PROBLEM 11.1

A snowboarder starts from rest at the top of a double black diamond hill. As he rides down the slope, GPS coordinates are used to determine his displacement as a function of time: $x = 0.5t^3 + t^2 + 2t$ where x and t are expressed in ft and seconds, respectively. Determine the position, velocity, and acceleration of the boarder when t = 5 seconds.

SOLUTION

Position: $x = 0.5t^3 + t^2 + 2t$

Velocity: $v = \dfrac{dx}{dt} = 1.5t^2 + 2t + 2$

Acceleration: $a = \dfrac{dv}{dt} = 3t + 2$

At $t = 5$ s, $x = 0.5(5)^3 + 5^2 + 2(5)$ $\qquad x = 97.5$ ft ◀

$v = 1.5(5)^2 + 2(5) + 2$ $\qquad v = 49.5$ ft/s ◀

$a = 3(5) + 2$ $\qquad a = 17$ ft/s^2 ◀

PROBLEM 11.2

The motion of a particle is defined by the relation $x = t^3 - 12t^2 + 36t + 30$, where x and t are expressed in feet and seconds, respectively. Determine the time, the position, and the acceleration of the particle when $v = 0$.

SOLUTION

$$x = t^3 - 12t^2 + 36t + 30$$

Differentiating,

$$v = \frac{dx}{dt} = 3t^2 - 24t + 36 = 3(t^2 - 8t + 12)$$
$$= 3(t-6)(t-2)$$

$$a = \frac{dv}{dt} = 6t - 24$$

So $v = 0$ at $t = 2$ s and $t = 6$ s.

At $t = 2$ s,

$$x_1 = (2)^3 - 12(2)^2 + 36(2) + 30 = 62$$

$t = 2.00$ s ◀

$$a_1 = 6(2) - 24 = -12$$

$x_1 = 62.00$ ft ◀

$a_1 = -12.00$ ft/s^2 ◀

At $t = 6$ s,

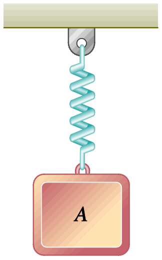
$$x_2 = (6)^3 - 12(6)^2 + 36(6) + 30 = 30$$

$t = 6.00$ s ◀

$x_2 = 30.00$ ft ◀

$$a_2 = (6)(6) - 24 = 12$$

$a_2 = 12.00$ ft/s^2 ◀



PROBLEM 11.3

The vertical motion of mass A is defined by the relation $x = \cos(10t) - 0.1\sin(10t)$, where x and t are expressed in mm and seconds, respectively. Determine (*a*) the position, velocity and acceleration of A when $t = 0.4$ s, (*b*) the maximum velocity and acceleration of A.

SOLUTION

$$x = \cos(10t) - 0.1\sin(10t)$$

$$v = \frac{dx}{dt} = -10\sin 10t - 0.1(10)\cos(10t)$$

$$a = \frac{dv}{dt} = -100\cos 10t + 0.1(100)\sin 10t$$

For trigonometric functions set calculator to radians:

(*a*) At $t = 0.4\,\text{s}$ $\quad x_1 = \cos 4 - 0.1\sin(4) = 0.578$ $\quad x_1 = 0.578$ mm ◀

$v_1 = -10\sin(4) - \cos(4) = 8.222$ $\quad v_1 = 8.22$ mm/s ◀

$a_1 = -100\cos(4) + 10\sin(4) = 57.80$ $\quad a_1 = 57.80\ \text{mm/s}^2$ ◀

(*b*) Maximum velocity occurs when $a = 0$.

$$-100\cos(10t) + 10\sin(10t) = 0$$

$$\tan 10t = 10,\ 10t = \tan^{-1}(10),\ t = 0.147\ \text{s}$$

So $\quad t = 0.147\,\text{s}$ and $0.147+\pi$ s for v_{max}

$$v_{max} = |-10\sin(10*0.147) - \cos(10*0.147)|$$
$$= 10.05$$

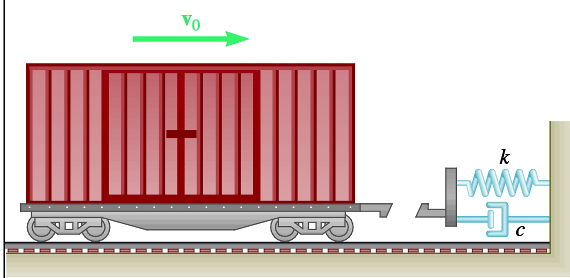
$v_{max} = 10.05$ mm/s ◀

Note that we could have also used $v_{max} = \sqrt{10^2 + 1^2} = 10.05$

by combining the sine and cosine terms. For a_{max} we can take the derivative and set equal to zero or just combine the sine and cosine terms.

$$a_{max} = \sqrt{100^2 + 10^2} = 100.5\ \text{mm/s}^2$$

$a_{max} = 100.5\ \text{mm/s}^2$ ◀



PROBLEM 11.4

A loaded railroad car is rolling at a constant velocity when it couples with a spring and dashpot bumper system. After the coupling, the motion of the car is defined by the relation $x = 60e^{-4.8t}\sin 16t$ where x and t are expressed in mm and seconds, respectively. Determine the position, the velocity and the acceleration of the railroad car when (*a*) $t = 0$, (*b*) $t = 0.3$ s.

SOLUTION

$$x = 60e^{-4.8t}\sin 16t$$

$$v = \frac{dx}{dt} = 60(-4.8)e^{-4.8t}\sin 16t + 60(16)e^{-4.8t}\cos 16t$$

$$v = -288e^{-4.8t}\sin 16t + 960e^{-4.8t}\cos 16t$$

$$a = \frac{dv}{dt} = 1382.4e^{-4.8t}\sin 16t - 4608e^{-4.8t}\cos 16t$$
$$- 4608e^{-4.8t}\cos 16t - 15360e^{-4.8t}\sin 16t$$

$$a = -13977.6e^{-4.8t}\sin 16t - 9216e^{-4.8}\cos 16t$$

(*a*) At $t = 0$,

$x_0 = 0$ $\qquad$ $x_0 = 0$ mm ◀

$v_0 = 960$ mm/s $\qquad$ $v_0 = 960$ mm/s $\longrightarrow$ ◀

$a_0 = -9216$ mm/s^2 $\qquad$ $a_0 = 9220$ mm/s^2 $\longleftarrow$ ◀

(*b*) At $t = 0.3$ s,

$$e^{-4.8t} = e^{-1.44} = 0.23692$$
$$\sin 16t = \sin 4.8 = -0.99616$$
$$\cos 16t = \cos 4.8 = 0.08750$$

$x_{0.3} = (60)(0.23692)(-0.99616) = -14.16$ $\qquad$ $x_{0.3} = 14.16$ mm $\longleftarrow$ ◀

$v_{0.3} = -(288)(0.23692)(-0.99616)$
$+ (960)(0.23692)(0.08750) = 87.9$ $\qquad$ $v_{0.3} = 87.9$ mm/s $\longrightarrow$ ◀

$a_{0.3} = -(13977.6)(0.23692)(-0.99616)$
$-(9216)(0.23692)(0.08750) = 3108$ $\qquad$ $a_{0.3} = 3110$ mm/s^2 ◀

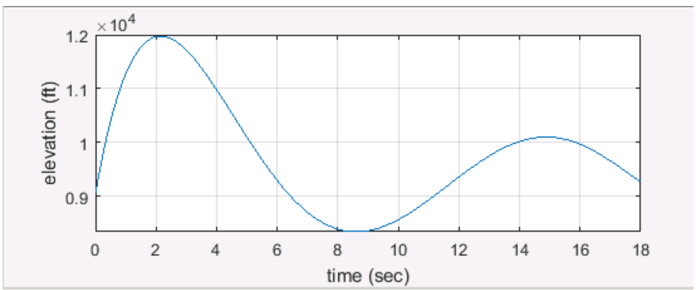
or 3.11 m/s^2 $\longrightarrow$ ◀

PROBLEM 11.5

A group of hikers uses a GPS while doing a 40 mile trek in Colorado. A curve fit to the data shows that their altitude can be approximated by the function, $y(t) = 0.12t^5 - 6.75t^4 + 135t^3 - 1120t^2 + 3200t + 9070$ where y and t are expressed in feet and hours, respectively. During the 18 hour hike, determine (a) the maximum altitude that the hikers reach, (b) the total feet they ascend, (c) the total feet they descend. *Hint*: You will need to use a calculator or computer to solve for the roots of a fourth order polynomial.

