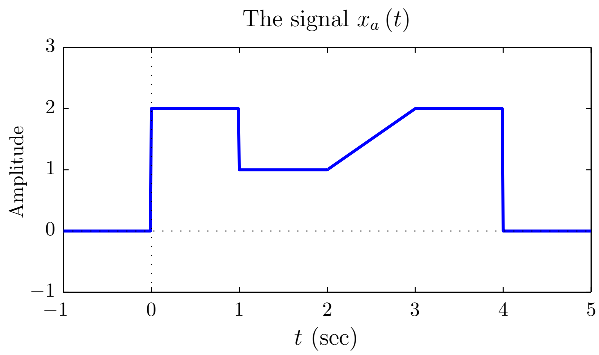
Chapter 1

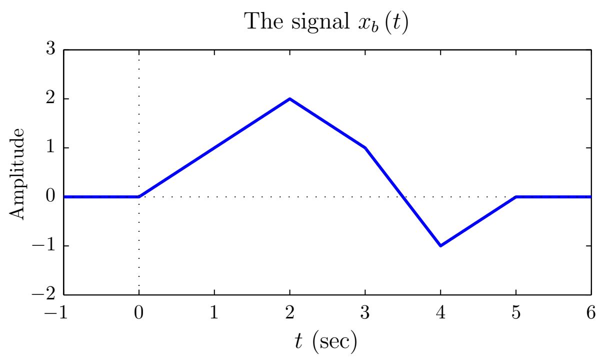
Signal Representation and Modeling

1.1.

a.



b.



1.2.

a.

$$x_a(t) = \begin{cases} 0\ , & t<-1 \text{ or } t>3 \\ 2t+2\ , & -1<t<0 \\ -t+2\ , & 0<t<1 \\ 1\ , & 1<t<2 \\ -t+3\ , & 2<t<3 \end{cases}$$

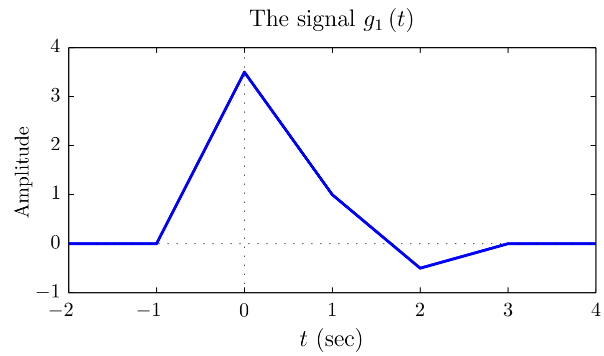
b.

$$x_a(t) = \begin{cases} 0\ , & t<-1 \text{ or } t>3 \\ 1.5t+1.5\ , & -1<t<0 \\ -1.5t+1.5\ , & 0<t<2 \\ 1.5t-4.5\ , & 2<t<3 \end{cases}$$

1.3.

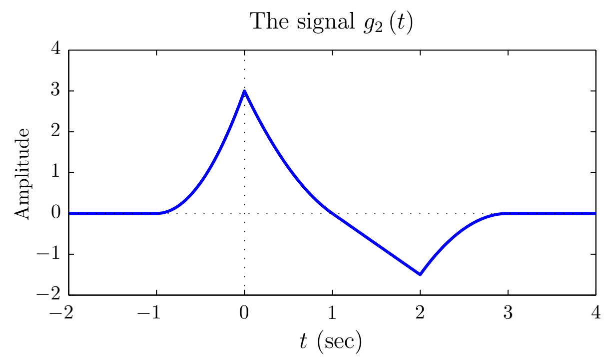
a.

$$g_1(t) = \begin{cases} 0\ , & t<-1 \text{ or } t>3 \\ 3.5t+3.5\ , & -1<t<0 \\ -2.5t+3.5\ , & 0<t<1 \\ -1.5t+2.5\ , & 1<t<2 \\ 0.5t-1.5\ , & 2<t<3 \end{cases}$$



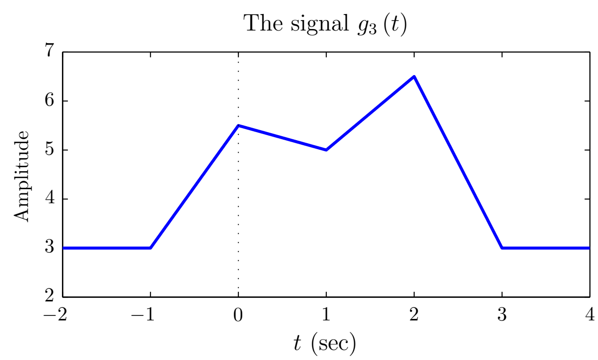
b.

$$g_2(t) = \begin{cases} 0\ , & t<-1 \text{ or } t>3 \\ 3t^2+6t+3\ , & -1<t<0 \\ 1.5t^2-4.5t+3\ , & 0<t<1 \\ -1.5t+1.5\ , & 1<t<2 \\ -1.5t^2+9t-13.5\ , & 2<t<3 \end{cases}$$



c.

$$g_3(t) = \begin{cases} 3\ , & t<-1 \text{ or } t>3 \\ 2.5t+5.5\ , & -1<t<0 \\ -0.5t+5.5\ , & 0<t<1 \\ 1.5t+3.5\ , & 1<t<2 \\ -3.5t+13.5\ , & 2<t<3 \end{cases}$$

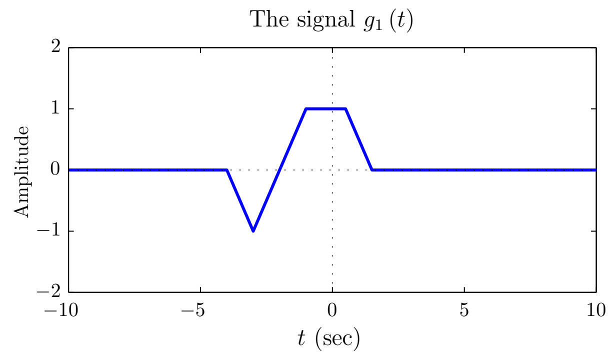


1.4.

a.

Time reversal

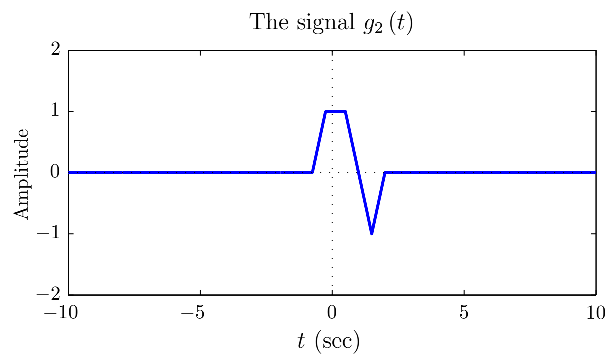
$$g_1(t) = x(-t)$$



b.

