$$

b. $x_p(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2n\pi t/T}$ where $X_n = \frac{5}{10} \text{Sinc}^2\left(\frac{n\pi}{4}\right)$

1.5.

$$y(t) = \sum_{n=-\infty}^{\infty} Y_n e^{j2n\pi t/T} \quad \text{where } Y_n = \frac{1}{T} \int_0^T y(t) e^{-j2n\pi t/T} dt = \frac{1}{T} \int_0^T x(t-t_0) e^{-j2n\pi t/T} dt$$

let $u = t - t_0 \Rightarrow du = dt$

$$\Rightarrow Y_n = \frac{1}{T} \int_{t_0}^{T+t_0} x(u) e^{j2n\pi(u-t_0)/T} du = e^{-j2n\pi t_0/T} \cdot \frac{1}{T} \int_{t_0}^{T+t_0} x(u) e^{-j2n\pi u/T} du = X_n e^{-j2n\pi t_0/T}$$

1.6. a. $R_u = \frac{c}{2f_r}$; $f_r = 200 \text{ Hz} \Rightarrow R_u = \frac{3 \times 10^8}{2 \times 200} = 750 \text{ Km}$

$f_r = 750 \text{ Hz} \Rightarrow R_u = \frac{3 \times 10^8}{2 \times 750} = 200 \text{ Km}$

b. $PRI = \frac{1}{f_r} \Rightarrow PRI_1 = \frac{1}{200} = 5.0 \text{ msec}$

$PRI_2 = \frac{1}{750} = 1.33 \text{ msec}$

1.7. a. for $f_r = 200 \text{ Hz}$; $d_c = \frac{c}{f_r}$; $T = 5.0 \text{ msec} \Rightarrow \tau = 0.3 \times 5.0 \times 10^{-3} = 1.5 \text{ msec}$

The average transmitted power is $P_{av} = d_c \cdot P_T = 0.3 \times 5 \times 10^3 = 1.5 \text{ kW}$

In 20 msec we have 4-periods \Rightarrow The average transmitted power in 20 msec is

$$P_{av_{20ms}} = 4 \times 1.5 \text{ kW} = 6 \text{ kW}$$

The single pulse energy is $E_P = \frac{1}{T} \tau = 5 \text{ kW} \times 1.5 \text{ msec} = 7.5 \text{ joules}$

and in 20 msec $\Rightarrow E_{20msec} = 4 \times 7.5 = 30 \text{ joules}$

b. for $f_r = 750 \text{ Hz} \Rightarrow \tau = \frac{3}{750} = 0.4 \text{ msec}$

As in 20 msec $\Rightarrow \frac{20}{1.33} = 15 \text{ periods} \Rightarrow P_{av} = 15 \times 1.5 = 22.5 \text{ kW}$

* $E_{20} = 15 * E_P = 15 \times 5 \text{ kW} \times 0.4 = 30 \text{ joules}$.

$$1.8. \quad DR = \frac{c\tau}{2} \Rightarrow DR = \frac{3 \times 10^8 \times 1 \times 10^{-6}}{2} = 150 \text{ m}$$

1.9.

$$R_u = \frac{c}{2f_r} = \frac{3 \times 10^8}{2 \times 3 \times 10^3} = 50 \text{ km}$$

$$DR = \frac{c}{2B} \Rightarrow B = \frac{c}{2\Delta R} = \frac{3 \times 10^8}{2 \times 30} = 5 \text{ MHz}$$

$$d_t = \frac{\tau}{T} = \tau f_r = \frac{f_r}{B} = \frac{3 \times 10^3}{5 \times 10^6} = 0.006$$

1.10.

$$f_d = \frac{2v_r}{\lambda} \Rightarrow \text{for } v_r = 100 \frac{\text{m}}{\text{s}} \Rightarrow f_{d1} = \frac{2 \times 100}{0.3} = 666.67 \text{ kHz}$$

$$\text{for } v_r = 200 \frac{\text{m}}{\text{s}} \Rightarrow f_{d2} = \frac{2 \times 200}{0.3} = 1.33 \text{ kHz}$$

$$\text{for } v_r = 350 \frac{\text{m}}{\text{s}} \Rightarrow f_{d3} = \frac{2 \times 350}{0.3} = 2.33 \text{ kHz}$$

1.11.