SOLUTION

You can graph the function to get the elevation profile.



Differentiate y(t) and set to zero to find when the hikers are ascending and descending.

$$\dot{x} = 0.12(5)t^4 - 6.75(4)t^3 + 135(3)t^2 - 1200(2)t + 3200 = 0$$

Use a your calculator to find the roots of this polynomial to get

t = 2.151, 8.606 and 14.884 hours

Next, evaluate y(t) at start, end and calculated times to find the elevations

x(0) = 9070 ft (begin to ascend)

x(2.151) = 11,976 ft (begin to descend) $\qquad y = 11{,}976 \text{ ft} \quad \text{at} \quad t = 2.151 \text{ hours}$ ◀

x(8.606) = 8,344 ft (begin to ascend)

x(14.884) = 10,103 ft (begin to descend)

x(18) = 9,270 ft

Add the accents to get the total ascent = 4,664 ft $\qquad \text{total ascent} = 4{,}660 \text{ ft}$ ◀

Add the descents to get the total descent = 4,464 ft $\qquad \text{total descent} = -4{,}460 \text{ ft}$ ◀

PROBLEM 11.6

The motion of a particle is defined by the relation $x = t^3 - 6t^2 + 9t + 5$, where x is expressed in feet and t in seconds. Determine (*a*) when the velocity is zero, (*b*) the position, acceleration, and total distance traveled when $t = 5$ s.

SOLUTION

Given: $$x = t^3 - 6t^2 + 9t + 5$$

Differentiate twice. $$v = \frac{dx}{dt} = 3t^2 - 12t + 9 \text{ and } a = \frac{dv}{dt} = 6t - 12$$

(*a*) When velocity is zero. $v = 0$, $3t^2 - 12t + 9 = 3(t-1)(t-3) = 0$, $t = 1\text{ s}$ and $t = 3\text{ s}$ ◀

(*b*) Position at $t = 5$ s. $$x_5 = (5)^3 - (6)(5)^2 + (9)(5) + 5$$ $x_5 = 25\text{ ft}$ ◀

Acceleration at $t = 5$ s. $$a_5 = (6)(5) - 12$$ $a_5 = 18\text{ ft/s}^2$ ◀

Position at $t = 0$. $$x_0 = 5\text{ ft}$$

Over $0 \le t < 1$ s: x is increasing.

Over $1\text{ s} < t < 3\text{ s}$: x is decreasing.

Over $3\text{ s} < t \le 5\text{ s}$: x is increasing.

Position at $t = 1$ s. $$x_1 = (1)^3 - (6)(1)^2 + (9)(1) + 5 = 9\text{ ft}$$

Position at $t = 3$ s. $$x_3 = (3)^3 - (6)(3)^2 + (9)(3) + 5 = 5\text{ ft}$$

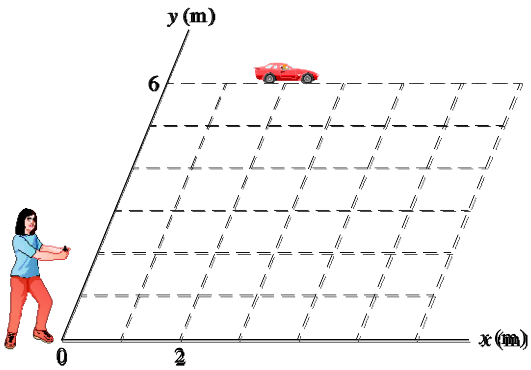
Distance traveled.

At $t = 1$ s $$d_1 = |x_1 - x_0| = |9 - 5| = 4\text{ ft}$$

At $t = 3$ s $$d_3 = d_1 + |x_3 - x_1| = 4 + |5 - 9| = 8\text{ ft}$$

At $t = 5$ s $$d_5 = d_3 + |x_5 - x_3| = 8 + |25 - 5| = 28\text{ ft}$$

$d_5 = 28\text{ ft}$ ◀



PROBLEM 11.7

A girl operates a radio-controlled model car in a vacant parking lot. The girl's position is at the origin of the xy coordinate axes, and the surface of the parking lot lies in the x-y plane. She drives the car in a straight line so that the x coordinate is defined by the relation $x(t) = 0.5t^3 - 3t^2 + 3t + 2$, where x and t are expressed in meters and seconds, respectively. Determine (a) when the velocity is zero, (b) the position and total distance travelled when the acceleration is zero.

SOLUTION

Position: $x(t) = 0.5t^3 - 3t^2 + 3t + 2$

Velocity: $v(t) = \dfrac{dx}{dt}$

$v(t) = 1.5t^2 - 6t + 3$

(a) Time when v = 0: $0 = 1.5t^2 - 6t + 3$

$$t = \frac{6 \pm \sqrt{6^2 - 4(1.5)(3)}}{2 * 1.5} \qquad t = 0.586 \text{ s} \quad \text{and} \quad t = 3.414 \text{ s} \blacktriangleleft$$

Acceleration: $a(t) = \dfrac{dv}{dt}$

$a(t) = 3t - 6$

Time when a = 0: $0 = 3t - 6$ so $t = 2$ s

(b) Position at t = 2 s: $x(2) = 0.5(2)^3 - 3(2)^2 + 3 * 2 + 2$

$$x(2) = 0 \text{ m} \blacktriangleleft$$

To find total distance note that car changes direction at t = 0.586 s

Position at t = 0 s: $x(0) = 0.5(0)^3 - 3(0)^2 + 3 * 0 + 2$

$x(0) = 2$

Position at t = 0.586 s: $x(0.586) = 0.5(0.586)^3 - 3(0.586)^2 + 3 * 0.586 + 2$

$x(0.586) = 2.828$ m

Distances traveled:

From t = 0 to t = 0.586 s: $|x(0.586) - x(0)| = 0.828$ m

From t = 0.586 to t = 2 s: $|x(2) - x(0.586)| = 2.828$ m

Total distance traveled $= 0.828 \text{ m} + 2.828 \text{ m}$ Total distance $= 3.656 \text{ m}$ ◀

PROBLEM 11.8

The motion of a particle is defined by the relation $x = t^2 - (t-2)^3$, where x and t are expressed in feet and seconds, respectively. Determine (*a*) the two positions at which the velocity is zero, (*b*) the total distance traveled by the particle from $t = 0$ to $t = 4$ s.

SOLUTION

Position: $x(t) = t^2 - \left(t-2\right)^3$

Velocity: $v(t) = \dfrac{dx}{dt}$

$$v(t) = 2t - 3(t-2)^2$$

$$v(t) = 2t - 3\left(t^2 - 4t + 4\right)$$

$$= -3t^2 + 14t - 12$$

Time when v(t) = 0 $\quad 0 = -3t^2 + 14t - 12$

$$t = \frac{-14 \pm \sqrt{14^2 - 4(-3)(-12)}}{2(-3)}$$

$$t = 1.131 \text{ s} \quad \text{and} \quad t = 3.535 \text{ s}$$

(*a*) Position at t = 1.131 s $\quad x(1.131) = (1.131)^2 - \left(1.1.31 - 2\right)^3$ $\quad x(1.131) = 1.935$ ft. ◀

Position at t = 3.535 s $\quad x(3.535) = (3.535)^2 - \left(3.535 - 2\right)^3$

$x(3.531) = 8.879$ ft. ◀

To find total distance traveled note that the particle changes direction at t = 1.131 s and again at t = 3.535 s.

Position at t = 0 s $\quad x(0) = (0)^2 - \left(0 - 2\right)^3$

$$x(0) = 8 \text{ ft}$$

Position at t = 4 s $\quad x(4) = (4)^2 - \left(4 - 2\right)^3$

$$x(4) = 8 \text{ ft}$$

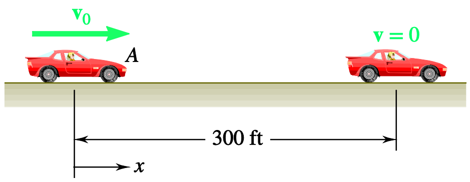
(*b*) Distances traveled:

From t = 0 to t = 1.131 s: $\quad \left|x(1.131) - x(0)\right| = 6.065$ ft.

From t = 1.131 to t = 3.531 s: $\quad \left|x(3.535) - x(1.131)\right| = 6.944$ ft.

From t = 3.531 to t = 4 s: $\quad \left|x(4) - x(3.535)\right| = 0.879$ ft.