Time scaling

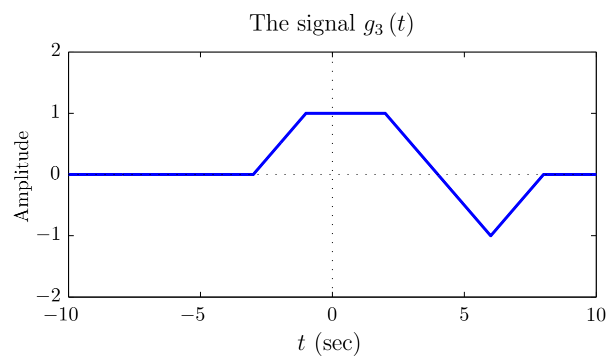
$$g_2(t) = x(2t)$$



c.

Time scaling

$$g_3(t) = x\left(\frac{t}{2}\right)$$



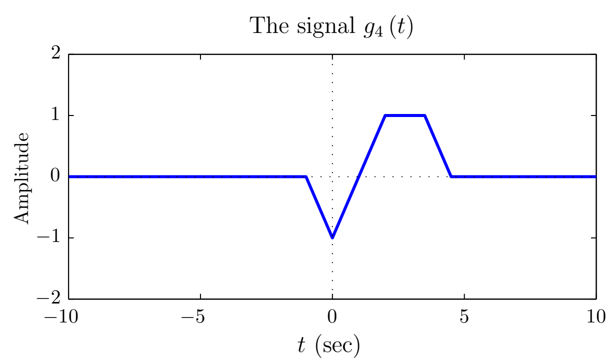
d.

Step 1: Time reversal

$$g_{4a}(t) = x(-t)$$

Step 2: Time shifting

$$g_4(t) = g_{4a}(t-3) = x(-t+3)$$



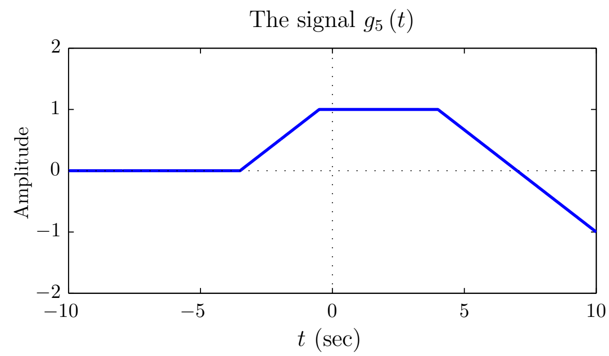
e.

Step 1: Time scaling

$$g_{5a}(t) = x\left(\frac{t}{3}\right)$$

Step 2: Time shifting

$$g_5(t) = g_{5a}(t-1) = x\left(\frac{(t-1)}{3}\right)$$



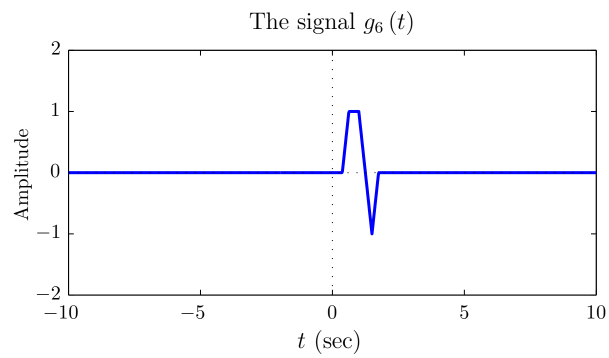
f.

Step 1: Time scaling

$$g_{6a}(t) = x(4t)$$

Step 2: Time shifting

$$g_6(t) = g_{6a}(t-3/4) = x(4t-3)$$



g.

Step 1: Time scaling

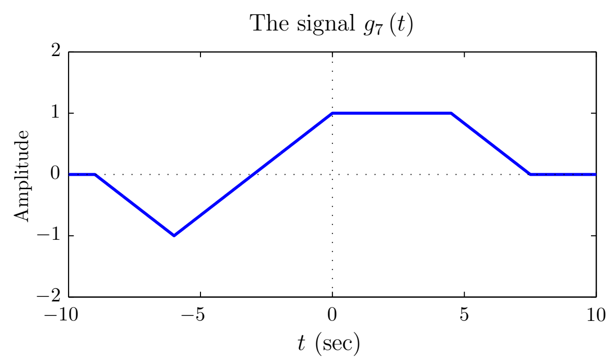
$$g_{7a}(t) = x\left(\frac{t}{3}\right)$$

Step 2: Time reversal

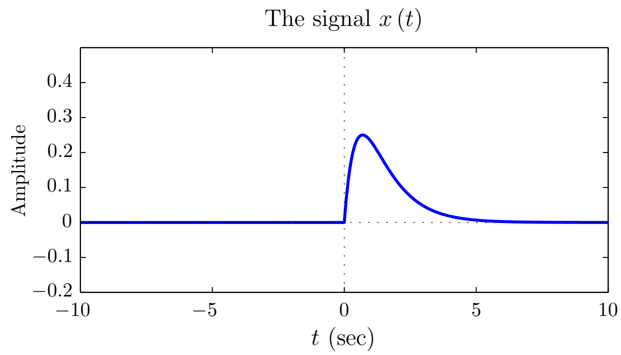
$$g_{7b}(t) = g_{7a}(-t) = x\left(= \frac{t}{3}\right)$$

Step 3: Time shifting

$$g_y(t) = g_{7b}(t-3) = x\left(1 - \frac{t}{3}\right)$$



1.5.



a.

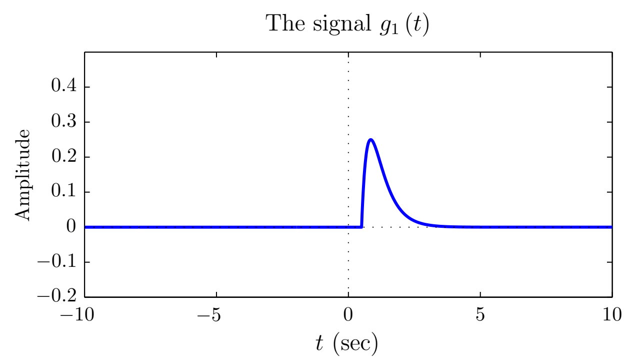
$$g_1(t) = x(2t-1)$$

Step 1: Time scaling

$$g_{1a}(t) = x(2t)$$

Step 2: Time shifting

$$g_1(t) = g_{1a}(t-0.5) = x(2t-1)$$



b.

