Chapter 1 Solutions

Solution 1.1-1

(a) $E = \int_0^2 (1)^2 dt + \int_2^3 (-1)^2 dt = 3$

(b) $E = \int_0^2 (-1)^2 dt + \int_2^3 (1)^2 dt = 3$

(c) $E = \int_0^2 (2)^2 dt + \int_2^3 (-2)^2 dt = 12$

(d) $E = \int_3^5 (1)^2 dt + \int_5^6 (-1)^2 dt = 3$

Comments: Changing the sign of a signal does not change its energy. Doubling a signal quadruples its energy. Shifting a signal does not change its energy. Multiplying a signal by a constant K increases its energy by a factor K^2.

Solution 1.1-2

$$E_x = \int_0^1 t^2 dt = \frac{1}{3}\, t^3\Big|_0^1 = \frac{1}{3}, \qquad E_{x_1} = \int_{-1}^0 (-t)^2 dt = \frac{1}{3}\, t^3\Big|_{-1}^0 = \frac{1}{3},$$

$$E_{x_2} = \int_0^1 (-t)^2 dt = \frac{1}{3}\, t^3\Big|_0^1 = \frac{1}{3}, \qquad E_{x_3} = \int_1^2 (t-1)^2 dt = \int_0^1 x^2 dx = \frac{1}{3},$$

$$E_{x_4} = \int_0^1 (2t)^2 dt = \frac{4}{3}\, t^3\Big|_{-1}^0 = \frac{4}{3}$$

Solution 1.1-3

(a)

$$E_x = \int_0^2 (1)^2 dt = 2, \qquad E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2,$$

$$E_{x+y} = \int_0^1 (2)^2 dt = 4, \qquad E_{x-y} = \int_1^2 (2)^2 dt = 4$$

Therefore $E_{x\pm y} = E_x + E_y$.

(b)

$$E_x = \int_0^{2\pi} \sin^2 t\, dt = \frac{1}{2}\int_0^{2\pi} (1) dt - \frac{1}{2}\int_0^{2\pi} \cos(2t) dt = \pi + 0 = \pi$$

$$E_y = \int_0^{2\pi} (1)^2\, dt = 2\pi$$

$$E_{x+y} = \int_0^{2\pi} (\sin t + 1)^2 \, dt = \int_0^{2\pi} \sin^2(t)dt + 2\int_0^{2\pi} \sin(t)dt + \int_0^{2\pi} (1)^2 dt = \pi + 0 + 2\pi = 3\pi$$

In both cases (a) and (b), $E_{x+y} = E_x + E_y$. Similarly we can show that for both cases $E_{x-y} = E_x + E_y$.

(c) As seen in part (a),

$$E_x = \int_0^{\pi} \sin^2 t \, dt = \pi/2$$

Furthermore,

$$E_y = \int_0^{\pi} (1)^2 \, dt = \pi$$

Thus,

$$E_{x+y} = \int_0^{\pi} (\sin t + 1)^2 \, dt = \int_0^{\pi} \sin^2(t)dt + 2\int_0^{\pi} \sin(t)dt + \int_0^{\pi} (1)^2 dt = \frac{\pi}{2} + 2(2) + \pi = \frac{3\pi}{2} + 4$$

Additionally,

$$E_{x-y} = \int_0^{\pi} (\sin t - 1)^2 \, dt = \pi/2 - 4 + \pi = \frac{3\pi}{2} - 4$$

In this case, $E_{x+y} \neq E_{x-y} \neq E_x + E_y$. Hence, we cannot generalize the conclusions observed in parts (a) and (b).

Solution 1.1-4

$P_x = \frac{1}{4}\int_{-2}^{2} (t^3)^2 dt = 64/7$

(a) $P_{-x} = \frac{1}{4}\int_{-2}^{2} (-t^3)^2 dt = 64/7$

(b) $P_{2x} = \frac{1}{4}\int_{-2}^{2} (2t^3)^2 dt = 4(64/7) = 256/7$

(c) $P_{cx} = \frac{1}{4}\int_{-2}^{2} (ct^3)^2 dt = 64c^2/7$

Comments: Changing the sign of a signal does not affect its power. Multiplying a signal by a constant c increases the power by a factor c^2.

Solution 1.1-5

In the original design, the 3-second duration 10-volt square pulse has energy

$$E_{\text{orig}} = \int_0^{3} (10)^2 \, dt = 300.$$

In the soft-start design, as shown in Fig. S1.1-5, the waveform follows a stair-step shape at the start, where each step increases by 1 volt from the previous and has a duration of 20 ms.

Let us call the initial 9 steps in this waveform $x_1(t)$, with duration $9(20) = 180$ ms, and the final step $x_2(t)$, with duration $T - 0.18$ s. The complete waveform $x(t) = x_1(t) + x_2(t)$ has total duration T seconds and, since $x_1(t)$ does not overlap with $x_2(t)$, energy $E_x = E_{x_1} + E_{x_2}$.

The nine steps before reaching the final 10 volt level have a combined energy of

$$E_{x_1} = \sum_{i=1}^{9} i^2(0.02) = 0.02\left[\frac{9(9+1)(2(9)+1)}{6}\right] = 5.7$$

The final step has energy

$$E_{x_2} = (T - 0.18)(10)^2 = 100T - 18$$

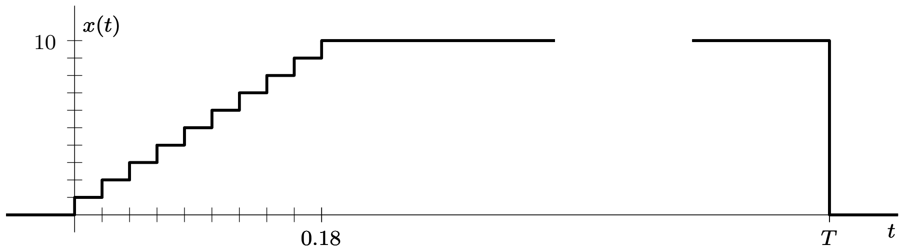

Figure S1.1-5

Combining, we see that

$$E_x = E_{x_1} + E_{x_2} = 5.7 + 100T - 18 = 300 = E_{\text{orig}}.$$

Solving for T, the duration of the soft-start signal is

$$T = \frac{300 - 5.7 + 18}{100} = 3.123 \text{ seconds.}$$

Solution 1.1-6

We can solve much of this problem by referring to Ex. 1.2 in the text.

(a) The power of a sinusoid of amplitude C is $C^2/2$ regardless of its frequency ($\omega \neq 0$) and phase. Therefore, in this case $P = 5^2 + (10)^2/2 = 75$.

(b) Power of a sum of sinusoids is equal to the sum of the powers of the sinusoids. Therefore, in this case $P = \frac{(10)^2}{2} + \frac{(16)^2}{2} = 178$.

(c) $(10 + 2\,\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 7t$. Hence $P = \frac{(10)^2}{2} + \frac{1}{2} + \frac{1}{2} = 51$.

(d) $10\cos 5t\cos 10t = 5(\cos 5t + \cos 15t)$. Hence $P = \frac{(5)^2}{2} + \frac{(5)^2}{2} = 25$.

(e) $10\sin 5t\cos 10t = 5(\sin 15t - \sin 5t)$. Hence $P = \frac{(5)^2}{2} + \frac{(-5)^2}{2} = 25$.

(f) The power of complex signal $e^{j\alpha t}\cos\omega_0 t$ is given as

$$\begin{aligned} P &= \lim_{T\to\infty} \frac{1}{T}\int_{T/2}^{T/2} |e^{j\alpha t}\cos\omega_0 t|^2\,dt \\ &= \lim_{T\to\infty} \frac{1}{T}\int_{T/2}^{T/2} e^{j\alpha t}\cos\omega_0 t e^{-j\alpha t}\cos\omega_0 t\,dt \\ &= \lim_{T\to\infty} \frac{1}{T}\int_{T/2}^{T/2} \cos^2\omega_0 t\,dt \end{aligned}$$

Clearly, the power of $e^{j\alpha t}\cos\omega_0 t$ of is the same as the power of $\cos\omega_0 t$. Thus, $P = 1/2$.