Total distance traveled $= 6.065 \text{ ft} + 6.944 \text{ ft} + 0.879 \text{ ft}$ $\quad$ Total distance $= 13.888$ ft. ◀



PROBLEM 11.9

The brakes of a car are applied, causing it to slow down at a rate of 10 ft/s². Knowing that the car stops in 300 ft, determine (*a*) how fast the car was traveling immediately before the brakes were applied, (*b*) the time required for the car to stop.

SOLUTION

$$a = -10 \text{ ft/s}^2$$

(*a*) Velocity at $x = 0$.

$$v\frac{dv}{dx} = a = -10$$

$$\int_{v_0}^{0} v dv = -\int_0^{x_f} (-10) dx$$

$$0 - \frac{v_0^2}{2} = -10x_f = -(10)(300)$$

$$v_0^2 = 6000 \qquad v_0 = 77.5 \text{ ft/s} \blacktriangleleft$$

(*b*) Time to stop.

$$\frac{dv}{dx} = a = -10$$

$$\int_{v_0}^{0} dv = -\int_0^{t_f} -10 dt$$

$$0 - v_0 = -10t_f$$

$$t_f = \frac{v_0}{10} = \frac{77.5}{10} \qquad t_f = 7.75 \text{ s} \blacktriangleleft$$

PROBLEM 11.10

The acceleration of a particle is defined by the relation $a = 3e^{-0.2t}$, where a and t are expressed in ft/s^2 and seconds, respectively. Knowing that $x = 0$ and $v = 0$ at $t = 0$, determine the velocity and position of the particle when $t = 0.5$ s.

SOLUTION

Acceleration: $a = 3e^{-0.2t}$ ft/s^2

Given: $v_0 = 0$ ft/s, $x_0 = 0$ ft

Velocity:

$$a = \frac{dv}{dt} \Rightarrow dv = adt$$

$$\int_{v_0}^{v} dv = \int_0^t adt$$

$$v - v_o = \int_0^t 3e^{-0.2t}dt \Rightarrow v - 0 = -15e^{-0.2t}\Big|_0^t$$

$$v = 15\left(1 - e^{-0.2t}\right) \text{ ft/s}$$

Position:

$$v = \frac{dx}{dt} \Rightarrow dx = vdt$$

$$\int_{x_0}^{x} dx = \int_0^t vdt$$

$$x - x_o = \int_0^t 15\left(1 - e^{-0.2t}\right)dt \Rightarrow \text{x} - 0 = 15\left(t + 5e^{-0.2t}\right)\Big|_0^t$$

$$x = 15\left(t + 5e^{-0.2t}\right) - 75 \text{ ft}$$

Velocity at t = 0.5 s

$$v = 15\left(1 - e^{-0.2*0.5}\right) \text{ ft/s}$$

$$v(0.5) = 1.427 \text{ ft/s} \blacktriangleleft$$

Position at t = 0.5 s

$$x = 15\left(0.5 + 5e^{-0.2*0.5}\right) - 75 \text{ ft}$$

$$x(0.5) = 0.363 \text{ ft.} \blacktriangleleft$$

PROBLEM 11.11

The acceleration of a particle is defined by the relation $a = 9 - 3t^2$, where a and t are expressed in ft/s^2 and seconds, respectively. The particle starts at $t = 0$ with $v = 0$ and x = 5 ft. Determine (*a*) the time when the velocity is again zero, (*b*) the position and velocity when $t = 4$ s, (*c*) the total distance traveled by the particle from $t = 0$ to $t = 4$ s.

SOLUTION

$$a = 9 - 3t^2$$

Separate variables and integrate. $\int_0^v dv = \int a\,dt = \int_0^t \left(9 - 3t^2\right)dt = 9$

$$v - 0 = 9t - t^3, \quad v = t\left(9 - t^2\right)$$

(*a*)When v is zero. $\quad t(9 - t^2) = 0$ so $t = 0$ and $t = 3$ s (2 roots) $\qquad t = 3\text{ s} \blacktriangleleft$

(*b*)Position and velocity at $t = 4$ s.

$$\int_5^x dx = \int_0^t v\,dt = \int_0^t \left(9t - t^3\right)dt$$

$$x - 5 = \frac{9}{2}t^2 - \frac{1}{4}t^4, \quad x = 5 + \frac{9}{2}t^2 - \frac{1}{4}t^4$$

At $t = 4$ s, $\quad x_4 = 5 + \left(\frac{9}{2}\right)(4)^2 - \left(\frac{1}{4}\right)(4)^4 \qquad x_4 = 13\text{ ft} \blacktriangleleft$

$v_4 = (4)\left(9 - 4^2\right) \qquad v_4 = -28\text{ ft/s} \blacktriangleleft$

(*c*)Distance traveled.

Over $0 < t < 3$ s, $\quad v$ is positive, so x is increasing.

Over $3\text{ s} < t \leq 4$ s, $\quad v$ is negative, so x is decreasing.

At $t = 3$ s, $\quad x_3 = 5 + \left(\frac{9}{2}\right)(3)^2 - \left(\frac{1}{4}\right)(3)^4 = 25.25$ ft

At $t = 3$ s $\quad d_3 = |x_3 - x_0| = |25.25 - 5| = 20.25$ ft

At $t = 4$ s $\quad d_4 = d_3 + |x_4 - x_3| = 20.25 + |13 - 25.25| = 32.5$ ft $\qquad d_4 = 32.5\text{ ft} \blacktriangleleft$

PROBLEM 11.12

Many car companies are performing research on collision avoidance systems. A small prototype applies engine braking that decelerates the vehicle according to the relationship $a = -k\sqrt{t}$, where a and t are expressed in m/s^2 and seconds, respectively. The vehicle is travelling 20 m/s when its radar sensors detect a stationary obstacle. Knowing that it takes the prototype vehicle 4 seconds to stop, determine (*a*) expressions for its velocity and position as a function of time, (*b*) how far the vehicle travelled before it stopped.

SOLUTION

Starting with the given function for acceleration,

$$a = -k\sqrt{t}$$

Differentiate with respect to time to get

$$v = -\frac{2k}{3}t^{3/2} + v_0$$

Let t = 4 and v0 = 20 to find k when v = 0

$$0 = -\frac{2k}{3}4^{3/2} + 20$$
$$k = 3.75$$

$$v = -\frac{2(3.75)}{3}t^{3/2} + 20$$
$$v = -2.5t^{3/2} + 20$$

$v = -2.5t^{3/2} + 20$ ◀

Integrate velocity with respect to time to get position

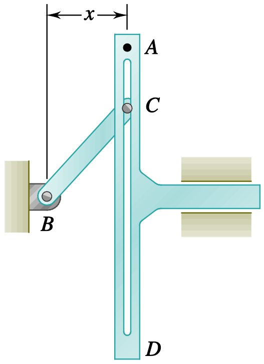
$$x(t) = -\frac{4}{15}(3.75)t^{5/2} + 20t$$
$$x(t) = -t^{5/2} + 20t$$

$x(t) = -t^{5/2} + 20t$ ◀

Find the distance when t = 4 seconds to find when it stops,

$$x(t) = -4^{5/2} + 20(4) = 48$$

$x = 48$ m ◀



PROBLEM 11.13

A Scotch yoke is a mechanism that transforms the circular motion of a crank into the reciprocating motion of a shaft (or vice versa). It has been used in a number of different internal combustion engines and in control valves. In the Scotch yoke shown, the acceleration of Point A is defined by the relation $a = -1.8\sin kt$, where a and t are expressed in m/s² and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0$ and $v = 0.6$ m/s when $t = 0$, determine the velocity and position of Point A when $t =$ 0.5 s.