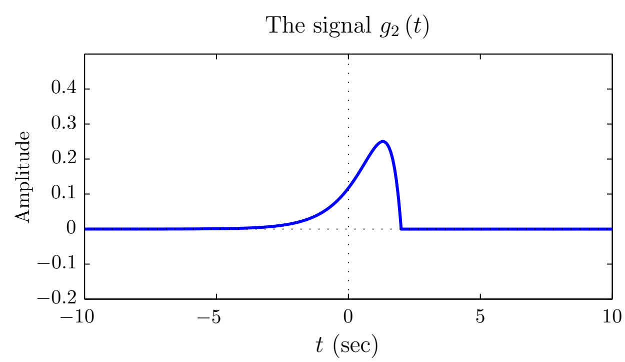
$$g_2(t) = x(-t+2)$$

Step 1: Time reversal

$$g_{2a}(t) = x(-t)$$

Step 2: Time shifting

$$g_2(t) = g_{2a}(t-2) = x(-t+2)$$



c.

$$g_3(t) = x(-3t+5)$$

Step 1: Time scaling

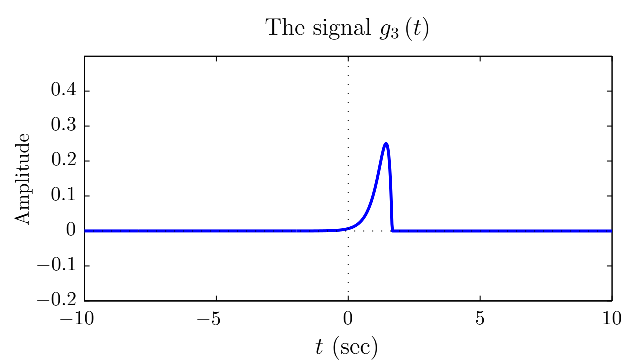
$$g_{3a}(t) = x(3t)$$

Step 2: Time reversal

$$g_{3b}(t) = g_{3a}(-t) = x(-3t)$$

Step 3: Time shifting

$$g_3(t) = g_{3b}(t-5/3) = x(-3t+5)$$



d.

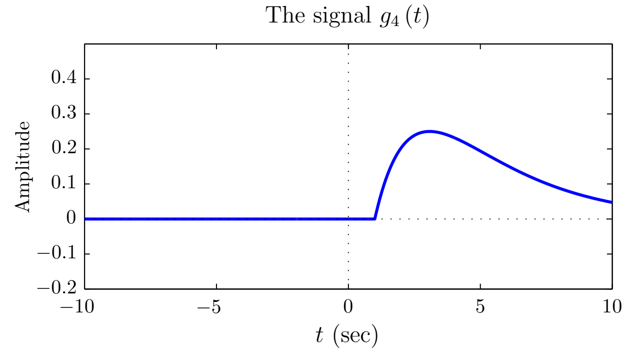
$$g_4(t) = x\left(\frac{t-1}{3}\right)$$

Step 1: Time scaling

$$g_{4a}(t) = x\left(\frac{t}{3}\right)$$

Step 2: Time shifting

$$g_4(t) = g_{4a}(t-1) = x\left(\frac{t-1}{3}\right)$$



1.6.

Let $q(t)$ be a rectangular pulse with height $1/a$ and width a.

$$q(t) = \frac{1}{a}\,\Pi\left(\frac{t}{a}\right)$$

and the unit-impulse function can be obtained through

$$\delta(t) = \lim_{a\to\infty}\left[q(t)\right] = \lim_{a\to\infty}\left[\frac{1}{a}\,\Pi\left(\frac{t}{a}\right)\right]$$

It follows that

$$\delta(bt) = \lim_{a\to\infty}\left[q(bt)\right] = \lim_{a\to\infty}\left[\frac{1}{a}\,\Pi\left(\frac{bt}{a}\right)\right]$$

Let $\tilde{a} = a/b$ so that

$$\frac{1}{a}\,\Pi\left(\frac{bt}{a}\right) = \frac{1}{\tilde{a}\,b}\,\Pi\left(\frac{t}{\tilde{a}}\right)$$

Therefore

$$\begin{aligned}\delta(bt) &= \lim_{a\to\infty}\left[\frac{1}{\tilde{a}\,b}\,\Pi\left(\frac{t}{\tilde{a}}\right)\right]\\ &= \frac{1}{b}\lim_{a\to\infty}\left[\frac{1}{\tilde{a}}\,\Pi\left(\frac{t}{\tilde{a}}\right)\right]\\ &= \frac{1}{b}\,\delta(t)\end{aligned}$$

1.7.

Given that

$$q(t) = \frac{1}{a}\,\Pi\left(\frac{t}{a}\right) = \begin{cases} \dfrac{1}{a}\,, & -\dfrac{a}{2} < t < \dfrac{a}{2} \\ 0\,, & \text{otherwise} \end{cases}$$

the time shifted pulse $q(t-t_1)$ is

$$q(t-t_1) = \frac{1}{a}\,\Pi\left(\frac{t-t_1}{a}\right) = \begin{cases} \dfrac{1}{a}\,, & t_1-\dfrac{a}{2} < t < t_1+\dfrac{a}{2} \\ 0\,, & \text{otherwise} \end{cases}$$

and the integral can be written as

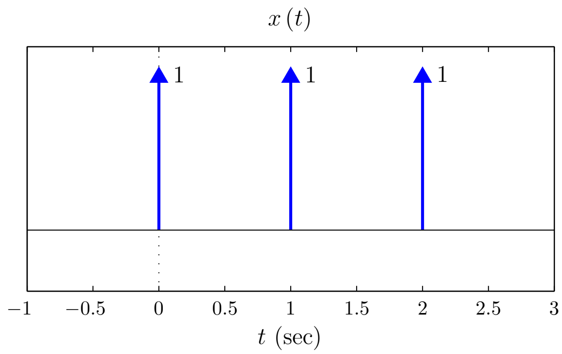
$$\int_{-\infty}^{\infty} f(t)\, q(t-t_1)\, dt = \frac{1}{a} \int_{t_1-a/2}^{t_1+a/2} f(t)\, dt$$

If $f(t)$ is continuous in the vicinity of $t = t_1$ then, for small values of a, the value of the integral above is approximately equal to the area of a rectangle with height equal to $f(t_1)$ and width equal to a, that is,

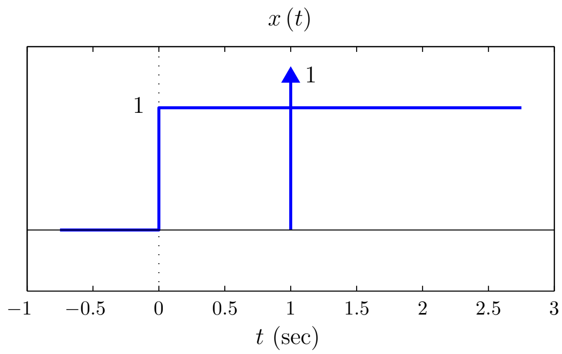
$$\frac{1}{a} \int_{t_1-a/2}^{t_1+a/2} f(t)\, dt \approx \frac{1}{a} \left[f(t_1)\, a \right] = f(t_1)$$

1.8.