Solution 1.1-7

First, $x(t) = \begin{cases} \frac{2A}{T}t & 0 \le t < \frac{T}{2} \\ 0 & \frac{T}{2} \le t < T \\ x(t+T) & \forall t \end{cases}$. Next, $P_x = \frac{1}{T}\int_0^{T/2}\left(\frac{2A}{T}t\right)^2 dt = \frac{4A^2}{T^2}\int_0^{T/2} t^2 dt = \frac{4A^2}{T^2}\frac{t^3}{3}\Big|_0^{T/2} = \frac{4A^2}{T^2}\frac{T^3}{3(8)} = \frac{A^2}{6}$. Since power is finite, energy must be infinite. Thus,

$$P_x = \frac{A^2}{6} \text{ and } E_x = \infty.$$

Solution 1.1-8

(a) Signal $x(t)$, shown in Fig. S1.1-8, is 1-periodic. Thus, $E_x = \infty$ and

$$P_x = \int_0^1 x^2(t)\,dt = \int_0^1 1\,dt = 1.$$

(b) Signal $y(t)$, shown in Fig. S1.1-8, is 1-periodic. Thus, $E_y = \infty$ and

$$P_y = \int_0^1 y^2(t)\,dt = \int_0^1 1\,dt = 1.$$

(c) Signal $f(t) = x(t) + jy(t)$ is also 1-periodic. Thus, $E_f = \infty$ and

$$P_f = \int_0^1 |f(t)|^2\,dt = \int_0^1 (x(t)+jy(t))(x(t)-jy(t))\,dt = \int_0^1 x^2(t)\,dt + \int_0^1 y^2(t)\,dt = 2.$$

(d) The energy of complex signal $f(t)$ requires integrating $|f(t)|^2$. If $f(t) = x(t) + jy(t)$, where $x(t)$ and $y(t)$ are real signals, then $|f(t)|^2 = (x(t)+jy(t))(x(t)-jy(t)) = x^2(t) + y^2(t)$, and the energy of $f(t)$ is just the sum the individual energies. That is,

$$\begin{aligned} E_f &= \int_{-\infty}^{\infty} |f(t)|^2\,dt = \int_{-\infty}^{\infty} (x(t)+jy(t))(x(t)-jy(t))\,dt \\ &= \int_{-\infty}^{\infty} (x^2(t)+y^2(t))\,dt = \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt \\ &= E_x + E_y \end{aligned}$$

Following a similar proof, the conclusion for the power of $f(t)$ is the same. Thus,

for real $x(t)$, real $y(t)$, and $f(t) = x(t) + jy(t)$, $E_f = E_x + E_y$ and $P_f = P_x + P_y$.

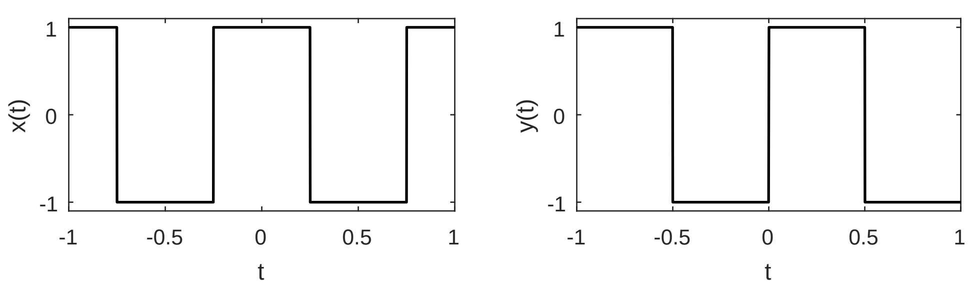

Figure S1.1-8

Solution 1.1-9

(a) By definition, $E[Tx_1(t)] = \int_{t=-\infty}^{\infty} (Tx_1(t))^2\,dt = \int_{t=-\infty}^{\infty} T^2 x_1^2(t)dt = T^2 \int_{t=-\infty}^{\infty} x_1^2(t)dt = T^2 E[x_1(t)]$.

$$E[Tx_1(t)] = T^2 E[x_1(t)].$$

(b) By definition, $E[x_1(t-T)] = \int_{t=-\infty}^{\infty} (x_1(t-T))^2\,dt$. Substituting $t' = t - T$ and $dt' = dt$ yields $\int_{t'=\infty}^{\infty} x_1^2(t')dt' = E[x_1(t)]$.

$$E[x_1(t-T)] = E[x_1(t)].$$

(c) By definition, $E[x_1(t)+x_2(t)] = \int_{t=-\infty}^{\infty}(x_1(t)+x_2(t))^2\,dt = \int_{t=-\infty}^{\infty}\left(x_1^2(t)+2x_1(t)x_2(t)+x_2^2(t)\right)dt$. However, $x_1(t)$ and $x_2(t)$ are non-overlapping so their product $x_1(t)x_2(t)$ must be zero. Thus, $E[x_1(t)+x_2(t)] = \int_{t=-\infty}^{\infty}\left(x_1^2(t)+x_2^2(t)\right)dt = \int_{t=-\infty}^{\infty}x_1^2(t)dt + \int_{t=-\infty}^{\infty}x_2^2(t)dt = E[x_1(t)]+E[x_2(t)]$.

$$\text{If } (x_1(t)\neq 0)\Rightarrow(x_2(t)=0) \text{ and } (x_2(t)\neq 0)\Rightarrow(x_1(t)=0),$$

$$\text{Then, } E[x_1(t)+x_2(t)] = E[x_1(t)]+E[x_2(t)].$$

(d) By definition, $E[x_1(Tt)] = \int_{t=-\infty}^{\infty}x_1^2(Tt)dt$.

First, consider the case $T>0$. Substituting $t'=Tt$ and $dt'=Tdt$ yields $E[x_1(Tt)] = \int_{t'=-\infty}^{\infty}x_1^2(t')\frac{dt'}{T} = \frac{1}{T}\int_{t'=-\infty}^{\infty}x_1^2(t')dt' = \frac{E[x_1(t)]}{T} = \frac{E[x_1(t)]}{|T|}$.

Next, consider the case $T<0$. Substituting $t'=Tt$ and $dt'=Tdt$ yields $E[x_1(Tt)] = \int_{t'=\infty}^{-\infty}x_1^2(t')\frac{dt'}{T} = \frac{-1}{T}\int_{t'=-\infty}^{\infty}x_1^2(t')dt' = \frac{E[x_1(t)]}{-T}$. For $T<0$, we know $T=-|T|$. Making this substitution yields $E[x_1(Tt)] = \frac{E[x_1(t)]}{|T|}$.

Since energy is the same whether $T<0$ or $T>0$, we know

$$E[x_1(Tt)] = \frac{E[x_1(t)]}{|T|}.$$

Solution 1.1-10

To solve this problem, we use the results of Prob. 1.1-9. Also, consider signal $y(t)=t(u(t)-u(t-1))$ which has energy equal to $E[y(t)] = \int_0^1 t^2dt = 1/3$.

To determine $E[x(t)]$, consider dividing $x(t)$ into three non-overlapping pieces: a first piece $x_{\mathrm{a}}(t)$ from $(-2\leq t<-1)$, a second piece $x_{\mathrm{b}}(t)$ from $(-1\leq t<0)$, and a third piece $x_{\mathrm{c}}(t)$ from $(0\leq t<3)$. Since the pieces are non-overlapping, the total energy $E[x(t)] = E[x_{\mathrm{a}}(t)]+E[x_{\mathrm{b}}(t)]+E[x_{\mathrm{c}}(t)]$.

Using the properties of energy, we know that shifting or reflecting a signal does not affect its energy. Notice that $y(t/3)$ is the same as a flipped and shifted version of $x_{\mathrm{c}}(t)$. Thus, $E[x_{\mathrm{c}}(t)] = E[y(t/3)] = 3(1/3) = 1$. Also, it is possible to combine $x_{\mathrm{a}}(t)$ with a flipped and shifted version of $x_{\mathrm{b}}(t)$ to equal a flipped and shifted version of $2y(t/2)$. Thus, $E[x_{\mathrm{a}}(t)+x_{\mathrm{b}}(t)] = E[2y(t/2)] = 4(2)(1/3) = 8/3$.

Thus,

$$E[x(t)] = 11/3.$$

Solution 1.1-11

(a)

$$P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}x(t)x^*(t)\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}\sum_{r=m}^{n}D_kD^*{}_re^{j(\omega_k-\omega_r)t}\,dt$$

The integrals of the cross-product terms (when $k\neq r$) are finite because the integrands are periodic signals (made up of sinusoids). These terms, when divided by $T\to\infty$, yield zero. The remaining terms $(k=r)$ yield

$$P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}|D_k|^2\,dt = \sum_{k=m}^{n}|D_k|^2$$

(b) Prob. 1.1-6(a)

$$\begin{aligned} x(t) &= 5+10\cos(100t+\pi/3) \\ &= 5+5e^{j(100t+\frac{\pi}{3})}+5e^{-j(100t+\frac{\pi}{3})} \\ &= 5+5e^{j\pi/3}e^{j100t}+5e^{-j\pi/3}e^{-j100t} \end{aligned}$$

Hence,

$$P_x = 5^2 + |5e^{j\pi/3}|^2 + |5e^{-j\pi/3}|^2 = 25 + 25 + 25 = 75.$$

Thought of another way, note that $D_0 = 5, D_{\pm 1} = 5$ and thus $P_x = 5^2 + 5^2 + 5^2 = 75$.

Prob. 1.1-6(b)

$$\begin{aligned} x(t) &= 10\cos(100t + \pi/3) + 16\sin(150t + \pi/5) \\ &= 5e^{j\pi/3}e^{j100t} + 5e^{-j\pi/3}e^{-j100t} - j8e^{j\pi/5}e^{j150t} + j8e^{-j\pi/5}e^{-j150t} \end{aligned}$$

Hence,

$$P_x = |5e^{j\pi/3}|^2 + |5e^{-j\pi/3}|^2 + |-j8e^{j\pi/5}|^2 + |j8e^{-j\pi/5}|^2 = 25 + 25 + 64 + 64 = 178.$$

Thought of another way, note that $D_{\pm 1} = 5$ and $D_{\pm 2} = 8$. Hence, $P_x = 5^2+5^2+8^2+8^2 = 178$.