SOLUTION

Acceleration: $a = -1.8\sin kt \text{ m/s}^2$

Given: $v_0 = 0.6 \text{ m/s}, \quad x_0 = 0, \quad k = 3 \text{ rad/s}$

Velocity: $a = \dfrac{dv}{dt} \Rightarrow \quad dv = adt \quad \Rightarrow \displaystyle\int_{v_0}^{v} dv = \int_0^t adt$

$$v - v_0 = \int_0^t a\,dt = -1.8\int_0^t \sin kt\,dt = \frac{1.8}{k}\cos kt\Big|_0^t$$

$$v - 0.6 = \frac{1.8}{3}(\cos kt - 1) = 0.6\cos kt - 0.6$$

$$v = 0.6\cos kt \text{ m/s}$$

Position: $v = \dfrac{dx}{dt} \Rightarrow \quad dx = vdt \quad \Rightarrow \displaystyle\int_{x_0}^{x} dx = \int_0^t vdt$

$$x - x_0 = \int_0^t v\,dt = 0.6\int_0^t \cos kt\,dt = \frac{0.6}{k}\sin kt\Big|_0^t$$

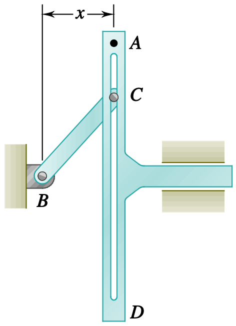
$$x - 0 = \frac{0.6}{3}(\sin kt - 0) = 0.2\sin kt$$

$$x = 0.2\sin kt \text{ m}$$

When t = 0.5 s, $kt = (3)(0.5) = 1.5 \text{ rad}$

$v = 0.6\cos 1.5 = 0.0424 \text{ m/s}$ $\qquad v = 42.4 \text{ mm/s}$ ◀

$x = 0.2\sin 1.5 = 0.1995 \text{ m}$ $\qquad x = 199.5 \text{ mm}$ ◀



PROBLEM 11.14

For the scotch yoke mechanism shown, the acceleration of Point A is defined by the relation $a = -1.08\sin kt - 1.44\cos kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0.16$ m and $v = 0.36$ m/s when $t = 0$, determine the velocity and position of Point A when $t = 0.5$ s.

SOLUTION

Acceleration: $a = -1.08\sin kt - 1.44\cos kt \text{ m/s}^2$

Given: $v_0 = 0.36 \text{ m/s}, \quad x_0 = 0.16, \quad k = 3 \text{ rad/s}$

Velocity: $$a = \frac{dv}{dt} \Rightarrow \quad dv = a\,dt \quad \Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt$$

Integrate: $$v - v_0 = -1.08\int_0^t \sin kt\,dt - 1.44\int_0^t \cos kt\,dt$$

$$v - 0.36 = \frac{1.08}{k}\cos kt\Big|_0^t - \frac{1.44}{k}\sin kt\Big|_0^t$$

$$= \frac{1.08}{3}(\cos 3t - 1) - \frac{1.44}{3}(\sin 3t - 0)$$

$$= 0.36\cos 3t - 0.36 - 0.48\sin 3t$$

$$v = 0.36\cos 3t - 0.48\sin 3t \text{ m/s}$$

Evaluate at $t = 0.5$ s $\quad = 0.36\cos 1.5 - 0.36 - 0.48\sin 1.5 \qquad v = -453$ mm/s ◀

Position: $$v = \frac{dx}{dt} \Rightarrow \quad dx = v\,dt \quad \Rightarrow \int_{x_0}^{x} dx = \int_0^t v\,dt$$

$$x - x_0 = \int_0^t v\,dt = 0.36\int_0^t \cos kt\,dt - 0.48\int_0^t \sin kt\,dt$$

$$x - 0.16 = \frac{0.36}{k}\sin kt\Big|_0^t + \frac{0.48}{k}\cos kt\Big|_0^t$$

$$= \frac{0.36}{3}(\sin 3t - 0) + \frac{0.48}{3}(\cos 3t - 1)$$

$$= 0.12\sin 3t + 0.16\cos 3t - 0.16$$

$$x = 0.12\sin 3t + 0.16\cos 3t \text{ m}$$

Evaluate at $t = 0.5$ s $\quad x = 0.12\sin 1.5 + 0.16\cos 1.5 = 0.1310$ m $\qquad x = 131.0$ mm ◀

PROBLEM 11.15

A piece of electronic equipment that is surrounded by packing material is dropped so that it hits the ground with a speed of 4 m/s. After contact the equipment experiences an acceleration of $a = -kx$, where k is a constant and x is the compression of the packing material. If the packing material experiences a maximum compression of 15 mm, determine the maximum acceleration of the equipment.

SOLUTION

$$a = \frac{v\,dv}{dx} = -kx$$

Separate and integrate.

$$\int_{v_0}^{v_f} v\,dv = -\int_0^{x_f} kx\,dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -\frac{1}{2}kx^2\Big|_0^{x_f} = -\frac{1}{2}kx_f^2$$

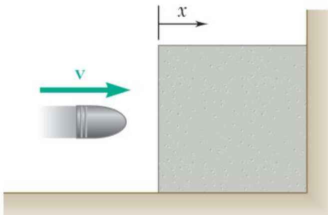
Use $v_0 = 4$ m/s, $x_f = 0.015$ m, and $v_f = 0$. Solve for k.

$$0 - \frac{1}{2}(4)^2 = -\frac{1}{2}k(0.015)^2 \qquad k = 71{,}111\ \text{s}^{-2}$$

Maximum acceleration.

$$a_{\max} = -kx_{\max}: \quad (-71{,}111)(0.015) = -1{,}067\ \text{m/s}^2$$

$$a = 1{,}067\ \text{m/s}^2 \uparrow \blacktriangleleft$$



PROBLEM 11.16

A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_0 = 1000$ ft/s and travels 3 in. before coming to rest. Assuming that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where v is expressed in ft/s and x is in feet, determine (*a*) the initial acceleration of the projectile, (*b*) the time required for the projectile to penetrate 2.5 in. into the resisting medium.

SOLUTION

When $x = \frac{3}{12}$ ft, $v = 0$:
$$0 = (1000 \text{ ft/s}) - k\left(\frac{3}{12} \text{ ft}\right)$$

Or
$$k = 4000 \frac{1}{\text{s}}$$

(a) We have
$$v = v_0 - kx, \quad a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv$$

or
$$a = -k(v_0 - kx)$$

At $t = 0$:
$$a = 4000 \frac{1}{\text{s}}(1000 \text{ ft/s} - 0)$$

$a_0 = -4.00 \times 10^6 \text{ ft/s}^2$ ◀

(*b*) We have
$$\frac{dx}{dt} = v = v_0 - kx$$

At $t = 0, x = 0$:
$$\int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$$

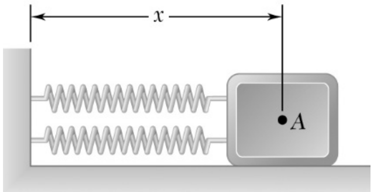
or
$$-\frac{1}{k}[\ln(v_0 - kx)]_0^x = t$$

or
$$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - kx}\right) = \frac{1}{k}\ln\left(\frac{1}{1 - \frac{k}{v_0}x}\right)$$

When $x = 2.5$ in.:
$$t = \frac{1}{4000 \frac{1}{\text{s}}}\ln\left[\frac{1}{1 - \frac{4000 \text{ 1/s}}{1000 \text{ ft/s}}\left(\frac{2.5}{12} \text{ ft}\right)}\right]$$

Or

$t = 4.48 \times 10^{-4}$ s ◀



PROBLEM 11.17

Point A oscillates with an acceleration $a = 100(0.25 - x)$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that the system starts at time $t = 0$ with $v = 0$ and $x = 0.2$ m, determine the position and the velocity of A when $t = 0.2$ s.

SOLUTION

a is a function of x: $$a = 100(0.25 - x)\,\text{m/s}^2$$

Use $v\,dv = a\,dx = 100(0.25 - x)dx$ with limits $v = 0$ when $x = 0.2$ m

$$\int_0^v v\,dv = \int_{0.2}^x 100(0.25 - x)dx$$

$$\frac{1}{2}v^2 - 0 = -\frac{1}{2}(100)(0.25 - x)^2\Big|_{0.2}^{x}$$

$$= -50\,(0.25 - x)^2 + 0.125$$

So $$v^2 = 0.25 - 100(0.25 - x)^2 \quad \text{or} \quad v = \pm 0.5\sqrt{1 - 400(0.25 - x)^2}$$

Use $$dx = v\,dt \quad \text{or} \quad dt = \frac{dx}{v} = \frac{dx}{\pm 0.5\sqrt{1 - 400(0.25 - x)^2}}$$

Integrate: $$\int_0^t dt = \pm\int_{0.2}^x \frac{dx}{0.5\sqrt{1 - 400(0.25 - x)^2}}$$

Let $u = 20(0.25 - x)$; when $x = 0.2$ $u = 1$ and $du = -20dx$

So $$t = \mp\int_1^u \frac{du}{10\sqrt{1 - u^2}} = \mp\frac{1}{10}\sin^{-1}u\Big|_1^u = \mp\frac{1}{10}\left(\sin^{-1}u - \frac{\pi}{2}\right)$$

Solve for u. $$\sin^{-1}u = \frac{\pi}{2} \mp 10t$$

$$u = \sin\left(\frac{\pi}{2} \mp 10t\right) = \cos(\pm 10t) = \cos 10t$$

PROBLEM 11.17 (CONTINUED)

$$u = \cos 10t = 20(0.25 - x)$$

Solve for x and v.