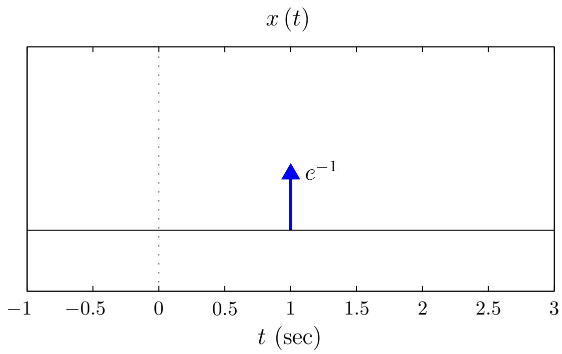
a.



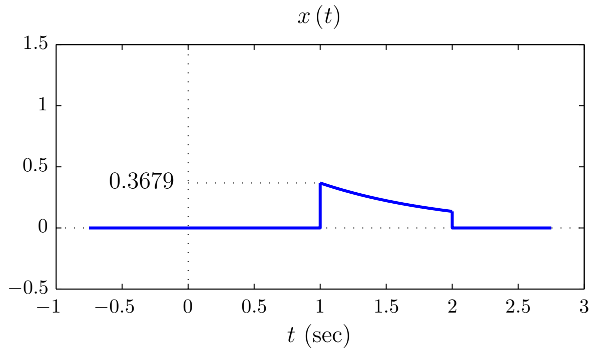
b.



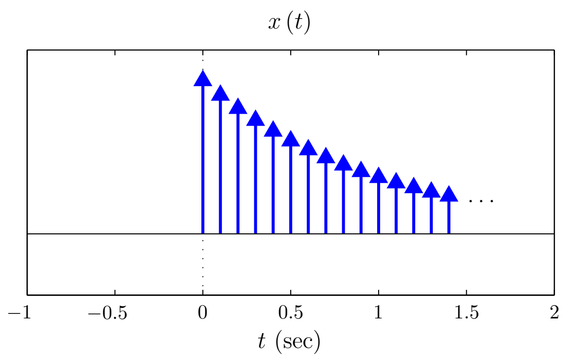
c.



d.

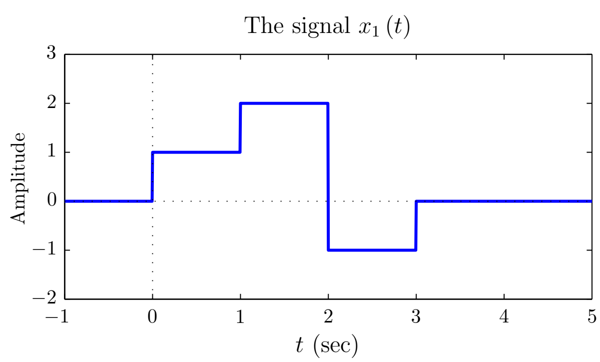

$x(t)$
1.5
1
0.5
0
−0.5
0.3679
−1
−0.5
0
0.5
1
1.5
2
2.5
3
t (sec)

e.

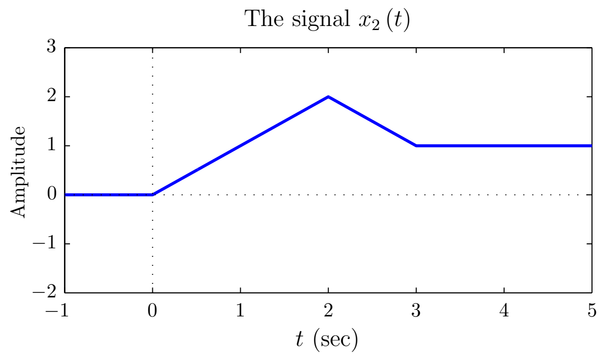

$x(t)$
...
−1
−0.5
0
0.5
1
1.5
2
t (sec)

1.9.

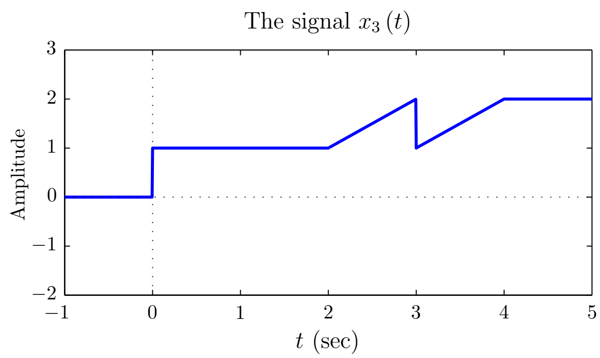
a.


The signal $x_1(t)$
Amplitude
3
2
1
0
−1
−2
−1
0
1
2
3
4
5
t (sec)

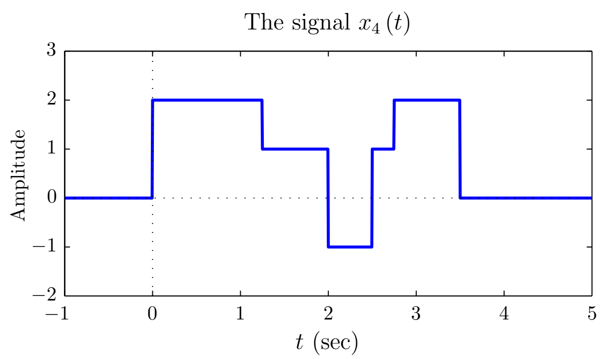
b.


The signal $x_2(t)$
Amplitude
t (sec)
3
2
1
0
−1
−2
−1
0
1
2
3
4
5

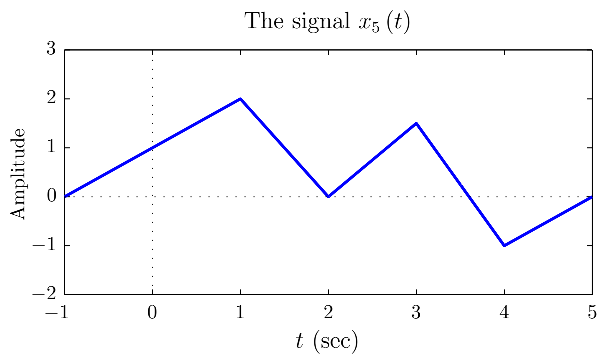
c.


The signal $x_3(t)$
Amplitude
t (sec)
3
2
1
0
−1
−2
−1
0
1
2
3
4
5

d.


The signal $x_4(t)$
Amplitude
t (sec)
3
2
1
0
−1
−2
−1
0
1
2
3
4
5

e.