Prob. 1.1-6(c)
To begin, we note that $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. In this case, $D_{\pm 1} = 5$, $D_{\pm 2} = 0.5$ and $D_{\pm 3} = 0.5$. Hence, $P = 5^2+5^2+(0.5)^2+(0.5)^2+(0.5)^2+(0.5)^2 = 51$.

Prob. 1.1-6(d)
To begin, we note that $10\cos 5t \cos 10t = 5(\cos 5t + \cos 15t)$. In this case, $D_{\pm 1} = 2.5$ and $D_{\pm 2} = 2.5$. Hence, $P = (2.5)^2 + (2.5)^2 + (2.5)^2 + (2.5)^2 = 25$.

Prob. 1.1-6(e)
$10\sin 5t \cos 10t = 5(\sin 15t - \sin 5t)$. In this case, $D_{\pm 1} = 2.5$ and $D_{\pm 2} = 2.5$. Hence, $P = (2.5)^2 + (2.5)^2 + (2.5)^2 + (2.5)^2 = 25$.

Prob. 1.1-6(f)
In this case, $e^{j\alpha t}\cos\omega_0 t = \frac{1}{2}\left[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t}\right]$. Thus, $D_{\pm 1} = 0.5$ and $P = (1/2)^2 + (1/2)^2 = 1/2$.

Solution 1.1-12

First, notice that $x(t) = x^2(t)$ and that the area of each pulse is one. Since $x(t)$ has an infinite number of pulses, the corresponding energy must also be infinite. To compute the power, notice that N pulses requires an interval of width $\sum_{i=0}^{N} 2(i+1) = N^2 + 3N$. As $N \to \infty$, power is computed by the ratio of area to width, or $P = \lim_{N\to\infty} \frac{N}{N^2+3N} = 0$. Thus,

$$P = 0 \text{ and } E = \infty.$$

Solution 1.2-1

Figure S1.2-1 shows **(a)** $x(-t)$, **(b)** $x(t+6)$, **(c)** $x(3t)$, and **(d)** $x(t/2)$.

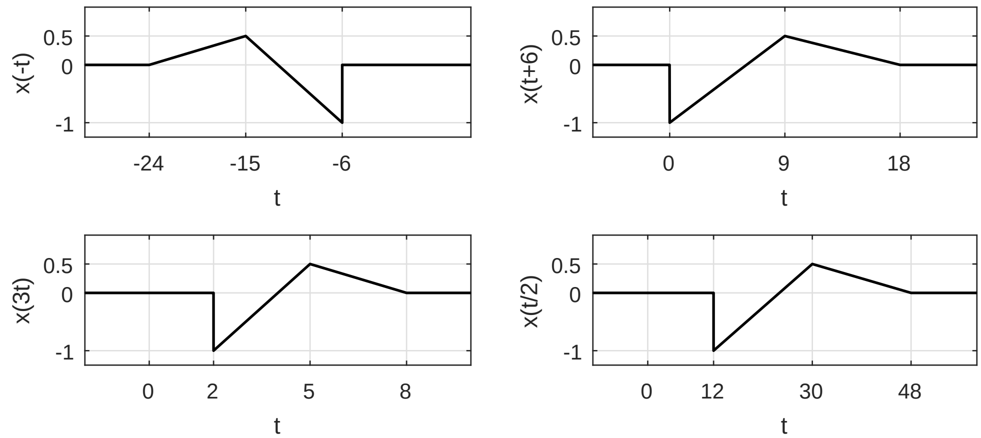

Figure S1.2-1

Solution 1.2-2

Figure S1.2-2 shows **(a)** $x(t-4)$, **(b)** $x(t/1.5)$, **(c)** $x(-t)$, **(d)** $x(2t-4)$, and **(e)** $x(2-t)$.

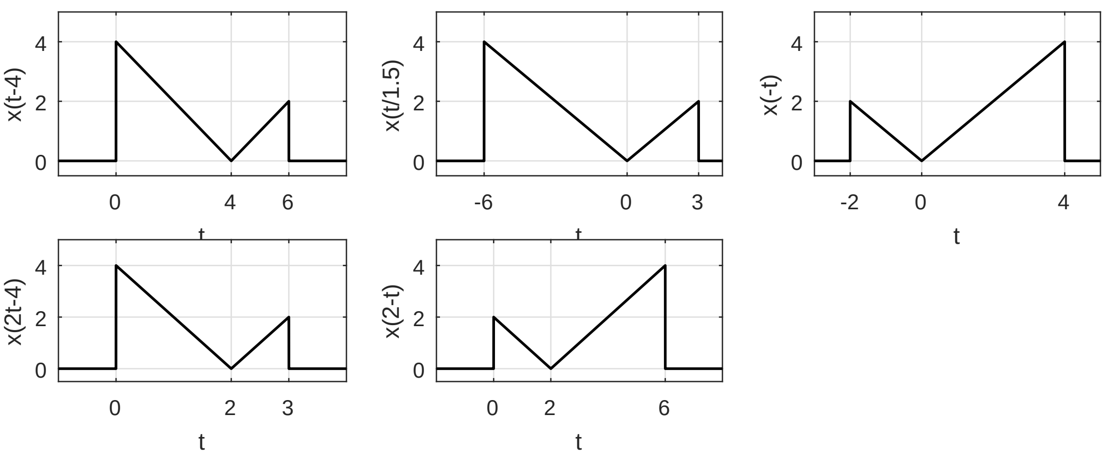

Figure S1.2-2

Solution 1.2-3

(a) $x_1(t)$ can be formed by shifting $x(t)$ to the left by 1 plus a time-inverted version of $x(t)$ shifted to left by 1. Thus,

$$x_1(t) = x(t+1) + x(-t+1) = x(t+1) + x(1-t).$$

(b) $x_2(t)$ can be formed by time-expanding $x(t)$ by factor 2 to obtain $x(t/2)$ Now, left-shift $x(t/2)$ by unity to obtain $x(\frac{t+1}{2})$. We now add to this a time-inverted version of $x(\frac{1-t}{2})$ to obtain $x_2(t)$. Thus,

$$x_2(t) = x(\tfrac{t+1}{2}) + x(\tfrac{1-t}{2}).$$

(c) Observe that $x_3(t)$ is composed of two parts:
First, a rectangular pulse to form the base is constructed by time-expanding $x_2(t)$ by a factor of

2. This is obtained by replacing t with $t/2$ in $x_2(t)$. Thus, we obtain $x_2(t/2) = x(\frac{t+2}{4}) + x(\frac{2-t}{4})$. Second, the two triangles on top of the rectangular base are constructed by time-expanded (factor of 2) and shifted versions of $x(t)$ according to $x(t/2) + x(-t/2)$. Thus,

$$x_3(t) = x(\tfrac{t+2}{4}) + x(\tfrac{2-t}{4}) + x(t/2) + x(-t/2).$$

(d) $x_4(t)$ can be obtained by time-expanding $x_1(t)$ by a factor 2 and then multiplying it by 4/3 to obtain $\frac{4}{3}x_1(t/2) = \frac{4}{3}\left[x(\frac{t+2}{2}) + x(\frac{2-t}{2})\right]$. From this, we subtract a rectangular pedestal of height 1/3 and width 4. This is obtained by time-expanding $x_2(t)$ by 2 and multiplying it by 1/3 to yield $\frac{1}{3}x_2(t/2) = \frac{1}{3}\left[x(\frac{t+2}{4}) + x(\frac{2-t}{4})\right]$. Hence,

$$x_4(t) = \tfrac{4}{3}\left[x(\tfrac{t+2}{2}) + x(\tfrac{2-t}{2})\right] - \tfrac{1}{3}\left[x(\tfrac{t+2}{4}) + x(\tfrac{2-t}{4})\right].$$

(e) $x_5(t)$ is a sum of three components: (i) $x_2(t)$ time-compressed by a factor 2, (ii) $x(t)$ left-shifted by 1.5, and (iii) $x(t)$ time-inverted and then right shifted by 1.5. Hence,

$$x_5(t) = x(t+0.5) + x(0.5-t) + x(t+1.5) + x(1.5-t).$$

Solution 1.2-4

$$E_{-x} = \int_{-\infty}^{\infty} [-x(t)]^2\,dt = \int_{-\infty}^{\infty} x^2(t)\,dt = E_x$$

$$E_{x(-t)} = \int_{-\infty}^{\infty} [x(-t)]^2\,dt = \int_{-\infty}^{\infty} x^2(x)\,dx = E_x$$

$$E_{x(t-T)} = \int_{-\infty}^{\infty} [x(t-T)]^2\,dt = \int_{-\infty}^{\infty} x^2(x)\,dx = E_x,$$

$$E_{x(at)} = \int_{-\infty}^{\infty} [x(at)]^2\,dt = \frac{1}{a}\int_{-\infty}^{\infty} x^2(x)\,dx = E_x/a$$

$$E_{x(at-b)} = \int_{-\infty}^{\infty} [x(at-b)]^2\,dt = \frac{1}{a}\int_{-\infty}^{\infty} x^2(x)\,dx = E_x/a,$$

$$E_{x(t/a)} = \int_{-\infty}^{\infty} [x(t/a)]^2\,dt = a\int_{-\infty}^{\infty} x^2(x)\,dt = aE_x$$

$$E_{ax(t)} = \int_{-\infty}^{\infty} [ax(t)]^2\,dt = a^2\int_{-\infty}^{\infty} x^2(t)\,dt = a^2E_x$$

Comment: Multiplying a signal by constant a increases the signal energy by a factor a^2.