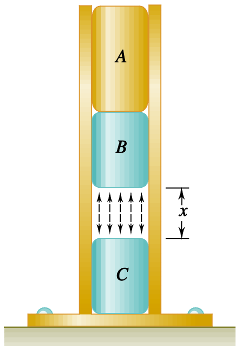
$$x = 0.25 - \frac{1}{20}\cos 10t$$

$$v = \frac{1}{2}\sin 10t$$

Evaluate at $t = 0.2$ s.

$$x = 0.25 - \frac{1}{20}\cos((10)(0.2))$$ $x = 0.271$ m ◀

$$v = \frac{1}{2}\sin((10)(0.2))$$ $v = 0.455$ m/s ◀



PROBLEM 11.18

A brass (nonmagnetic) block A and a steel magnet B are in equilibrium in a brass tube under the magnetic repelling force of another steel magnet C located at a distance $x = 0.004$ m from B. The force is inversely proportional to the square of the distance between B and C. If block A is suddenly removed, the acceleration of block B is $a = -9.81 + k/x^2$, where a and x are expressed in m/s^2 and m, respectively, and $k = 4\times10^{-4}$ m^3/s^2. Determine the maximum velocity and acceleration of B.

SOLUTION

The maximum velocity occurs when $a = 0$. $\qquad 0 = -9.81 + \dfrac{k}{x_m^2}$

$$x_m^2 = \frac{k}{9.81} = \frac{4\times10^{-4}}{9.81} = 40.775\times10^{-6}\,\text{m}^2 \qquad x_m = 0.0063855\ \text{m}$$

The acceleration is given as a function of x.

$$v\frac{dv}{dx} = a = -9.81 + \frac{k}{x^2}$$

Separate variables and integrate:

$$vdv = -9.81dx + \frac{k\,dx}{x^2}$$

$$\int_0^v vdv = -9.81\int_{x_0}^x dx + k\int_{x_0}^x \frac{dx}{x^2}$$

$$\frac{1}{2}v^2 = -9.81(x - x_0) - k\left(\frac{1}{x} - \frac{1}{x_0}\right)$$

$$\frac{1}{2}v_m^2 = -9.81(x_m - x_0) - k\left(\frac{1}{x_m} - \frac{1}{x_0}\right)$$

$$= -9.81(0.0063855 - 0.004) - (4\times10^{-4})\left(\frac{1}{0.0063855} - \frac{1}{0.004}\right)$$

$$= -0.023402 + 0.037358 = 0.013956\ \text{m}^2/\text{s}^2$$

Maximum velocity: $\qquad v_m = 0.1671$ m/s $\qquad\qquad v_m = 167.1$ mm/s $\uparrow$ ◀

The maximum acceleration occurs when x is smallest, that is, $x = 0.004$ m.

$$a_m = -9.81 + \frac{4\times10^{-4}}{(0.004)^2} \qquad\qquad a_m = 15.19\ \text{m/s}^2 \uparrow \blacktriangleleft$$

PROBLEM 11.19

Based on experimental observations, the acceleration of a particle is defined by the relation $a = -(0.1 + \sin x/b)$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that $b = 0.8$ m and that $v = 1$ m/s when $x = 0$, determine (*a*) the velocity of the particle when $x = -1$ m, (*b*) the position where the velocity is maximum, (*c*) the maximum velocity.

SOLUTION

We have
$$v\frac{dv}{dx} = a = -\left(0.1 + \sin\frac{x}{0.8}\right)$$

When $x = 0$, $v = 1$ m/s:
$$\int_1^v v\,dv = \int_0^x -\left(0.1 + \sin\frac{x}{0.8}\right)dx$$

or
$$\frac{1}{2}(v^2 - 1) = -\left[0.1x - 0.8\cos\frac{x}{0.8}\right]_0^x$$

or
$$\frac{1}{2}v^2 = -0.1x + 0.8\cos\frac{x}{0.8} - 0.3$$

(*a*) When $x = -1$ m:
$$\frac{1}{2}v^2 = -0.1(-1) + 0.8\cos\frac{-1}{0.8} - 0.3$$

or $v = \pm 0.323$ m/s ◀

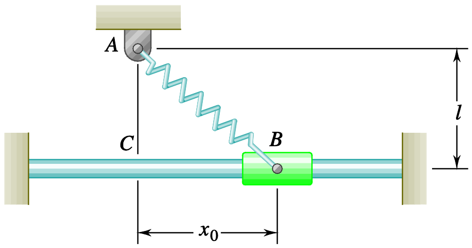
(*b*) When $v = v_{max}$, $a = 0$: $-\left(0.1 + \sin\dfrac{x}{0.8}\right) = 0$

or $x = -0.080\,134$ m $\qquad x = -0.0801$ m ◀

(*c*) When $x = -0.080\,134$ m:

$$\frac{1}{2}v_{max}^2 = -0.1(-0.080\,134) + 0.8\cos\frac{-0.080\,134}{0.8} - 0.3$$
$$= 0.504 \text{ m}^2/\text{s}^2$$

or $v_{max} = 1.004$ m/s ◀



PROBLEM 11.20

A spring AB is attached to a support at A and to a collar. The unstretched length of the spring is l. Knowing that the collar is released from rest at $x = x_0$ and has an acceleration defined by the relation $a = -100(x - lx/\sqrt{l^2 + x^2})$, determine the velocity of the collar as it passes through Point C.

SOLUTION

Since a is function of x,

$$a = v\frac{dv}{dx} = -100\left(x - \frac{lx}{\sqrt{l^2 + x^2}}\right)$$

Separate variables and integrate:

$$\int_{v_0}^{v_f} v dv = -100\int_{x_0}^{0}\left(x - \frac{lx}{\sqrt{l^2 + x^2}}\right)dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -100\left(\frac{x^2}{2} - l\sqrt{l^2 + x^2}\right)\Bigg|_{x_0}^{0}$$

$$\frac{1}{2}v_f^2 - 0 = -100\left(-\frac{x_0^2}{2} - l^2 + l\sqrt{l^2 + x_0^2}\right)$$

$$\frac{1}{2}v_f^2 = \frac{100}{2}(-l^2 + x_0^2 - l^2 - 2l\sqrt{l^2 + x_0^2})$$

$$= \frac{100}{2}(\sqrt{l^2 + x_0^2} - l)^2$$

$$v_f = 10(\sqrt{l^2 + x_0^2} - l) \blacktriangleleft$$

PROBLEM 11.21

The acceleration of a particle is defined by the relation $a = k(1 - e^{-x})$, where k is a constant. Knowing that the velocity of the particle is $v = +9$ m/s when $x = -3$ m and that the particle comes to rest at the origin, determine (*a*) the value of k, (*b*) the velocity of the particle when $x = -2$ m.

SOLUTION

Acceleration: $a = k(1 - e^{-x})$

Given: at $x = -3$ m, $v = 9$ m/s

at $x = 0$ m, $v = 0$ m/s

$$adx = vdv$$

$$\mathrm{k}(1 - e^{-x})dx = vdv$$

Integrate using x = −3 m and v = 9 m/s as the lower limits of the integrals

$$\int_{-3}^{x} \mathrm{k}(1 - e^{-x})dx = \int_{9}^{v} vdv$$

$$k(x + e^{-x})\Big|_{-3}^{x} = \frac{1}{2}v^2\Big|_{9}^{v}$$

Velocity: $$k\left(x + e^{-x} - (-3 + e^3)\right) = \frac{1}{2}v^2 - \frac{1}{2}(9)^2 \qquad (1)$$

(*a*) Now substitute v = 0 m/s and x = 0 m into (1) and solve for k

$$k\left(0 + e^{-0} - (-3 + e^3)\right) = \frac{1}{2}0^2 - \frac{1}{2}(9)^2$$

$k = 2.52$ m^2/s^2 ◀

(*b*) Find velocity when x = −2 m using the equation (1) and the value of k

$$2.518\left(-2 + e^2 - (-3 + e^3)\right) = \frac{1}{2}v^2 - \frac{1}{2}(9)^2$$

$v = 4.70$ m/s ◀

PROBLEM 11.22

Starting from $x = 0$ with no initial velocity, a particle is given an acceleration, $a = 0.8\sqrt{v^2 + 49}$ where a and v are expressed in ft/s^2 and ft/s, respectively. Determine (*a*) the position of the particle when $v = 24$ ft/s, (*b*) the speed of the particle when $x = 40$ ft.