1.10.

a. $x_a(t) = u(t+2) + 0.5\,u(t+1) - 2\,u(t-1.5) + 0.5\,u(t)$

b. $x_b(t) = 1.5\,u(t+1) - 2.5\,u(t) + u(t-1)$

1.11.

a. $x_a(t) = \Pi(t+1.5) + 1.5\,\Pi\left(\frac{t-0.25}{2.5}\right) - 0.5\,\Pi\left(\frac{t-2.25}{1.5}\right)$

b. $x_b(t) = 1.5\,\Pi(t+0.5) - \Pi(t-0.5)$

1.12.

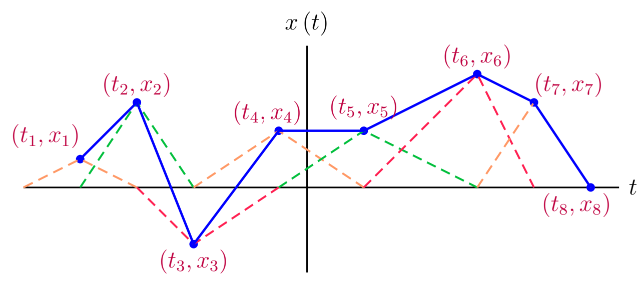
a. $x(t) = -r(t+1) + 3.5\,r(t) - 3\,r(t-1) - 0.5\,r(t-2) + r(t-3)$

b. $x(t) = -\Lambda(t) + 1.5\,\Lambda(t-1) + \Lambda(t-2)$

1.13.

For each data point (t_i, x_i) use a triangle $x_i\,\Lambda_s(t, a_i, b_i)$ with parameters $a_i = t_i - t_{i-1}$ and $b_i = t_{i+1} - t_i$. The signal $x(t)$ can be expressed as

$$x(t) = \sum_i x_i\,\Lambda_s(t, a_i, b_i)$$
$$= \sum_i x_i\,\Lambda_s(t, t_i - t_{i-1}, t_{i+1} - t_i)$$



1.14.

Integration by parts:

$$\int_a^b u(t)\, dv(t) = u(t)\, v(t)\Big|_a^b - \int_a^b v(t)\, du(t)$$

Let $u(t) = f(t)$ and $dv(t) = \delta'(t)\, dt$:

$$\int_{-\infty}^{\infty} f(t)\, \delta'(t)\, dt = f(t)\, \delta'(t)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(t)\, f'(t)\, dt$$

Using the sifting property of the unit impulse function yields

$$\int_{-\infty}^{\infty} f(t)\, \delta'(t)\, dt = -f'(0)$$

1.15.

Using

$$u(t) = \begin{cases} 1\,, & t > 0 \\ 0\,, & t < 0 \end{cases} \qquad \text{and} \qquad u(-t) = \begin{cases} 1\,, & t < 1 \\ 0\,, & t > 0 \end{cases}$$

the signum function can be written as

$$\text{sgn}(t) = -u(-t) + u(t)$$

or as

$$\text{sgn}(t) = -1 + 2\, u(t)$$

1.16.

a.

$$e^{ja} = \cos(a) + j\, \sin(a)$$

$$e^{-ja} = \cos(a) - j\, \sin(a)$$

Therefore

$$\frac{1}{2}\, e^{ja} + \frac{1}{2}\, e^{-ja} = \cos(a)$$

b.

$$e^{ja} = \cos(a) + j\, \sin(a)$$

$$e^{-ja} = \cos(a) - j\, \sin(a)$$

Therefore

$$\frac{1}{2}\, e^{ja} - \frac{1}{2}\, e^{-ja} = j\, \sin(a) \qquad \Rightarrow \qquad \sin(a) = \frac{1}{2j}\, e^{ja} - \frac{1}{2j}\, e^{-ja}$$

c.

$$\frac{d}{da}\left[\frac{1}{2}e^{ja}+\frac{1}{2}e^{-ja}\right]=\frac{j}{2}e^{ja}-\frac{j}{2}e^{-ja}=-\frac{1}{2j}e^{ja}+\frac{j}{2j}e^{-ja}=-\sin(a)$$

d.

$$\begin{aligned}\cos(a+b)=&\frac{1}{2}e^{j(a+b)}+\frac{1}{2}e^{-j(a+b)}\\=&\frac{1}{2}\left[e^{ja}e^{jb}+e^{-ja}e^{-jb}\right]\\=&\frac{1}{2}\Big[\left[\cos(a)+j\sin(a)\right]\left[\cos(b)+j\sin(b)\right]+\left[\cos(a)-j\sin(a)\right]\left[\cos(b)-j\sin(b)\right]\Big]\\=&\cos(a)\cos(b)-\sin(a)\sin(b)\end{aligned}$$

e.

$$\frac{d}{da}\left[\frac{1}{2}e^{ja}+\frac{1}{2}e^{-ja}\right]=\frac{j}{2}e^{ja}-\frac{j}{2}e^{-ja}=-\frac{1}{2j}e^{ja}+\frac{j}{2j}e^{-ja}=-\sin(a)$$

d.

$$\begin{aligned}\sin(a+b)=&\frac{1}{2j}e^{j(a+b)}-\frac{1}{2j}e^{-j(a+b)}\\=&\frac{1}{2j}\left[e^{ja}e^{jb}-e^{-ja}e^{-jb}\right]\\=&\frac{1}{2j}\Big[\left[\cos(a)+j\sin(a)\right]\left[\cos(b)+j\sin(b)\right]-\left[\cos(a)-j\sin(a)\right]\left[\cos(b)-j\sin(b)\right]\Big]\\=&\sin(a)\cos(b)+\cos(a)\sin(b)\end{aligned}$$

f.

$$\begin{aligned}\cos^2(a)=&\left[\frac{1}{2}e^{ja}+\frac{1}{2}e^{-ja}\right]^2\\=&\frac{1}{4}e^{j2a}+\frac{1}{2}+\frac{1}{4}e^{-j2a}\\=&\frac{1}{2}+\frac{1}{2}\cos(2a)\end{aligned}$$

1.17.

a. Periodic.

$$2\pi f_0=2 \quad\Rightarrow\quad f_0=\frac{1}{\pi}\ \text{Hz}\,,\qquad T_0=\frac{1}{f_0}=\pi\ \text{sec}$$

b. Periodic.

$$2\pi f_0=\sqrt{20} \quad\Rightarrow\quad f_0=\frac{\sqrt{20}}{2\pi}=\frac{\sqrt{5}}{\pi}\ \text{Hz}\,,\qquad T_0=\frac{1}{f_0}=\frac{\pi}{\sqrt{5}}\ \text{sec}$$

c. Not periodic due to the factor $u(t)$.

d. Periodic.

$$2\pi f_0 = 3 \quad \Rightarrow \quad f_0 = \frac{3}{2\pi} \text{ Hz}, \quad T_0 = \frac{1}{f_0} = \frac{2\pi}{3} \text{ sec}$$

e. Not periodic due to the factor $e^{-|t|}$.

f. Not periodic.

g. Periodic.

$$x(t) = cos(2t + \pi/10) + j\,sin(2t + \pi/10)$$

$$2\pi f_0 = 2 \quad \Rightarrow \quad f_0 = \frac{1}{\pi} \text{ Hz}, \quad T_0 = \frac{1}{f_0} = \pi \text{ sec}$$

h. Not periodic.