Solution 1.2-5

(a) Calling $y(t) = 2x(-3t+1) = t(u(-t-1) - u(-t+1))$, MATLAB is used to sketch $y(t)$.

```
>>  u = @(t) 1.0*(t>=0); t = -1.5:.001:1.5; y = @(t) t.*(u(-t-1)-u(-t+1));
>>  subplot(121); plot(t,y(t)); axis([-1.5 1.5 -1.1 1.1]);
>>  xlabel('t'); ylabel('2x(-3t+1)'); grid on
```

(b) Since $y(t) = 2x(-3t+1)$, $0.5*y(-t/3+1/3) = 0.5(2)x(-3(-t/3+1/3)+1) = x(t)$. MATLAB is used to sketch $x(t)$.

```
>>  t = -3:.001:5; x = @(t) 0.5*y(-t/3+1/3);
>>  subplot(122); plot(t,x(t)); axis([-3 5 -1.1 1.1]);
>>  xlabel('t'); ylabel('x(t)'); grid on
```

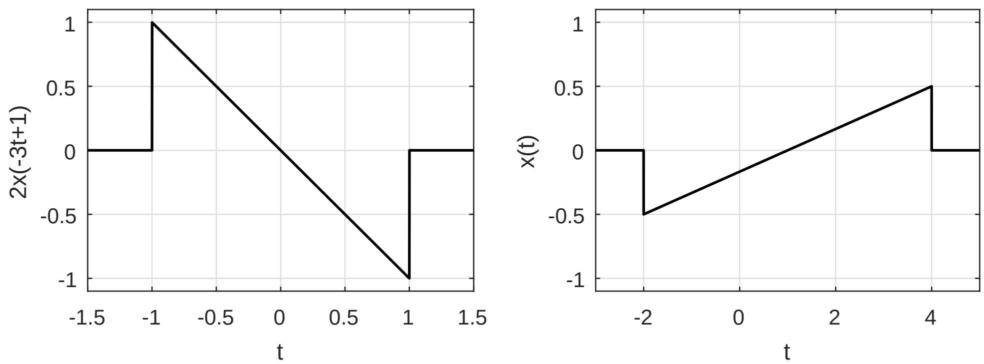

Figure S1.2-5

Solution 1.2-6

MATLAB is used to compute each sketch. Notice that the unit step is in the exponent of the function $x(t)$.

(a)
```
>> u = @(t) 1.0*(t>=0); t = [-1:.001:1]; x = @(t) 2.^(-t.*u(t));
>> subplot(121); plot(t,x(t),'k'); grid on;
>> axis([-1 1 0 1.1]); xlabel('t'); ylabel('x(t)');
```

(b)
```
>> subplot(122); plot(t,0.5*x(1-2*t),'k'); grid on;
>> axis([-1 1 0 1.1]); xlabel('t'); ylabel('y(t)');
```

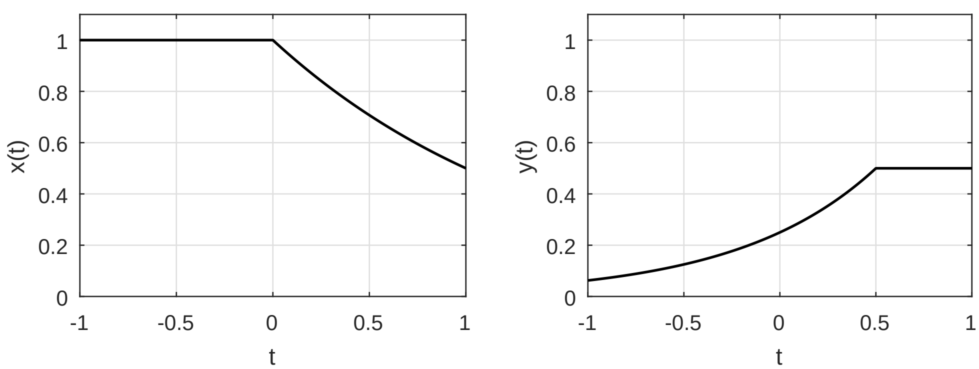

Figure S1.2-6

Solution 1.2-7

(a) Here, we are looking for constants a, b, and c to produce $z(t) = ax(bt+c)$. Signal $z(t)$ is three times taller than $y(t)$, which is $x(t)$ scaled by $-\frac{1}{2}$. Thus, $3(-\frac{1}{2}) = a$ or $a = -\frac{3}{2}$. Next, we pick two corresponding points of $y(t)$ and $z(t)$ to determine b and c.

$$-3t+2|_{t=-1} = bt+c|_{t=8} \quad \Rightarrow \quad 8b+c=5$$

and

$$-3t+2|_{t=0} = bt+c|_{t=2} \quad \Rightarrow \quad 2b+c=2$$

This system of equations is easily solved with MATLAB.

```
>> inv([8 1;2 1])*[5;2]
   ans = 0.5
         1.0
```

Thus,

$$a = -\frac{3}{2}, \quad b = \frac{1}{2}, \quad c = 1.$$

(b) Differentiating the plot of $z(t)$, we obtain a signal $v(t)$ such that $z(t) = \int_{-\infty}^{t} v(\tau)\, d\tau$. The result, shown in Fig. S1.2-7, is expressed mathematically as

$$v(t) = \frac{1}{2}u(t+4) - \frac{1}{2}u(t-2) - 3\delta(t-8).$$

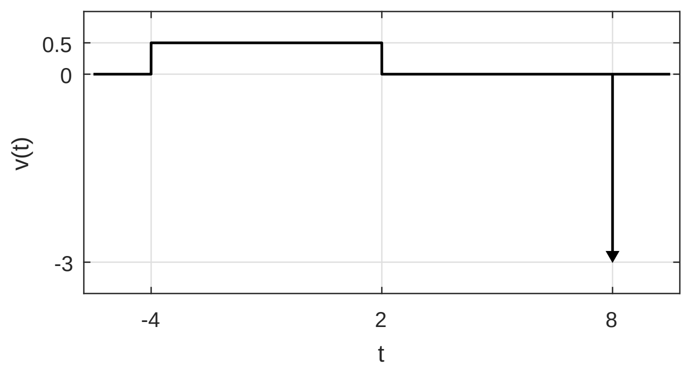

Figure S1.2-7

Solution 1.3-1

There are an infinite number of possible answers to this problem. Let us consider a simple example to demonstrate the overall logic of the problem.

Consider a simple circuit where a series of three AA batteries connect to an LED through a push-button switch. A simple real-world signal is the voltage $v(t)$ measured at the LED when a person presses the button for 1 second. Assuming the button is pressed at time $t = 0$, signal $v(t)$ is reasonably modeled as

$$v(t) = 4.5\left[u(t) - u(t-1)\right].$$

By inspection, we see that

$v(t)$ is (a) continuous-time, (b) analog, (c) aperiodic, (d) energy, (e) causal, and (f) deterministic.

It is not possible to devise a real-world signal that is opposite in all six of the characteristics of $v(t)$. To understand why, we note that any practical real-world signal must be finite in duration (nothing lasts forever in the physical world) and of finite energy (it is impractical to generate the infinite energy needed for a power signal). Since practical signals are finite duration, no real-world signal is truly periodic. Thus, we cannot devise a real-world signal that has the needed (opposite to $v(t)$) characteristics of being periodic and a power signal. It is possible for a real-world signal to have the other four opposite characteristics of discrete, digital, noncausal, and random. For example, recording the average yearly temperature, rounded to the nearest degree Celsius, from 1000 BC to 2000 AD would result in a discrete-time, digital, aperiodic, energy, noncausal, and random signal.