$$\Delta t = \frac{2L}{c} \Rightarrow \text{for } L_1 = 30 \text{ km} \Rightarrow \Delta t_1 = \frac{2 \times 30 \times 10^3}{3 \times 10^8} = 200 \mu\text{sec}$$

$$\text{for } L_2 = 80 \text{ km} \Rightarrow \Delta t_2 = \frac{2 \times 80 \times 10^3}{3 \times 10^8} = 533.33 \mu\text{sec}$$

$$\text{for } L_3 = 150 \text{ km} \Rightarrow \Delta t_3 = \frac{2 \times 150 \times 10^3}{3 \times 10^8} = 1.0 \text{ msec}$$

1.12.

$$\text{for S-Band radar where } f = 3 \text{ GHz} \Rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}$$

$$\Rightarrow \text{for } v_r = 50 \frac{\text{m}}{\text{sec}} \Rightarrow f_{d1} = \frac{2 \times 50}{0.1} = 1 \text{ kHz}$$

$$\text{for } v_r = 200 \frac{\text{m}}{\text{sec}} \Rightarrow f_{d2} = \frac{2 \times 200}{0.1} = 4 \text{ kHz}$$

$$\text{for } v_r = 250 \frac{\text{m}}{\text{sec}} \Rightarrow f_{d3} = \frac{2 \times 250}{0.1} = 5 \text{ kHz}$$

$$1.13. \quad \text{for } f = 10 \text{ GHz} \Rightarrow \lambda = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m} = 30 \text{ mm}$$

$$\Rightarrow \text{for } v_r = 50 \frac{\text{m}}{\text{sec}} \Rightarrow f_{d1} = \frac{2 \times 50}{0.03} = 3.33 \text{ kHz}$$

$$f_{d2} = 200 \frac{m}{sec} \Rightarrow f_{d2} = \frac{2 \times 200}{.03} = 13.33 \text{ kHz}$$

$$f_{d3} = 250 \frac{m}{sec} \Rightarrow f_{d3} = \frac{2 \times 250}{.03} = 22.0 \text{ kHz}$$

1.14.

$$f_{dmax} = \frac{2v_{max}}{\lambda}; \text{ we know that } f_{dmin} \geq 2 \frac{2v_{max}}{\lambda}$$

$$\Rightarrow v_{max} = \frac{f_{dmin} \lambda}{4}; \text{ \& } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^9} = 0.2 \text{ m}$$

$$\Rightarrow v_{max} = 10 \times 10^3 \cdot \frac{.2}{4} = 500 \frac{m}{sec} \Rightarrow f_{dmax} = \frac{2 \times 500}{.2} = 5 \text{ kHz}$$

1.15.

Start with

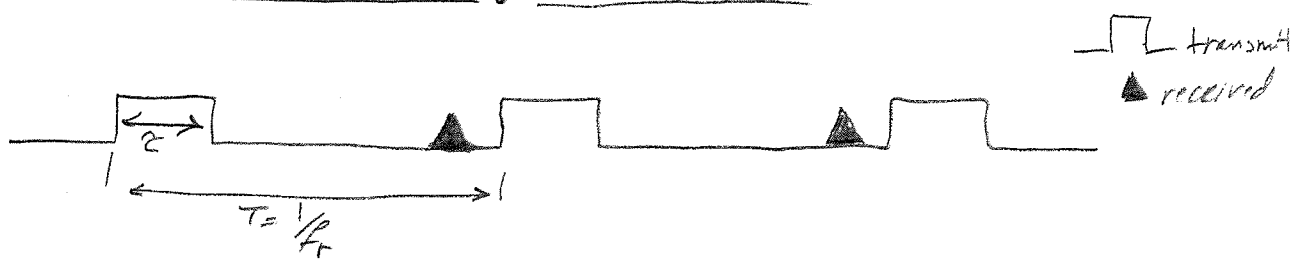
$$R(t) = R_0 + v(t-t_0) \text{ \& follow same steps outlined in text.}$$

1.16.

$$f_d = \frac{2vr}{\lambda} \cos \theta_0 \cos \theta_a \Rightarrow f_d = \frac{2 \times 150}{0.1} \cos 30^\circ \cos 15^\circ = 2.51 \text{ kHz}$$

$$DR = \frac{c\tau}{2} \Rightarrow 0.3 = \frac{3 \times 10^8 \tau}{2} \Rightarrow \tau = 2 \text{ nsec} \Rightarrow B = \frac{1}{\tau} = 500 \text{ MHz}$$