1.18.

a.

$$f_1 = \frac{5}{2\pi} \text{ Hz}, \quad f_2 = \frac{5}{2\pi} \text{ Hz} \quad \Rightarrow \quad f_0 = \frac{5}{2\pi} \text{ Hz}, \quad T_0 = \frac{2\pi}{5} \text{ sec}$$

b.

$$f_1 = 5 \text{ Hz}, \quad f_2 = 15 \text{ Hz} \quad \Rightarrow \quad f_0 = 5 \text{ Hz}, \quad T_0 = \frac{1}{5} = 0.2 \text{ sec}$$

c.

$$f_1 = \frac{\sqrt{2}}{2\pi} \text{ Hz}, \quad f_2 = \frac{2}{2\pi} \text{ Hz}$$

For periodicity we require two integers m_1 and m_2 to be found such that

$$\frac{m_1}{f_1} = \frac{m_2}{f_2} \quad \Rightarrow \quad \frac{m_1}{\sqrt{2}} = \frac{m_2}{2}$$

No two integers can be found; therefore the signal is not periodic.

d.

$$f_1 = 22.5 \text{ Hz}, \quad f_2 = 27.5 \text{ Hz} \quad \Rightarrow \quad f_0 = 2.5 \text{ Hz}, \quad T_0 = \frac{1}{2.5} = 0.4 \text{ sec}$$

1.19.

a. The energy of the signal $x(t)$ is

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt < \infty$$

The energy of $g(t)$ is found as

$$E_g = \int_{-\infty}^{\infty} = \left| g(t) \right|^2 dt = \int_{-\infty}^{\infty} |A\,x(t)|^2 \, dt = |A|^2 \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Thus we have

$$E_g = |A|^2 \, E_x$$

If $x(t)$ is an energy signal, $x(t)$ is also an energy signal.

b. The power in the signal $x(t)$ is

$$P_x = \left\langle \left| x(t) \right|^2 \right\rangle$$

The power in $g(t)$ is found as

$$P_g = \left\langle \left| g(t) \right|^2 \right\rangle = \left\langle \left| A\, x(t) \right|^2 \right\rangle = |A|^2 \left\langle \left| x(t) \right|^2 \right\rangle$$

Thus we have

$$P_g = |A|^2 \, P_x$$

If $x(t)$ is a power signal, $x(t)$ is also a power signal.

1.20.

a. We will assume that parameters A and B are real-valued. Given that $E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$, attempting to compute the normalized energy of the signal $g(t)$ results in

$$E_g = \int_{-\infty}^{\infty} \left| A\, x(t) + B \right|^2 \, dt$$

which does not converge for $B \neq 0$. Therefore $g(t)$ is a power signal.

b. The normalized average power in $g(t)$ is

$$\begin{aligned} P_g &= \left\langle \left| g(t) \right|^2 \right\rangle \\ &= \left\langle \, \big(A\, x(t) + B\big) \big(A\, x(t) + B\big)^* \right\rangle \\ &= A^2 \left\langle \, |x(t)|^2 \right\rangle + B^2 + 2AB \left\langle \, \text{Re}\{x(t)\} \right\rangle \end{aligned}$$

Since $x(t)$ is an energy signal, we have

$$P_g = B^2$$

1.21.

a. The signal $x(t)$ can be written as

$$x(t) = e^{-2|t|} = \begin{cases} e^{2t}, & t < 0 \\ e^{-2t}, & t > 0 \end{cases}$$

The energy of the signal is

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{0} e^{4t} \, dt + \int_{0}^{\infty} e^{-4t} \, dt = \frac{1}{2}$$

b.

$$E_x = \int_{-\infty}^{\infty} \left|e^{-2t}\right|^2 u(t)\, dt = \int_{0}^{\infty} e^{-4t}\, dt = \frac{1}{4}$$

c.

$$\begin{aligned} |x(t)|^2 &= \left|e^{-2t}\right|^2 \left|\cos(5t)\right|^2 u(t) \\ &= e^{-4t} \cos^2(5t)\, u(t) \end{aligned}$$

Remembering that

$$\cos^2(5t) = \frac{1}{2} + \frac{1}{2}\cos(10t)$$

we have

$$\begin{aligned} |x(t)|^2 =& \frac{1}{2} e^{-4t} u(t) + \frac{1}{2} e^{-4t} \cos(10t)\, u(t) \\ =& \frac{1}{2} e^{-4t} u(t) + \frac{1}{4} e^{-4t} e^{j10t} u(t) + \frac{1}{4} e^{-4t} e^{-j10t} u(t) \\ =& \frac{1}{2} e^{-4t} u(t) + \frac{1}{4} e^{(-4+j10)t} u(t) + \frac{1}{4} e^{(-4-j10)t} u(t) \end{aligned}$$

The normalized energy is

$$\begin{aligned} E_x &= \int_{-\infty}^{\infty} |x(t)|^2\, dt \\ &= \frac{1}{2} \int_0^{\infty} e^{-4t}\, dt + \frac{1}{4} \int_0^{\infty} e^{(-4+j10)t}\, dt + \frac{1}{4} \int_0^{\infty} e^{(-4-j10)t}\, dt \\ &= \frac{1}{8} + \frac{1}{58} = 0.1422 \end{aligned}$$

1.22.

a.

$$x_a(t) = \begin{cases} 2t+2\,, & -1 < t < 0 \\ -t+2\,, & 0 < t < 1 \\ 1\,, & 1 < t < 2 \\ -t+3\,, & 2 < t < 3 \\ 0;\,, & \text{otherwise} \end{cases}$$

$$E_x = \int_{-1}^{0} (2t+2)^2\, dt + \int_0^1 (-t+2)^2\, dt + \int_1^2 (1)^2\, dt + \int_2^3 (-t+3)^2\, dt = 5$$

b.

$$x_b(t) = \begin{cases} 1.5t+1.5\,, & -1 < t < 0 \\ -1.5t+1.5\,, & 0 < t < 2 \\ 1.5t-4.5\,, & 2 < t < 3 \\ 0\,, & \text{otherwise} \end{cases}$$

$$E_x = \int_{-1}^{0} (1.5t+1.5)^2 \, dt + \int_{0}^{2} (-1.5t+1.5)^2 \, dt + \int_{2}^{3} (1.5t-4.5)^2 \, dt = 3$$

1.23.

a.

$$P_x = \left\langle \left|x(t)\right|^2 \right\rangle = \int_0^{0.5} (1)^2 \, dt = \frac{1}{3}$$

b.

$$P_x = \left\langle \left|x(t)\right|^2 \right\rangle = \int_0^{1} t^2 \, dt = \left. \frac{t^3}{3} \right|_0^1 = 0.5$$

c.

$$P_x = \int_0^1 \sin^2(\pi t) \, dt = \int_0^1 \left(\frac{1}{2} - \frac{1}{2}\cos(2\pi t) \right) dt = \frac{1}{2}$$

1.24.