Solution 1.3-2

(a) The leftmost edge of the right-sided signal $x(t)$ occurs when the argument t is equal to 1, $t = 1$. The same edge occurs for signal $x(-2t + a)$ when its argument $-2t + a$ also equals 1, or $a = 1 + 2t$. To be borderline anticausal, the edge of the left-sided signal $x(-2t + a)$ must occur at $t = 0$, which requires $a = 1 + 2t|_{t=0} = 1$. Thus,

$$a = 1 \text{ causes } x(-2t + a) \text{ to be borderline anticausal.}$$

(b) To test for periodicity, we look to see if $y(t+T_y) = y(t)$ for some value T_y. To begin, we notice that

$$y(t + T_y) = \sum_{k=-\infty}^{\infty} x(0.5t + 0.5T_y - 10k).$$

For $T_y = 20l$ and any integer l, we see that

$$y(t + T_y) = \sum_{k=-\infty}^{\infty} x(0.5t + 10l - 10k) = \sum_{k=-\infty}^{\infty} x(0.5t - 10(k - l)).$$

Letting $k' = k - l$, we obtain

$$y(t + T_y) = \sum_{k'=-\infty}^{\infty} x(0.5t - 10k') = y(t).$$

Thus, $y(t)$ is periodic. Setting $l = 1$ yields the fundamental period of $T_y = 20$. In summary,

$$y(t) \text{ is periodic with fundamental period } T_y = 20.$$

Solution 1.3-3

(a) False. Figure 1.11b is an example of a signal that is continuous-time but digital.

(b) False. Figure 1.11c is discrete-time but analog.

(c) False. e^{-t} is neither an energy nor a power signal.

(d) False. $e^{-t}u(t)$ has infinite duration but is an energy signal.

(e) False. $u(t)$ is a power signal that is causal.

(f) True. A periodic signal, which repeats for all t, cannot be 0 for $t > 0$ like an anticausal signal.

Solution 1.3-4

(a) True. Every bounded periodic signal is a power signal.

(b) False. Signals with bounded power are not necessarily periodic. For example, $x(t) = \cos(t)u(t)$ is non-periodic but has a bounded power of $P_x = 0.25$.

(c) True. If an energy signal $x(t)$ has energy E, then the energy of $x(at)$ is $\frac{E}{a}$ (a real and positive).

(d) False. If a power signal $x(t)$ has power P, then the power of $x(at)$ is generally not $\frac{P}{a}$. A counter-example provides a simple proof. Consider the case of $x(t) = u(t)$, which has $P = 0.5$. Letting $a = 2$, $x(at) = x(2t) = u(2t) = u(t)$, which still has power $P = 0.5$ and not $P/a = P/2$.

Solution 1.3-5

(a) For periodicity, $x_1(t) = \cos(t) = \cos(t+T_1) = x_1(t+T_1)$. Since cosine is a 2π-periodic function, $T_1 = 2\pi$. Similarly, $x_2(t) = \sin(\pi t) = \sin(\pi t + \pi T_2) = \sin(\pi(t + T_2)) = x_2(t + T_2)$. Thus, $\pi T_2 = 2\pi k$. The smallest possible value is $T_2 = 2$. Thus,

$$T_1 = 2\pi \text{ and } T_2 = 2.$$

(b) Periodicity requires $x_3(t) = x_3(t+T_3)$ or $\cos(t) + \sin(\pi t) = \cos(t+T_3) + \sin(\pi t + \pi T_3)$. This requires $T_3 = 2\pi k_1$ and $\pi T_3 = 2\pi k_2$ for some integers k_1 and k_2. Combining, periodicity thus requires $T_3 = 2\pi k_1 = 2k_2$ or $\pi = k_1/k_2$. However, π is irrational. Thus, no suitable k_1 and k_2 exist, and $x_3(t)$ cannot be periodic.

(c)

$$P_{x_1} = \frac{1}{2\pi}\int_0^{2\pi} \cos^2(t)dt = \frac{1}{2\pi}\left(0.5(t+\sin(t)\cos(t))|_{t=0}^{2\pi}\right) = \frac{1}{2\pi}\left(\frac{1}{2}\right)2\pi = \frac{1}{2}$$

$$P_{x_2} = \frac{1}{2}\int_0^{2} \sin^2(\pi t)dt = \frac{1}{2}\left(\frac{1}{2\pi}(\pi t - \sin(\pi t)\cos(\pi t))\Big|_{t=0}^{2}\right) = \frac{1}{2}\left(\frac{1}{2\pi}\right)2\pi = \frac{1}{2}$$

$$\begin{aligned}
P_{x_3} &= \lim_{t\to\infty}\frac{1}{2T}\int_{-T}^{T}\left(\cos(t)+\sin(\pi t)\right)^2 dt \\
&= \lim_{t\to\infty}\frac{1}{2T}\int_{-T}^{T}\left(\cos^2(t)+\sin^2(t)+\cos(t)\sin(\pi t)\right) dt \\
&= P_{x_1} + Px_2 + \lim_{t\to\infty}\frac{1}{2T}\int_{-T}^{T} 0.5\left(\sin(\pi t - t)+\sin(\pi t + t)\right) dt \\
&= P_{x_1} + Px_2 + 0 = 1
\end{aligned}$$

Thus,

$$P_{x_1} = \frac{1}{2}, \quad P_{x_2} = \frac{1}{2}, \quad \text{and} \quad P_{x_3} = 1.$$

Solution 1.3-6

No, $f(t) = \sin(\omega t)$ is not guaranteed to be a periodic function for an arbitrary constant ω. Specifically, if ω is purely imaginary then $f(t)$ is in the form of hyperbolic sine, which is not a periodic function. For example, if $\omega = j$ then $f(t) = j\sinh(t)$. Only when ω is constrained to be real will $f(t)$ be periodic.

Solution 1.3-7

(a) $E_{y_1} = \int_{-\infty}^{\infty} y_1^2(t)dt = \int_{-\infty}^{\infty}\frac{1}{9}x^2(2t)dt$. Performing the change of variable $t' = 2t$ yields $\int_{-\infty}^{\infty}\frac{1}{9}x^2(t')\frac{dt'}{2} = \frac{E_x}{18}$. Thus,

$$E_{y_1} = \frac{E_x}{18} \approx \frac{1.0417}{18} = 0.0579.$$

(b) Since $y_2(t)$ is just a $(T_{y_2} = 4)$-periodic replication of $x(t)$, the power is easily obtained as

$$P_{y_2} = \frac{E_x}{T_{y_2}} = \frac{E_x}{4} \approx= 0.2604.$$

(c) Notice, $T_{y_3} = T_{y_2}/2 = 2$. Thus, $P_{y_3} = \frac{1}{2}\int_{T_{y_3}} y_3^2(t)dt = \frac{1}{2}\int_{T_{y_3}}\frac{1}{9}y_2^2(2t)dt$. Performing the change of variable $t' = 2t$ yields $P_{y_3} = \frac{1}{2}\int_{T_{y_2}}\frac{1}{9}y_2^2(t')\frac{dt'}{2} = \frac{1}{36}\int_0^4 x^2(t')dt' = \frac{E_x}{36}$. Thus,

$$P_{y_3} = \frac{E_x}{36} \approx 0.0289.$$

Solution 1.3-8

For all parts, $y_1(t) = y_2(t) = t^2$ over $0 \leq t \leq 1$.

(a) To ensure $y_1(t)$ is even, $y_1(t) = t^2$ over $-1 \leq t \leq 0$. Since $y_1(t)$ is $(T_1 = 2)$-periodic, $y_1(t) = y_1(t+2)$ for all t. Thus, $y_1(t) = \begin{cases} t^2 & -1 \leq t \leq 1 \\ y_1(t+2) & \forall t \end{cases}$. $P_{y_1} = \frac{1}{T_1}\int_{-1}^{1}(t^2)^2 dt = 0.5\frac{t^5}{5}\Big|_{t=-1}^{1} = 1/5$. Thus,

$$P_{y_1} = 1/5.$$

A sketch of $y_1(t)$ over $-3 \leq t \leq 3$ is created using MATLAB and shown in Fig. S1.3-8.

```
>> t = [-3:.001:3]; y_1 = @(mt) (mt<=1).*(mt.^2) + (mt>1).*((mt-2).^2);
>> subplot(121); plot(t,y_1(mod(t,2)),'k'); grid on
>> xlabel('t'); ylabel('y_1(t)'); axis([-3 3 -.1 1.1]);
```

(b) Let

$$y_2(t) = \begin{cases} k & 1 \leq t < 1.5 \\ t^2 & 0 \leq t < 1 \\ -y_2(-t) & \forall t \\ y_2(t+3) & \forall t \end{cases}.$$

With this form, $y_2(t)$ is odd and $(T_2 = 3)$-periodic. The constant k is determined by constraining the power to be unity, $P_{y_2} = 1 = \frac{1}{3}\left(k^2 + \frac{2}{5}t^5\Big|_{t=0}^{1}\right)$. Solving for k yields $k^2 = 3 - 2/5 = 13/5$ or $k = \sqrt{13/5}$. Thus,

$$y_2(t) = \begin{cases} \sqrt{13/5} & 1 \leq t < 1.5 \\ t^2 & 0 \leq t < 1 \\ -y_2(-t) & \forall t \\ y_2(t+3) & \forall t \end{cases}.$$