SOLUTION

$$a = 0.8\sqrt{v^2 + 49}$$

$$v\,dv = a\,dx \qquad dx = \frac{v\,dv}{a} = \frac{v\,dv}{0.8\sqrt{v^2 + 49}}$$

Integrating using $x = 0$ when $v = 0$,

$$\int_0^x dx = \frac{1}{0.8}\int_0^v \frac{v\,dv}{\sqrt{v^2 + 49}} = \frac{1}{0.8}\sqrt{v^2 + 49}\,\Big]_0^v$$

$$x = 1.25\left(\sqrt{v^2 + 49} - 7\right) \qquad (1)$$

(a) When $v = 24$ ft/s,

$$x = 1.25\left(\sqrt{24^2 + 49} - 7\right) \qquad x = 22.5 \text{ ft} \blacktriangleleft$$

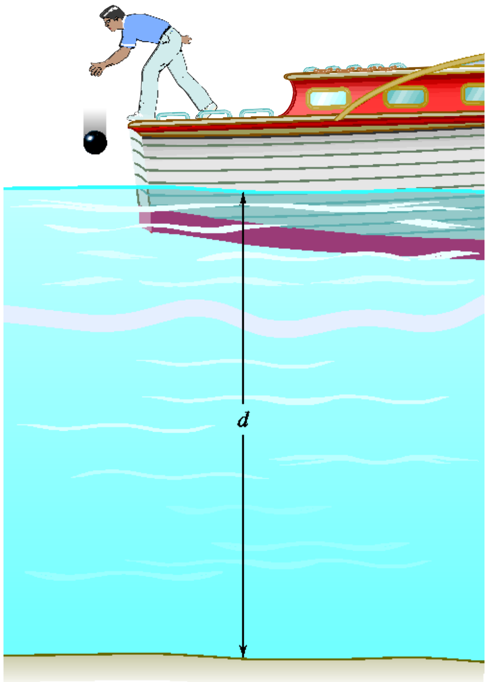
(*b*) Solving equation (1) for v^2,

$$\sqrt{v^2 + 49} = 7 + 0.8x$$

$$v^2 = (7 + 0.8x)^2 - 49$$

When $x = 40$ ft,

$$v^2 = \left[7 + (0.8)(40)\right]^2 - 49 = 1472 \text{ ft}^2/\text{s}^2 \qquad v = 38.4 \text{ ft/s} \blacktriangleleft$$



PROBLEM 11.23

A ball is dropped from a boat so that it strikes the surface of a lake with a speed of 16.5 ft/s. While in the water the ball experiences an acceleration of $a = 10 - 0.8v$, where a and v are expressed in ft/s^2 and ft/s, respectively. Knowing the ball takes 3 s to reach the bottom of the lake, determine (*a*) the depth of the lake, (*b*) the speed of the ball when it hits the bottom of the lake.

SOLUTION

$$a = \frac{dv}{dt} = 10 - 0.8v$$

Separate and integrate:

$$\int_{v_0}^{v} \frac{dv}{10 - 0.8v} = \int_0^t dt$$

$$-\frac{1}{0.8}\ln(10 - 0.8v)\Big|_{v_0}^{v} = t$$

$$\ln\left(\frac{10 - 0.8v}{10 - 0.8v_0}\right) = -0.8t$$

$$10 - 0.8v = (10 - 0.8v_0)e^{-0.8t}$$

or

$$0.8v = 10 - (10 - 0.8v_0)e^{-0.8t}$$

$$v = 12.5 - (12.5 - v_0)e^{-0.8t}$$

With $v_0 = 16.5\,\text{ft/s}$

$$v = 12.5 + 4e^{-0.8t}$$

Integrate to determine x as a function of t.

$$v = \frac{dx}{dt} = 12.5 + 4e^{-0.8t}$$

$$\int_0^x dx = \int_0^t (12.5 + 4e^{-0.8t})dt$$

PROBLEM 11.23 (CONTINUED)

$$x = 12.5t - 5e^{-0.8t}\Big|_0^t = 12.5t - 5e^{-0.8t} + 5$$

(*a*) At $t = 35$ s,

$$x = 12.5(3) - 5e^{-2.4} + 5 = 42.046 \text{ ft}$$

$x = 42.0$ ft ◀

(*b*) $v = 12.5 + 4e^{-2.4} = 12.863$ ft/s

$v = 12.86$ ft/s ◀

PROBLEM 11.24

The acceleration of a particle is defined by the relation $a = -k\sqrt{v}$, where k is a constant. Knowing that $x = 0$ and $v = 81$ m/s at $t = 0$ and that $v = 36$ m/s when $x = 18$ m, determine (*a*) the velocity of the particle when $x = 20$ m, (*b*) the time required for the particle to come to rest.

SOLUTION

(*a*) We have $$v\frac{dv}{dx} = a = -k\sqrt{v}$$

so that $$\sqrt{v}\,dv = -k\,dx$$

When $x = 0, v = 81$ m/s: $$\int_{81}^{v} \sqrt{v}\,dv = \int_0^x -k\,dx$$

or $$\frac{2}{3}[v^{3/2}]_{81}^{v} = -kx$$

or $$\frac{2}{3}[v^{3/2} - 729] = -kx$$

When $x = 18$ m, $v = 36$ m/s: $$\frac{2}{3}(36^{3/2} - 729) = -k(18)$$

or $$k = 19\sqrt{\text{m/s}^2}$$

Finally

When $x = 20$ m: $$\frac{2}{3}(v^{3/2} - 729) = -19(20)$$

or $$v^{3/2} = 159$$

$v = 29.3$ m/s ◀

(*b*) We have $$\frac{dv}{dt} = a = -19\sqrt{v}$$

At $t = 0, v = 81$ m/s: $$\int_{81}^{v} \frac{dv}{\sqrt{v}} = \int_0^t -19\,dt$$

or $$2[\sqrt{v}]_{81}^{v} = -19t$$

or $$2(\sqrt{v} - 9) = -19t$$

When $v = 0$: $$2(-9) = -19t$$

or $t = 0.947$ s ◀

PROBLEM 11.25

The acceleration of a particle is defined by the relation $a = -kv^{2.5}$, where k is a constant. The particle starts at $x = 0$ with a velocity of 16 mm/s, and when $x = 6$ mm the velocity is observed to be 4 mm/s. Determine (*a*) the velocity of the particle when $x = 5$ mm, (*b*) the time at which the velocity of the particle is 9 mm/s.

SOLUTION

Acceleration: $a = -kv^{2.5}$

Given: at t = 0, $x = 0$ mm, $v = 16$ mm/s

at $x = 6$ mm, $v = 4$ mm/s

$$adx = vdv$$

$$-kv^{2.5}dx = vdv$$

Separate variables $-kdx = v^{-3/2}dv$

Integrate using x = −0 m and v = 16 mm/s as the lower limits of the integrals

$$\int_0^x -\text{k}dx = \int_{16}^v v^{-3/2}dv$$

$$-kx\Big|_0^x = -2v^{-1/2}\Big|_{16}^v$$

Velocity and position: $kx = 2v^{-1/2} - \frac{1}{2}$ (1)

Now substitute v = 4 mm/s and x = 6 mm into (1) and solve for k

$$k(6) = 4^{-1/2} - \frac{1}{2}$$

$$k = 0.0833 \text{ mm}^{-3/2}s^{1/2}$$

(*a*) Find velocity when x = 5 mm using the equation (1) and the value of k

$$k(5) = 2v^{-1/2} - \frac{1}{2}$$

$v = 4.76$ mm/s ◀

(*b*) $a = \frac{dv}{dt}$ or $adt = dv$

$$a = \frac{dv}{dt} \text{ or } adt = dv$$

Separate variables $-kdt = v^{-2.5}dv$

PROBLEM 11.25 (CONTINUED)

Integrate using t = 0 and v = 16 mm/s as the lower limits of the integrals

$$\int_0^t -\mathrm{k}dt = \int_{16}^v v^{-5/2} dv$$

$$-kt\Big|_0^t = -\frac{2}{3} v^{-3/2}\Big|_{16}^v$$

Velocity and time: $$kt = \frac{2}{3} v^{-3/2} - \frac{1}{96} \qquad (2)$$

Find time when v = 9 mm/s using the equation (2) and the value of k

$$kt = \frac{2}{3}(9)^{-3/2} - \frac{1}{96}$$

$t = 0.171$ s ◀

PROBLEM 11.26

A human powered vehicle (HPV) team wants to model the acceleration during the 260 *m* sprint race (the first 60 *m* is called a flying start) using $a = A - Cv^2$, where a is acceleration in m/s^2 and v is the velocity in m/s. From wind tunnel testing, they found that C = 0.0012 m^{-1}. Knowing that the cyclist is going 100 km/h at the 260 meter mark, what is the value of A?