The two terms with the same frequency need to be combined. Consider that

$$A\cos\left(2\pi f_1 t + \theta\right) = A\cos(\theta)\cos\left(2\pi f_1 t\right) - A\sin(\theta)\sin\left(2\pi f_1 t\right)$$

Let $A\cos(\theta) = 2$ and $-A\sin(\theta) = 3$. Solving the two equations we get

$$A = 3.6056 \qquad \text{and} \qquad \theta = -0.9828 \text{ radians}$$

The signal is

$$x(t) = 3.6056\cos\left(2\pi f_1 t - 0.9858\right) + 6\cos\left(2\pi f_2 t\right)$$

and its RMS value is

$$x_{RMS} = \sqrt{\frac{3.6056^2}{2} + \frac{6^2}{2}} = 4.9497$$

1.25.

a. Even

b. Odd

c. Neither even nor odd

d. Even

e. Odd

f. Neither even nor odd

1.26.

a.

$$\int_{-\lambda}^{\lambda} x(t)\,dt = \int_{-\lambda}^{0} x(t)\,dt \int_{0}^{\lambda} x(t)\,dt$$

For the first integral, apply the variable change $t = -\alpha$ to obtain

$$\begin{aligned}\int_{-\lambda}^{\lambda} x(t)\,dt &= \int_{\lambda}^{0} x(-\alpha)\,(-d\alpha) + \int_{0}^{\lambda} x(t)\,dt \\ &= -\int_{\lambda}^{0} x(-\alpha)\,d\alpha + \int_{0}^{\lambda} x(t)\,dt \\ &= \int_{0}^{\lambda} x(-\alpha)\,d\alpha + \int_{0}^{\lambda} x(t)\,dt\end{aligned}$$

Since $x(t)$ is even, $x(-\alpha) = x(\alpha)$ and

$$\begin{aligned}\int_{-\lambda}^{\lambda} x(t)\,dt &= \int_{0}^{\lambda} x(\alpha)\,d\alpha + \int_{0}^{\lambda} x(t)\,dt \\ &= 2\int_{0}^{\lambda} x(t)\,dt\end{aligned}$$

b. From part (a) we have

$$\int_{-\lambda}^{\lambda} x(t)\,dt = \int_{0}^{\lambda} x(-\alpha)\,d\alpha + \int_{0}^{\lambda} x(t)\,dt$$

Since $x(t)$ is odd, $x(-\alpha) = -x(\alpha)$ and

$$\begin{aligned}\int_{-\lambda}^{\lambda} x(t)\,dt &= -\int_{0}^{\lambda} x(\alpha)\,d\alpha + \int_{0}^{\lambda} x(t)\,dt \\ &= 0\end{aligned}$$

1.27.

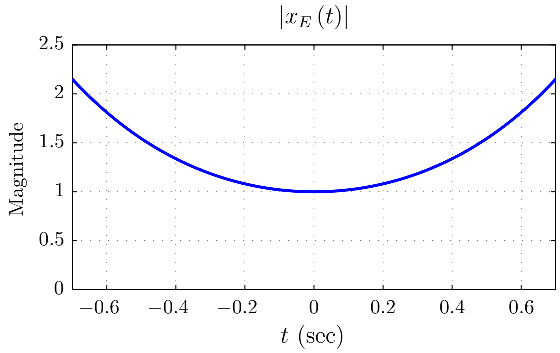
a. $x(-t) = x_1(-t)\,x_2(-t) = x_1(t)\,x_2(t) = x(t)$

b. $x(-t) = x_1(-t)\,x_2(-t) = [-x_1(t)]\,[-x_2(t)] = x_1(t)\,x_2(t) = x(t)$

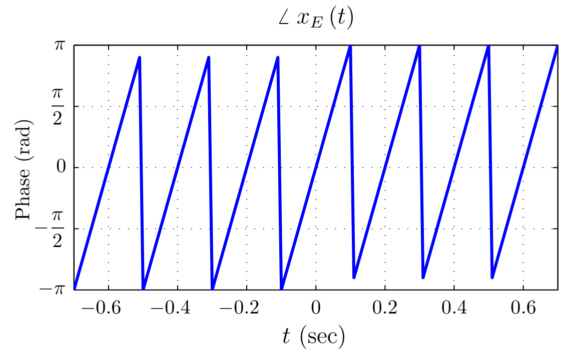
c. $x(-t) = x_1(-t)\,x_2(-t) = x_1(t)\,[-x_2(t)] = -x_1(t)\,x_2(t) = -x(t)$

1.28.

$$\begin{aligned}x_E(t) &= \frac{1}{2}\,e^{(-2+j10\pi)t} + \frac{1}{2}\,e^{(-2-j10\pi)(-t)} \\ &= \frac{1}{2}\left(e^{-2t} + e^{2t}\right) e^{j10\pi t}\end{aligned}$$



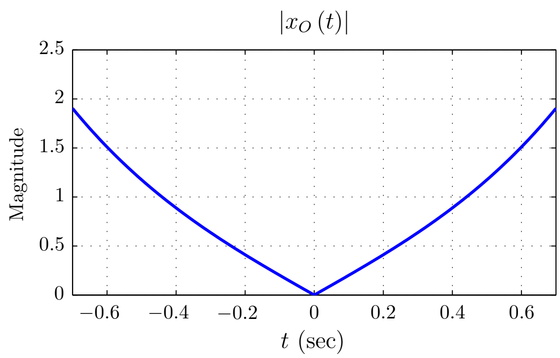
(a)



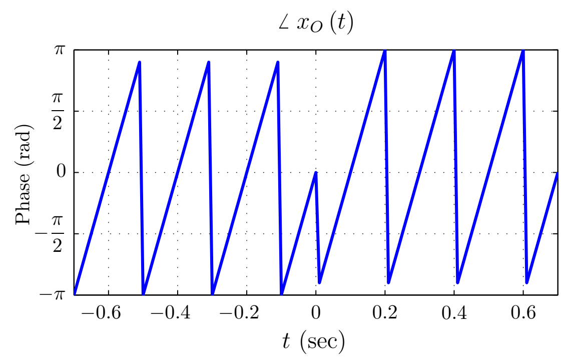
(b)

$$x_O(t) = \frac{1}{2}\,e^{(-2+j10\pi)t} - \frac{1}{2}\,e^{(-2-j10\pi)(-t)}$$

$$= \frac{1}{2}\left(e^{-2t} - e^{2t}\right)e^{j10\pi t}$$



(c)