A sketch of $y_2(t)$ over $-3 \leq t \leq 3$ is created using MATLAB and shown in Fig. S1.3-8.

```
>> y_2 = @(mt) (mt<1).*(mt.^2)-(mt>=2).*((mt-3).^2)+...
>>        ((mt>=1)&(mt<1.5))*sqrt(13/5)-...
>>        ((mt>=1.5)&(mt<2))*sqrt(13/5);
>> subplot(122); plot(t,y_2(mod(t,3)),'k'); grid on
>> xlabel('t'); ylabel('y_2(t)'); axis([-3 3 -2 2]);
```

(c) Define $y_3(t) = y_1(t) + jy_2(t)$. To be periodic, $y_3(t)$ must equal $y_3(t+T_3)$ for some value T_3. This implies that $y_1(t) = y_1(t+T_3)$ and $y_2 = y_2(t+T_3)$. Since $y_1(t)$ is $(T_1 = 2)$-periodic, T_3 must be an integer multiple of T_1. Similarly, since $y_2(t)$ is $(T_2 = 3)$-periodic, T_3 must be an integer multiple of T_2. Thus, periodicity of $y_3(t)$ requires $T_3 = T_1k_1 = 2k_1 = T_2k_2 = 3k_2$, which is satisfied letting $k_1 = 3$ and $k_2 = 2$. Thus,

$$y_3(t) \text{ is periodic with } T_3 = 6.$$

(d) Noting $y_3(t)y_3^*(t) = y_1^2(t) + y_2^2(t)$, $P_{y_3} = \frac{1}{T_3}\int_{T_3}\left(y_1^2(t) + y_2^2(t)\right)dt = P_{y_1} + P_{y_2}$. Thus,

$$P_{y_3} = 1 + \frac{1}{5} = \frac{6}{5}.$$

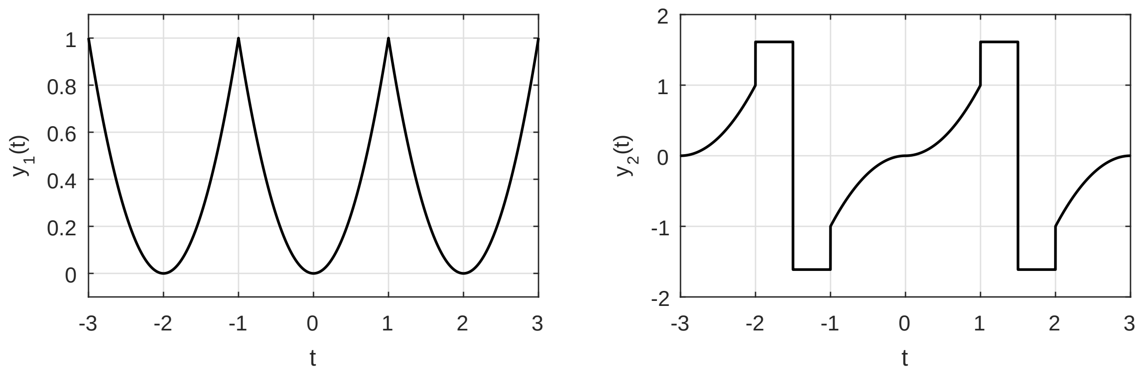

Figure S1.3-8

Solution 1.4-1

Figure S1.4-1 shows **(a)** $u(t-5) - u(t-7)$, **(b)** $u(t-5) + u(t-7)$, **(c)** $t^2[u(t-1) - u(t-2)]$, and **(d)** $(t-4)[u(t-2) - u(t-4)]$.

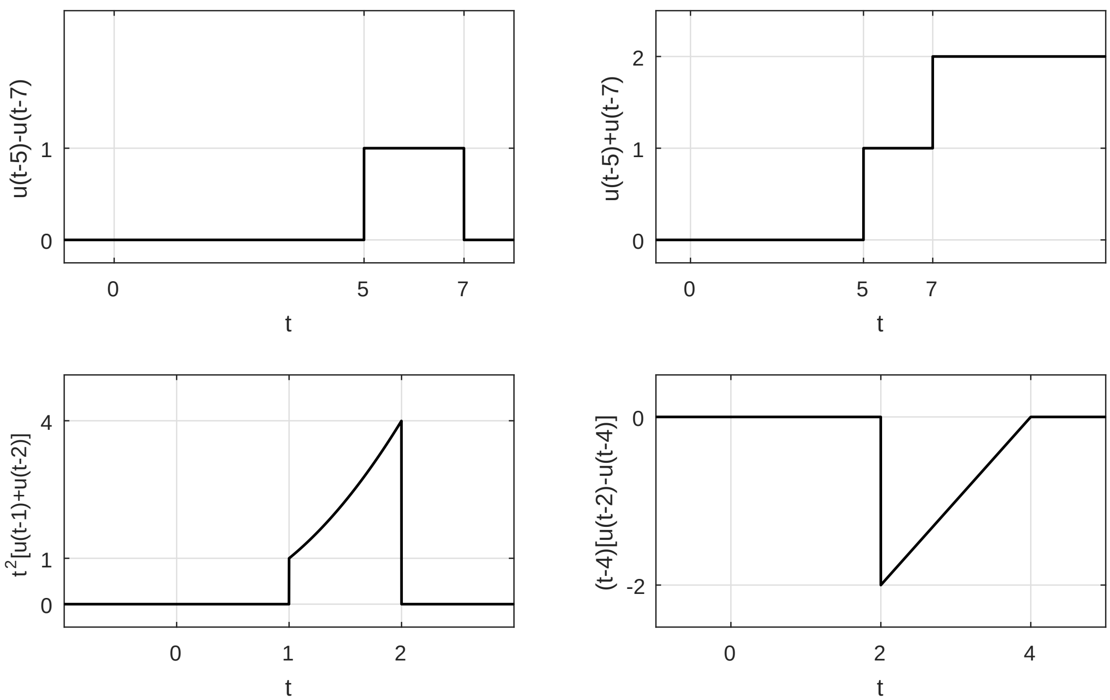

Figure S1.4-1

Solution 1.4-2

(a)

$$\begin{aligned} x_1(t) &= (4t+1)[u(t+1) - u(t)] + (-2t+4)[u(t) - u(t-2)] \\ &= (4t+1)u(t+1) - 6tu(t) + 3u(t) + (2t-4)u(t-2) \end{aligned}$$

(b)

$$\begin{aligned} x_2(t) &= t^2[u(t) - u(t-2)] + (2t-8)[u(t-2) - u(t-4)] \\ &= t^2u(t) - (t^2 - 2t + 8)u(t-2) - (2t-8)u(t-4) \end{aligned}$$

Solution 1.4-3

(a) Signal $w(t)$ is a unit-duration ramp. Signal $x(t)$ is a periodic replication of a compressed-by-2 version of $w(t)$ interlaced with a periodic replication of a compressed-by-2, negated, and shifted version of $w(t)$. Both $w(t)$ and $x(t)$ are shown in Fig. S1.4-3. By inspection, it is clear that the fundamental period of $x(t)$ is $T_0 = 1$.

(b) To sketch $y(t) = \frac{d}{dt}x(1-0.5t)$, we first plot $x(1-0.5t)$ and then graphically differentiate the waveform to obtain $y(t)$. Both $x(1-0.5t)$ and $y(t)$ are shown in Fig. S1.4-3.

(c) To assist in finding the energy E_z and power P_z of the signal $z(t) = x(0.5 - 1.5t)\,[u(t) - u(t-1)]$, we first sketch $z(t)$. As shown in Fig. S1.4-3, $z(t)$ is a finite-duration signal, which means it is an energy signal and $P_z = 0$. Further, $z(t)$ is comprised of three triangular pieces. The energy of the first triangular piece is

$$E_{\text{tri}} = \int_0^{\frac{1}{3}} (1-3t)^2\,dt = \left.\frac{(1-3t)^3}{-9}\right|_{t=0}^{\frac{1}{3}} = \frac{1}{9}.$$

Since shifting and reflecting a signal do not impact energy, the energy of the third triangle equals the energy of the first. The second triangle, which is like the first scaled by $-\frac{1}{2}$, has energy $(-\frac{1}{2})^2 = \frac{1}{4}$ as large as the first triangle. Since the triangles are non-overlapping, E_z is just the sum of the energies of the three pieces. That is, $E_z = \frac{1}{9} + \frac{1}{36} + \frac{1}{9} = \frac{1}{4}$. Thus,

signal $z(t)$ has energy $E_z = \frac{1}{4}$ and power $P_z = 0$.