SOLUTION

Acceleration: $a = A - Cv^2 \text{ m/s}^2$

Given: $C = 0.0012 \text{ m}^{-1}, \ v_f = 100 \text{ km/hr when } x_f = 260 \text{ m}$

Note: 100 km/hr = 27.78 m/s

$$adx = vdv$$

$$(A - Cv^2)dx = vdv$$

Separate variables $$dx = \frac{vdv}{A - Cv^2}$$

Integrate starting from rest and traveling a distance x_f with a final velocity v_f.

$$\int_0^{x_f} dx = \int_0^{v_f} \frac{vdv}{A - Cv^2}$$

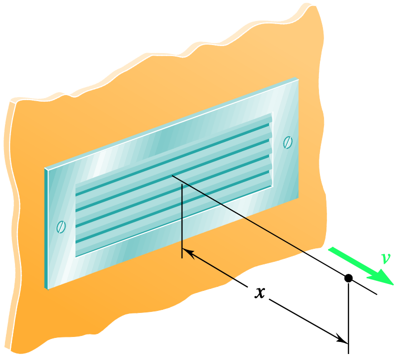
$$x\Big|_0^{x_f} = -\frac{1}{2C}\ln\left(A - Cv^2\right)\Big|_0^{v_f}$$

$$x_f = -\frac{1}{2C}\ln\left(A - v_f^2\right) + \frac{1}{2C}\ln(A)$$

$$x_f = \frac{1}{2C}\ln\left(\frac{A}{A - Cv_f^2}\right)$$

Next solve for A $$A = -\frac{Cv_f^2 e^{2Cx_f}}{1 - e^{2Cx_f}}$$

Now substitute values of C, x_f and v_f and solve for A $\qquad A = 1.995 \text{ m/s}^2$ ◀



PROBLEM 11.27

Experimental data indicate that in a region downstream of a given louvered supply vent the velocity of the emitted air is defined by $v = 0.18v_0/x$, where v and x are expressed in m/s and meters, respectively, and v_0 is the initial discharge velocity of the air. For $v_0 = 3.6$ m/s, determine (*a*) the acceleration of the air at $x = 2$ m, (*b*) the time required for the air to flow from $x = 1$ to $x = 3$ m.

SOLUTION

(*a*) We have
$$a = v\frac{dv}{dx}$$
$$= \frac{0.18v_0}{x}\frac{d}{dx}\left(\frac{0.18v_0}{x}\right)$$
$$= -\frac{0.0324v_0^2}{x^3}$$

When $x = 2$ m:
$$a = -\frac{0.0324(3.6)^2}{(2)^3}$$

or
$$a = -0.0525 \text{ m/s}^2 \blacktriangleleft$$

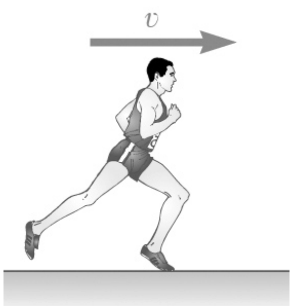
(*b*) We have
$$\frac{dx}{dt} = v = \frac{0.18v_0}{x}$$

From $x = 1$ m to $x = 3$ m:
$$\int_1^3 x\,dx = \int_{t_1}^{t_3} 0.18v_0\,dt$$

or
$$\left[\frac{1}{2}x^2\right]_1^3 = 0.18v_0(t_3 - t_1)$$

or
$$(t_3 - t_1) = \frac{\frac{1}{2}(9-1)}{0.18(3.6)}$$

or
$$t_3 - t_1 = 6.17 \text{ s} \blacktriangleleft$$



PROBLEM 11.28

Based on observations, the speed of a jogger can be approximated by the relation $v = 7.5(1-0.04x)^{0.3}$, where v and x are expressed in km/h and kilometers, respectively. Knowing that $x = 0$ at $t = 0$, determine (*a*) the distance the jogger has run when $t = 1$ h, (*b*) the jogger's acceleration in m/s^2 at $t = 0$, (*c*) the time required for the jogger to run 6 km.

SOLUTION

Given: $v = 7.5(1-0.04x)^{0.3}$ with units km and km/h

(a) Distance at $t = 1$ hr.

$$\text{Using } dx = v\,dt, \text{ we get } dt = \frac{dx}{v} = \frac{dx}{7.5(1-0.04x)^{0.3}}$$

Integrating, using $t = 0$ when $x = 0$,

$$\int_0^t dt = \frac{1}{7.5}\int_0^x \frac{dx}{(1-0.04)^{0.3}} \quad \text{or} \quad [t]_0^t = \frac{1}{(7.5)}\cdot\frac{-1}{(0.7)(0.04)}\left\{1-0.04x^{0.7}\right\}\Big|_0^x$$

$$t = 4.7619\left\{1-(1-0.04x)^{0.7}\right\} \tag{1}$$

Solving for x, $\quad x = 25\left\{1-(1-0.210t)^{1/0.7}\right\}$

When t = 1 h $\quad x = 25\left\{1-\left[1-(0.210)(1)\right]^{1/0.7}\right\}$ $\quad x = 7.15 \text{ km}$ ◀

(b) Acceleration when $t = 0$.

$$\frac{dv}{dx} = (7.5)(0.3)(-0.04)(1-0.04x)^{-0.7} = -0.0900(1-0.04x)^{-0.7}$$

When $t = 0$ and $x = 0$, $\quad v = 7.5$ km/h, $\quad \frac{dv}{dx} - 0.0900 \text{ h}^{-1}$

$$a = v\frac{dv}{dx} = (7.5)(-0.0900) = -0.675 \text{ km/h}^2$$

PROBLEM 11.28 (Continued)

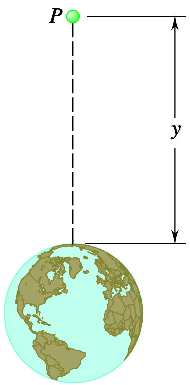
$$= -\frac{(0.675)(1000)}{(3600)^2} \text{ m/s}^2 \qquad a = -52.1 \times 10^{-6} \text{ m/s}^2 ◀$$

(c) Time to run 6 km.

Using $x = 6$ km in equation (1),

$$t = 4.7619\left\{1 - \left[1 - (0.04)(6)\right]^{0.7}\right\} = 0.8323 \text{ h}$$

$$t = 49.9 \text{ min} ◀$$



PROBLEM 11.29

The acceleration due to gravity at an altitude y above the surface of the earth can be expressed as

$$a = \frac{-32.2}{[1+(y/20.9\times 10^6)]^2}$$

where a and y are expressed in ft/s^2 and feet, respectively. Using this expression, compute the height reached by a projectile fired vertically upward from the surface of the earth if its initial velocity is (*a*) 1800 ft/s, (*b*) 3000 ft/s, (*c*) 36,700 ft/s.