(d)

1.29.

a.

$$x_e(t) = \frac{1}{2}\,e^{-5t}\sin(t)\,u(t) + \frac{1}{2}\,e^{5t}\sin(-t)\,u(-t)$$

$$= \frac{1}{2}\sin(t)\left[e^{-5t}\,u(t) - e^{5t}\,u(-t)\right]$$

$$x_o(t) = \frac{1}{2}\,e^{-5t}\sin(t)\,u(t) - \frac{1}{2}\,e^{5t}\sin(-t)\,u(-t)$$

$$= \frac{1}{2}\sin(t)\left[e^{-5t}\,u(t) + e^{5t}\,u(-t)\right]$$

b.

$$x_e(t) = \frac{1}{2}\,e^{-3|t|}\cos(t) + \frac{1}{2}\,e^{-3|-t|}\cos(-t)$$
$$= e^{-3|t|}\cos(t)$$

$$x_o(t) = \frac{1}{2}\,e^{-3|t|}\cos(t) - \frac{1}{2}\,e^{-3|-t|}\cos(-t)$$
$$= 0$$

c.

$$x_e(t) = \frac{1}{2}\,e^{-3|t|}\sin(t) + \frac{1}{2}\,e^{-3|-t|}\sin(-t)$$
$$= 0$$

$$x_o(t) = \frac{1}{2}\,e^{-3|t|}\sin(t) - \frac{1}{2}\,e^{-3|-t|}\sin(-t)$$
$$= e^{-3|t|}\sin(t)$$

d.

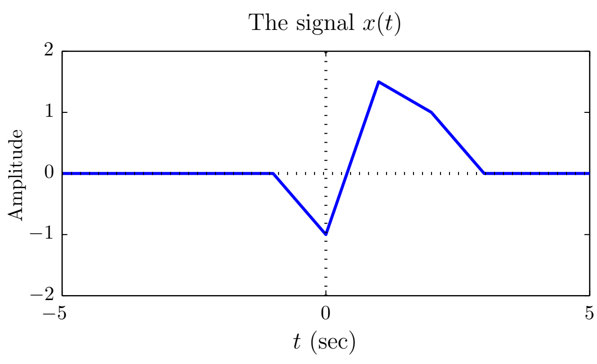
$$x_e(t) = \frac{1}{2}\left(t\,e^{-3t}+2\right)u(t) + \frac{1}{2}\left(-t\,e^{3t}+2\right)u(-t)$$
$$= \frac{1}{2}\,te^{-3t}\,u(t) - \frac{1}{2}\,te^{3t}\,u(-t) + 1$$
$$x_o(t) = \frac{1}{2}\left(t\,e^{-3t}+2\right)u(t) - \frac{1}{2}\left(-t\,e^{3t}+2\right)u(-t)$$

e.

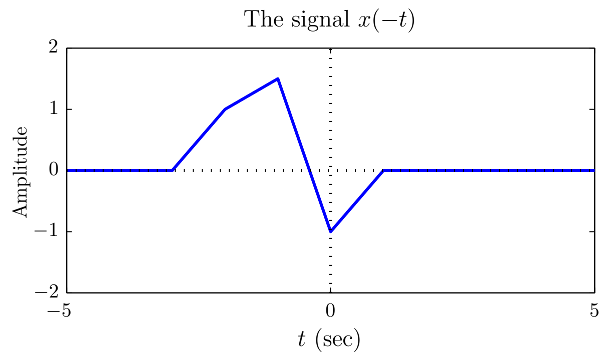
$$x_e(t) = \frac{1}{2}\,e^{-2|t-1|} + \frac{1}{2}\,e^{-2|-t-1|}$$
$$x_o(t) = \frac{1}{2}\,e^{-2|t-1|} - \frac{1}{2}\,e^{-2|-t-1|}$$

1.30.

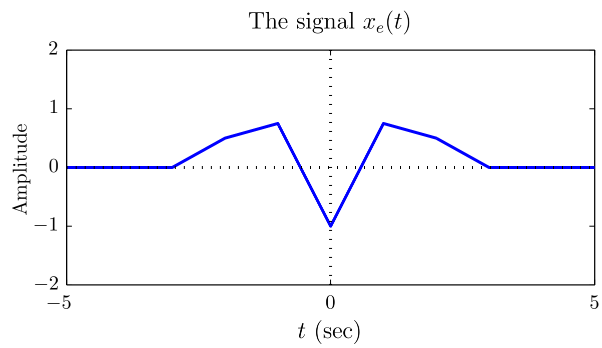
$$x(t) = \begin{cases} -t-1\,, & -1<t<0 \\ 2.5\,t-1\,, & 0<t<1 \\ -0.5\,t+2\,, & 1<t<2 \\ -t+3\,, & 2<t<3 \\ 0\,, & \text{otherwise} \end{cases}$$



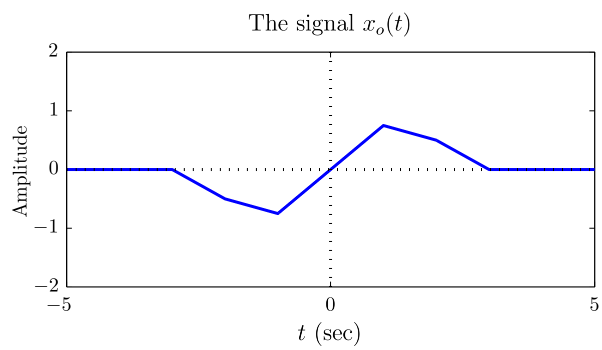
$$x(-t) = \begin{cases} t+3\,, & -3 < t < -2 \\ 0.5\,t+2\,, & -2 < t < -1 \\ -2.5\,t-1\,, & -1 < t < 0 \\ t-1\,, & 0 < t < 1 \\ 0\,, & \text{otherwise} \end{cases}$$



$$x_e(t) = \frac{x(t)+x(-t)}{2}$$



$$x_o(t) = \frac{x(t)-x(-t)}{2}$$



1.31.

a. $\boldsymbol{X} = 3\,e^{j\,0^\circ}\,, \quad \omega_0 = 200\pi \text{ rad/s}$

b. $x(t) = 7\cos(100\pi t - \pi/2) \quad \Rightarrow \quad \boldsymbol{X} = 7\,e^{-j\pi/2}\,, \quad \omega_0 = 100\pi \text{ rad/s}$

c.

$$x(t) = 2\cos(10\pi t - \pi/2) + 5\cos(10\pi t + \pi)$$

$$\begin{aligned} \boldsymbol{X} &= 2\,e^{-j\pi/2} + 5\,e^{j\pi} \\ &= 5.3852\,e^{-j\,2.7611}\,, \quad \omega_0 = 10\pi \text{ rad/s} \end{aligned}$$