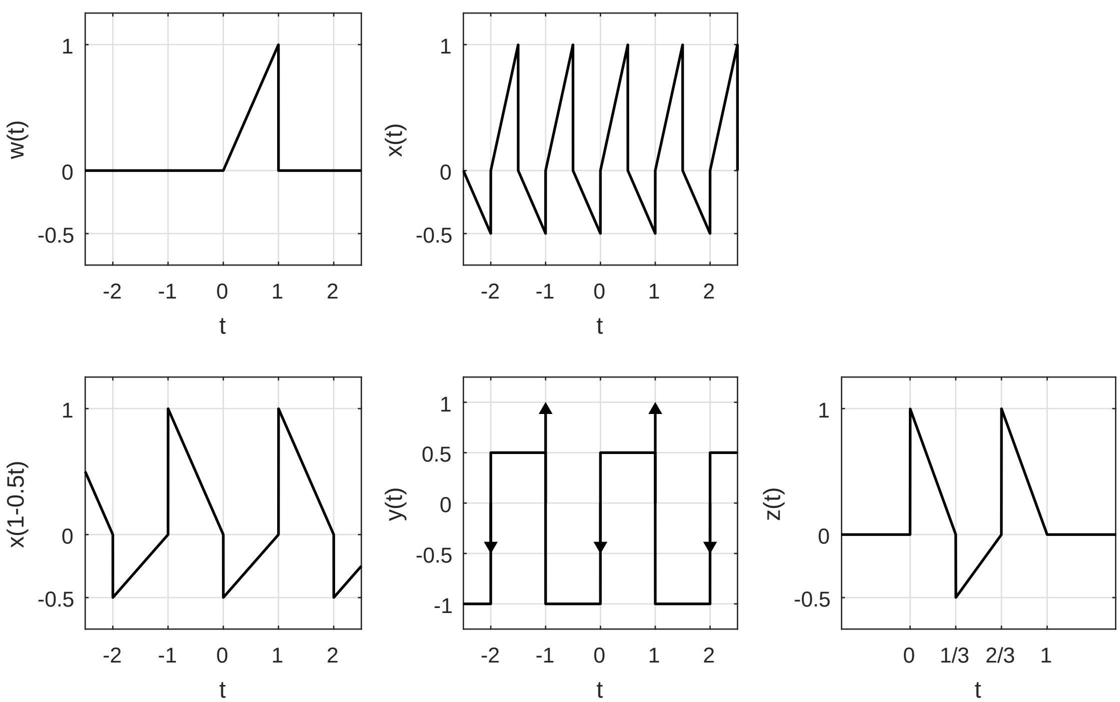

Figure S1.4-3

Solution 1.4-4

(a) To sketch $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$, we first sketch $x(t)$ (see Fig. S1.4-4). Now, $y(t)$ can be obtained graphically as an accumulation of area of $x(t)$ as we move from left to right. Alternatively, we can analytically compute and then plot $y(t)$.

$$y(t) = \begin{cases} 0 & t < 1 \\ \int_1^t d\tau = t - 1 & 1 \le t < 2.5 \\ 1.5 & 2.5 \le t < 4 \\ 1.5 - \int_{-\infty}^{4} 2\delta(\tau - 4)\, d\tau = 1.5 - 2 = -0.5 & 4 \le t < 6 \\ -0.5 + \int_{-\infty}^{6} 2\delta(\tau - 6)\, d\tau = -0.5 + 1 = 0.5 & t \ge 6 \end{cases}$$

(b) If we adjust the weight of the right-most delta function from 1 to $\frac{1}{2}$, then $y(t)$ will be end at $t = 6$ and have finite duration and finite energy. That is,

if $\delta(t-6)$ in $x(t)$ is changed to $\frac{1}{2}\delta(t-6)$, then $y(t)$ will have finite energy.

(c) Signal $z(t) = \int_{t}^{\infty} x(\tau)\, d\tau$, which is the accumulation of area of $x(t)$ as we move from right to left, can be sketched by inspection of $x(t)$ (see Fig. S1.4-4). Alternatively, we can analytically compute and then plot $z(t)$.

$$z(t) = \begin{cases} 0 & t > 6 \\ \int_6^{\infty} \delta(\tau - 6) d\tau = 1 & 4 < t \le 6 \\ 1 - \int_4^{-\infty} 2\delta(\tau - 4)\, d\tau = 1 - 2 = -1 & 2.5 < t \le 4 \\ -1 + \int_t^{2.5} d\tau = -1 + (2.5 - t) = 1.5 - t & 1 < t \le 2.5 \\ 0.5 & t \le 1 \end{cases}$$

(d) There are two ways to determine real constants A and B so that $w(t) = x\left(\frac{t-A}{B}\right)$ has a region of support $[-2, 2]$. In the first way, we map the leftmost edge of $x(t)$ to the leftmost edge of $w(t)$, and map the rightmost edge of $x(t)$ to the rightmost edge of $w(t)$. This requires that

$$z(-2) = x(\tfrac{-2-A}{B}) = x(1) \quad \Rightarrow \quad A + B = -2$$

and

$$z(2) = x(\tfrac{2-A}{B}) = x(6) \quad \Rightarrow \quad A + 6B = 2.$$

Solving this pair of equations yields

$$A = -\frac{14}{5} \quad \text{and} \quad B = \frac{4}{5}.$$

The resulting signal $w(t)$ is shown in Fig. S1.4-4.

The second way to solve this problem is to map the leftmost edge of $x(t)$ to the rightmost edge of $w(t)$, and map the rightmost edge of $x(t)$ to the leftmost edge of $w(t)$. This requires that

$$z(2) = x(\tfrac{2-A}{B}) = x(1) \quad \Rightarrow \quad A + B = 2$$

and

$$z(-2) = x(\tfrac{-2-A}{B}) = x(6) \quad \Rightarrow \quad A + 6B = -2.$$

Solving this pair of equations yields

$$A = \frac{14}{5} \quad \text{and} \quad B = -\frac{4}{5}.$$

The resulting $w(t)$ is just a reflection of the first solution.

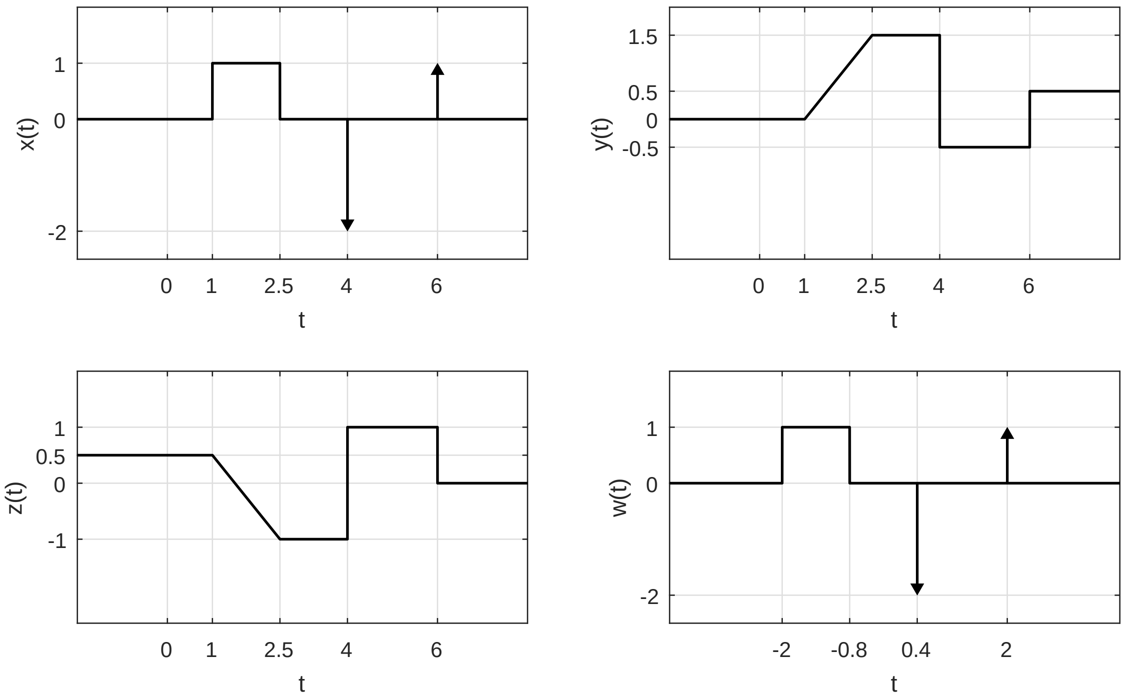

Figure S1.4-4

Solution 1.4-5

Using the fact that $f(x)\delta(x) = f(0)\delta(x)$, we have

(a) 0

(b) $\frac{2}{9}\delta(\omega)$

(c) $\frac{1}{2}\delta(t)$

(d) $-\frac{1}{5}\delta(t-1)$

(e) $\frac{1}{2-j3}\delta(\omega+3)$

(f) $k\delta(\omega)$ (use L' Hôpital's rule)

Solution 1.4-6

In these problems remember that impulse $\delta(x)$ is located at $x = 0$. Thus, an impulse $\delta(t-\tau)$ is located at $\tau = t$, and so on.

(a) The impulse is located at $\tau = t$ and $x(\tau)$ at $\tau = t$ is $x(t)$. Therefore

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau = x(t).$$

(b) The impulse $\delta(\tau)$ is at $\tau = 0$ and $x(t-\tau)$ at $\tau = 0$ is $x(t)$. Therefore

$$\int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)\,d\tau = x(t).$$

Using similar arguments, we obtain

(c) 1

(d) 0

(e) e^3

(f) 5

(g) $x(-1)$

(h) $-e^2$

Solution 1.4-7

In this problem we assume that the constant a is real and positive constant a. Letting $t' = at$ and $dt' = adt$, we see that

$$\int_{-\infty}^{\infty} \delta(at)\,dt = \int_{-\infty}^{\infty} \delta(t')\frac{dt'}{a} = \frac{1}{a}$$

In this way, we see that time-scaling a delta function by (positive) factor a reciprocally changes the strength of the delta function as $\frac{1}{a}$.