1.18



The minimum PRF must be chosen so that the target range

is unambiguous \Rightarrow

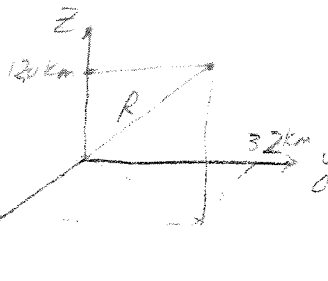
$$f_{dmin} \geq 2 \frac{2v_{max}}{\lambda} \geq 2 f_{dmax} \Rightarrow$$

$$f_{dmin} = 4 \frac{400}{.2} = 8 \text{ kHz}$$

$$\& R_u = \frac{c}{2f_r} = \frac{3 \times 10^8}{2 \times 8 \times 10^3} = 18.75 \text{ kHz}$$

1.19.

$$R = \sqrt{x^2 + y^2 + z^2}$$



$$\Rightarrow a. R = \sqrt{120^2 + 25^2 + 32^2} = \sqrt{179.3} = 42.344 \text{ Km.}$$

$$b. v_x = -250 \text{ m/sec}, v_y = v_z = 0$$

$$\dot{R} = \text{Range rate} = \frac{dR}{dt} = \frac{DR}{Dt} = \frac{Dv \cdot R}{DR} = \frac{Dv_x \cdot R_x + Dv_y \cdot R_y + Dv_z \cdot R_z}{DR}$$

$$= \frac{-250 \cdot 25 + 32 \cdot 0 + 12 \cdot 0}{42.344} \Rightarrow \dot{R} = -147 \text{ m/sec.}$$

$$c. \text{ Round trip delay is } \Delta t = \frac{2 \cdot 42.344 \times 10^3}{3 \times 10^8} = 282.29 \mu\text{sec}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^9} = 33.33 \times 10^{-3} \text{ m}$$

\Rightarrow

$$f_d = \frac{2 \dot{R}}{\lambda} = \frac{2 \cdot -147}{33.33 \times 10^{-3}} = -8.82 \text{ KHz (opening target).}$$

1.20

$$\Delta R = \frac{c \tau}{2} \Rightarrow \tau = \frac{2 \times 10^2}{3 \times 10^8} = 66.67 \mu\text{sec}$$

$$R_u = \frac{c}{2f_r} \Rightarrow f_r = \frac{2 \times 100 \times 10^3}{3 \times 10^8} = 1.5 \text{ KHz}$$

$$P_{\text{av}} = P_e \tau = P_e \tau f_r = 500 \times 66.67 \times 10^{-6} \times 1.5 \times 10^3 = 0.5 \text{ KW} \Leftrightarrow 27 \text{ dB}$$



$$2.1. \quad x(t) = \text{Rect}\left(\frac{t}{T}\right) \cos(\omega_0 t + \beta t^2/2T)$$

$$x_I(t) = \text{Rect}\left(\frac{t}{T}\right) \cos \frac{\beta}{2T} t^2 \quad \& \quad x_Q(t) = \text{Rect}\left(\frac{t}{T}\right) \sin\left(\frac{\beta t^2}{2T}\right)$$

$$2.2. \quad \text{for } \tau = 15 \mu\text{sec} \quad \& \quad \beta = 10 \text{ MHz} \Rightarrow$$

$$x_I(t) = \text{Rect}\left(\frac{t}{15\mu}\right) \cos(333.336 t^2) \quad \& \quad x_Q(t) = \text{Rect}\left(\frac{t}{15\mu}\right) \sin(333.336 t^2)$$

2.3.