Solution 1.4-8

(a) Recall that the derivative of a function at the jump discontinuity is equal to an impulse of strength equal to the amount of discontinuity. Hence, dx/dt contains impulses $4\delta(t+4)$ and $2\delta(t-2)$. In addition, the derivative is -1 over the interval $(-4,\ 0)$, and is 1 over the interval $(0,\ 2)$. The derivative is zero for $t < -4$ and $t > 2$. The result dx/dt is shown in Fig. S1.4-8.

(b) Graphically differentiating Fig. P1.4-2a we obtain

$$\frac{dx_1(t)}{dt} = 4[u(t+1) - u(t)] - 2[u(t) - u(t-2)] = 4u(t+1) - 6u(t) + 2u(t-2).$$

Differentiating again, we obtain,

$$\frac{d^2x_1(t)}{dt^2} = 4\delta(t+1) - 6\delta(t) + 2\delta(t-2).$$

This result is also shown in Fig. S1.4-8.

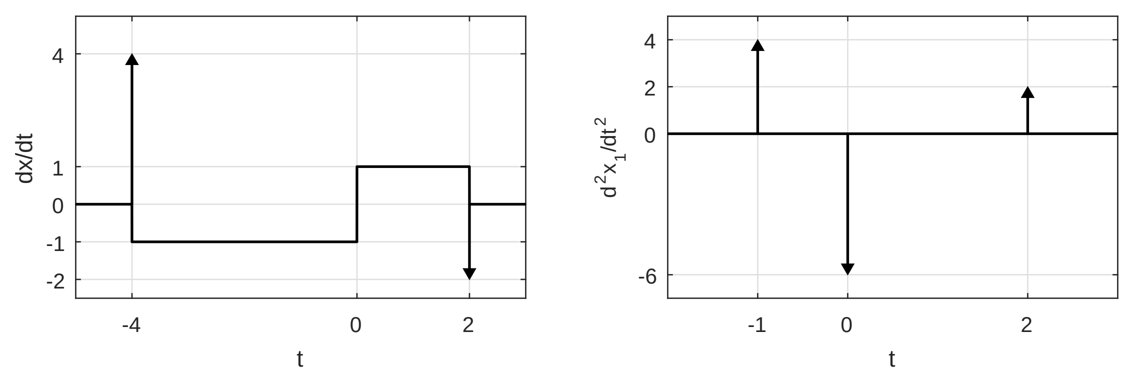

Figure S1.4-8

Solution 1.4-9

For convenience, define $y(t) = \int_{-\infty}^{t} x(t)\,dt$. For sketches, refer to Fig. S1.4-9.

(a) Recall that the area under an impulse of strength k is k. Over the interval $0 \le t < 1$, we have

$$y_{\text{a}}(t) = \int_0^t 1\,dx = t \qquad 0 \le t < 1.$$

Over the interval $0 \le t < 3$, we have

$$y_{\text{a}}(t) = \int_0^1 1\,dx + \int_1^t (-1)\,dx = 2 - t \qquad 1 \le t < 3.$$

At $t = 3$, the impulse (of strength unity) yields an additional term of unity. Thus (assuming $\epsilon \to 0$),

$$y_{\text{a}}(t) = \int_0^1 1\,dx + \int_1^{3-\epsilon} (-1)\,dx + \int_{3-\epsilon}^{t} \delta(x-3)\,dx = 1 + (-2) + 1 = 0 \qquad t > 3$$

Putting the pieces together, we have

$$y_{\text{a}}(t) = \begin{cases} t & 0 \le t < 1 \\ 2 - t & 1 \le t < 3 \\ 0 & t \ge 3 \end{cases}$$

(b) By inspection,

$$y_{\text{b}}(t) = \int_0^t [1 - \delta(x-1) - \delta(x-2) - \delta(x-3) - \cdots]\,dx = tu(t) - u(t-1) - u(t-2) - u(t-3) - \ldots$$

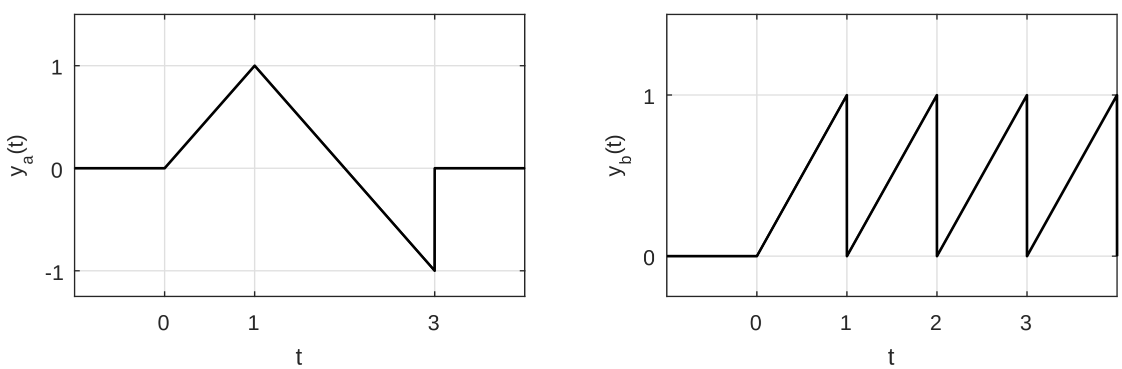

Figure S1.4-9

Solution 1.4-10

Changing the variable t to $-x$, we obtain

$$\int_{-\infty}^{\infty} \phi(t)\delta(-t)\,dt = -\int_{\infty}^{-\infty} \phi(-x)\delta(x)\,dx = \int_{-\infty}^{\infty} \phi(-x)\delta(x)\,dx = \phi(0).$$

This shows that

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt = \int_{-\infty}^{\infty} \phi(t)\delta(-t)\,dt = \phi(0).$$

Therefore

$$\delta(t) = \delta(-t).$$

Solution 1.4-11

Letting $at = x$, we obtain (for $a > 0$)

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{a}\int_{-\infty}^{\infty} \phi(\frac{x}{a})\delta(x)\,dx = \frac{1}{a}\phi(0)$$

Similarly for $a < 0$, we show that this integral is $-\frac{1}{a}\phi(0)$. Therefore

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{|a|}\phi(0) = \frac{1}{|a|}\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt$$

Therefore

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Solution 1.4-12

$$\begin{aligned}\int_{-\infty}^{\infty} \dot{\delta}(t)\phi(t)\,dt &= \phi(t)\delta(t)|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \dot{\phi}(t)\delta(t)\,dt \\ &= 0 - \int \dot{\phi}(t)\delta(t)\,dt = -\dot{\phi}(0)\end{aligned}$$

Solution 1.4-13

For sketches, refer to Fig. S1.4-13.

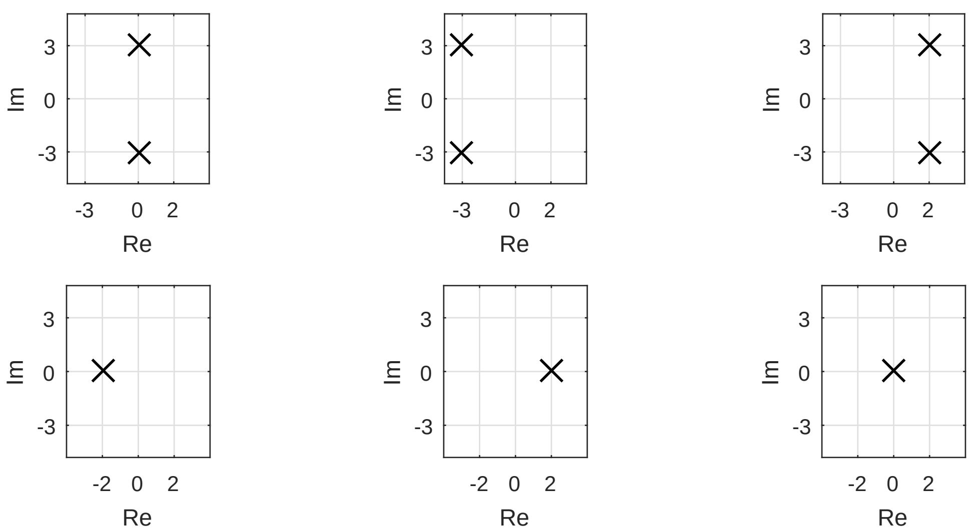

Figure S1.4-13

(a) $s_{1,2} = \pm j3$

(b) $e^{-3t}\cos 3t = 0.5[e^{-(3+j3)t} + e^{-(3-j3)t}]$. Therefore the frequencies are $s_{1,2} = -3 \pm j3$.

(c) Using the argument in (b), we find the frequencies $s_{1,2} = 2 \pm j3$

(d) $s = -2$