$$f_i(t) = \frac{1}{2\pi} \text{Im} \left\{ \frac{d}{dt} \ln \psi(t) \right\}, \quad \psi(t) \text{ is the analytic signal}$$

$$\Rightarrow f_i(t) = \frac{1}{2\pi} \text{Im} \left\{ \frac{\psi'(t)}{|\psi(t)|^2} \right\} \quad \text{where } \psi' \text{ indicates derivative w.r.t } t$$

$$\text{but } \psi(t) = \tilde{x}(t) e^{j2\pi f_0 t} \quad \tilde{x}(t) \text{ is the complex envelope for } x(t)$$

$$\Rightarrow f_i(t) = \frac{1}{2\pi |\psi(t)|^2} \left[\tilde{x}'(t) x(t) - x'(t) \tilde{x}(t) \right] \quad \text{where } \tilde{x} \text{ is the Hilbert transform for } x$$

$$\text{Recall } \tilde{x} = x_I + j x_Q \quad \& \quad x = x_I \cos 2\pi f_0 t - x_Q \sin 2\pi f_0 t$$

$$\text{so when } \psi(t) = \text{Rect}\left(\frac{t}{T}\right) \cos\left(2\pi f_0 t + \frac{\beta}{2T} t^2\right) \quad \text{we get}$$

$$\ln[\psi(t)] = C + j\left(2\pi f_0 t + \frac{\beta}{2T} t^2\right)$$

$$\Rightarrow f_i(t) = \frac{1}{2\pi} \text{Im} \left\{ \frac{d}{dt} \ln \psi(t) \right\} = f_0 + \frac{1}{2\pi} \frac{\beta}{T} t$$

2.4.

$$h(t) = \delta(t) - \frac{\omega_0}{\omega_d} e^{-\zeta t} \sin \omega_d t \quad t \geq 0$$

$$\text{In general } h(t) = h_I(t) \cos \omega_0 t - h_Q(t) \sin \omega_0 t$$

\Rightarrow by inspection

$$h_I(t) = \delta(t) \quad \& \quad h_Q(t) = \frac{\omega_0}{\omega_d} e^{-\zeta t}$$

2.8

7

a) REPLACE $D(y)$ BY THE CONSTANT $d_0 \Rightarrow$

$$E(f) = d_0 \int_{-f/2}^{f/2} \exp\left(2\pi y j \frac{\sin f}{\lambda}\right) dy$$

$$= d_0 \left[\frac{\exp\left(2\pi y j \frac{\sin f}{\lambda}\right)}{j 2\pi \frac{\sin f}{\lambda}} \right] \Big|_{-f/2}^{f/2}$$

\Rightarrow

$$E(f) = \frac{d_0}{j 2\pi \frac{\sin f}{\lambda}} \left[\exp\left(\pi j \frac{\sin f}{\lambda} r\right) - \exp\left(-\pi j \frac{\sin f}{\lambda} r\right) \right]$$

USING THE RELATION $\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ AND $\text{Sa}(\theta) = \frac{\sin \theta}{\theta} \Rightarrow$

$$E(f) = d_0 r \text{Sa}\left(\pi \frac{\sin f}{\lambda} r\right) \Rightarrow |E(f)|^2 = d_0^2 a^2 \text{Sa}^2\left(\pi \frac{\sin f}{\lambda} r\right)$$

$$b) P(f) = \frac{E(f)}{E(0)} = \frac{\text{Sa}^2\left(\pi \frac{\sin f}{\lambda} r\right)}{\text{Sa}^2\left(\pi \frac{\sin 0}{\lambda} r\right)} = \text{Sa}^2(x), \quad x = \pi \frac{\sin f}{\lambda} r$$

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$6\pi/8$	$7\pi/8$	π
$P(x)$	1	0.949	0.8105	0.61499	0.40528	0.2214	0.090	0.019	0
$P(x)_{dB}$	0	-0.2244	-0.9121	-2.111	-3.9223	-6.548	-10.45	-17.12	$-\infty$

2.9 a.

$$P_{\text{rad}} = \iint |F(\theta, \varphi)|^2 d\Omega = \int_0^{\pi} \int_0^{2\pi} |\cos^n \theta| \sin \theta d\varphi d\theta = 2\pi \int_0^{\pi} \cos^n \theta \sin \theta d\theta$$

$$\Rightarrow \text{for } n=1 \quad P_{\text{rad}} = 4\pi \int_0^{\pi/2} \cos \theta \sin \theta d\theta = 4\pi \left(\frac{1}{2} \sin^2 \theta \Big|_0^{\pi/2} \right) = 2\pi$$

$$\text{for } n=2 \quad P_{\text{rad}} = 4\pi \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta = 4\pi \left(-\frac{\cos^3 \theta}{3} \Big|_0^{\pi/2} \right) = \frac{4\pi}{3}$$