SOLUTION

We have
$$v\frac{dv}{dy} = a = -\frac{32.2}{\left(1+\frac{y}{20.9\times 10^6}\right)^2}$$

When
$$y = 0, \qquad v = v_0$$

provided that v does reduce to zero,
$$y = y_{\max}, \quad v = 0$$

Then
$$\int_{v_0}^{0} v\,dv = \int_0^{y_{\max}} \frac{-32.2}{\left(1+\frac{y}{20.9\times 10^6}\right)^2}\,dy$$

or
$$-\frac{1}{2}v_0^2 = -32.2\left[-20.9\times 10^6\,\frac{1}{1+\frac{y}{20.9\times 10^6}}\right]_0^{y_{\max}}$$

or
$$v_0^2 = 1345.96\times 10^6\left(1-\frac{1}{1+\frac{y_{\max}}{20.9\times 10^6}}\right)$$

or
$$y_{\max} = \frac{v_0^2}{64.4-\frac{v_0^2}{20.9\times 10^6}}$$

(*a*) $v_0 = 1800$ ft/s:
$$y_{\max} = \frac{(1800)^2}{64.4-\frac{(1800)^2}{20.9\times 10^6}}$$

or
$$y_{\max} = 50.4\times 10^3 \text{ ft} \blacktriangleleft$$

PROBLEM 11.29 (Continued)

(*b*) $v_0 = 3000$ ft/s:

$$y_{\max} = \frac{(3000)^2}{64.4 - \frac{(3000)^2}{20.9 \times 10^6}}$$

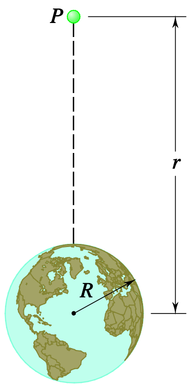
or $$y_{\max} = 140.7 \times 10^3 \text{ ft} \blacktriangleleft$$

(*c*) $v_0 = 36{,}700$ ft/s:

$$y_{\max} = \frac{(36{,}700)^2}{64.4 - \frac{(36{,}700)^2}{20.9 \times 10^6}} = -3.03 \times 10^{10} \text{ ft}$$

This solution is invalid since the velocity does not reduce to zero. The velocity 36,700 ft/s is above the escape velocity v_R from the earth. For v_R and above.

$$y_{\max} \longrightarrow \infty \blacktriangleleft$$



PROBLEM 11.30

The acceleration due to gravity of a particle falling toward the earth is $a = -gR^2/r^2$, where r is the distance from the *center* of the earth to the particle, R is the radius of the earth, and g is the acceleration due to gravity at the surface of the earth. If $R = 3960$ mi, calculate the *escape velocity,* that is, the minimum velocity with which a particle must be projected vertically upward from the surface of the earth if it is not to return to the earth. (*Hint:* $v = 0$ for $r = \infty$.)

SOLUTION

We have
$$v\frac{dv}{dr} = a = -\frac{gR^2}{r^2}$$

When
$$r = R, \quad v = v_e$$
$$r = \infty, \quad v = 0$$

then
$$\int_{v_e}^{0} v\,dv = \int_R^\infty -\frac{gR^2}{r^2}dr$$

or
$$-\frac{1}{2}v_e^2 = gR^2\left[\frac{1}{r}\right]_R^\infty$$

or
$$v_e = \sqrt{2gR}$$
$$= \left(2\times 32.2\text{ ft/s}^2 \times 3960\text{ mi}\times\frac{5280\text{ ft}}{1\text{ mi}}\right)^{1/2}$$

or
$$v_e = 36.7\times 10^3\text{ ft/s} \blacktriangleleft$$

PROBLEM 11.31

The velocity of a particle is $v = v_0[1 - \sin(\pi t/T)]$. Knowing that the particle starts from the origin with an initial velocity v_0, determine (*a*) its position and its acceleration at $t = 3T$, (*b*) its average velocity during the interval $t = 0$ to $t = T$.

SOLUTION

(*a*) We have
$$\frac{dx}{dt} = v = v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]$$

At $t = 0, x = 0$:
$$\int_0^x dx = \int_0^t v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]dt$$

$$x = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right)\right]_0^t = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right) - \frac{T}{\pi}\right] \qquad (1)$$

At $t = 3T$:
$$x_{3T} = v_0\left[3T + \frac{T}{\pi}\cos\left(\frac{\pi \times 3T}{T}\right) - \frac{T}{\pi}\right] = v_0\left(3T - \frac{2T}{\pi}\right) \qquad x_{3T} = 2.36\, v_0 T \blacktriangleleft$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]\right\} = -v_0\frac{\pi}{T}\cos\frac{\pi t}{T}$$

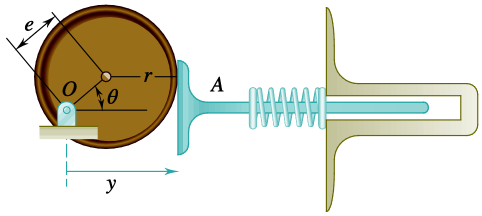
At $t = 3T$:
$$a_{3T} = -v_0\frac{\pi}{T}\cos\frac{\pi \times 3T}{T} \qquad a_{3T} = \frac{\pi v_0}{T} \blacktriangleleft$$

(*b*) Using Eq. (1)

At $t = 0$:
$$x_0 = v_0\left[0 + \frac{T}{\pi}\cos(0) - \frac{T}{\pi}\right] = 0$$

At $t = T$:
$$x_T = v_0\left[T + \frac{T}{\pi}\cos\left(\frac{\pi T}{T}\right) - \frac{T}{\pi}\right] = v_0\left(T - \frac{2T}{\pi}\right) = 0.363 v_0 T$$

Now
$$v_{\text{ave}} = \frac{x_T - x_0}{\Delta t} = \frac{0.363 v_0 T - 0}{T - 0} \qquad v_{\text{ave}} = 0.363 v_0 \blacktriangleleft$$



Problem 11.32

An eccentric circular cam, which serves a similar function as the Scotch yoke mechanism in Problem 11.13, is used in conjunction with a flat face follower to control motion in pumps and in steam engine valves. Knowing that the eccentricity is denoted by e, the maximum range of the displacement of the follower is d_{max}, and the maximum velocity of the follower is v_{max}, determine the displacement, velocity, and acceleration of the follower.

SOLUTION

Constraint: $$y = r + e\cos\theta \qquad (1)$$

Differentiate: $$\dot{y} = -e\dot{\theta}\sin\theta \qquad (2)$$

Differentiate again: $$\ddot{y} = -e\ddot{\theta}\sin\theta - e\dot{\theta}^2\cos\theta \qquad (3)$$

y_{max} occurs when $\cos\theta = 1$ and y_{min} occurs when $\cos\theta = -1$

$$d_{max} = y_{max} - y_{min}$$
$$d_{max} = r + e - (r - e)$$
$$d_{max} = 2e$$
$$e = \frac{d_{max}}{2} \qquad (4)$$

Substitute (4) into (1) to get Position $$y = r + \frac{d_{max}}{2}\cos\theta \blacktriangleleft$$

Max Velocity occurs when $\sin\theta = \pm 1$

$$v_{max} = \mp e\dot{\theta}$$

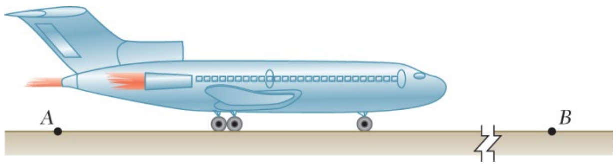
$$\dot{\theta} = \frac{v_{max}}{\mp e} \qquad (5)$$

Substitute (5) into (2) to get velocity and assume cw rotation. $$\dot{y} = -v_{max}\sin\theta$$

Substitute (4) and (5) into (3)

$$\ddot{y} = -\frac{d_{max}}{2}\ddot{\theta}\sin\theta - \frac{d_{max}}{2}\left(\frac{4v_{max}^2}{d_{max}^2}\right)\cos\theta$$

<u>Acceleration:</u> $$\ddot{y} = -\frac{d_{max}}{2}\ddot{\theta}\sin\theta - \frac{2v_{max}^2}{d_{max}}\cos\theta \blacktriangleleft$$



PROBLEM 11.33

An airplane begins its take-off run at A with zero velocity and a constant acceleration a. Knowing that it becomes airborne 30 s later at B with a take-off velocity of 270 km/h, determine (a) the acceleration a, (b) distance AB.

SOLUTION

Since it is constant acceleration you can find the acceleration from the distance over time

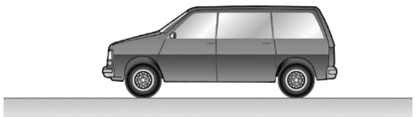
$$a = dv / dt$$
$$270\,\text{km} = 75\,\text{m/s}$$
$$a = \frac{75\,\text{m/s}}{30\,\text{s}} = 2.5\,\text{m/s}^2$$

$$a = 2.50\ \ \text{m/s}^2 \blacktriangleleft$$

Next find the distance

$$x = \frac{1}{2}at^2 + v_o t + x_0$$
$$x = \frac{1}{2}2.5(30)^2 + 0t + 0 = 1125$$

$$x = 1125\text{ m} \blacktriangleleft$$



PROBLEM 11.34

A minivan is tested for acceleration and braking. In the street-start acceleration test, elapsed time is 8.2 s for a velocity increase from 10 km/h to 100 km/h. In the braking test, the distance traveled is 44 m during braking to a stop from 100 km/h. Assuming constant values of acceleration and deceleration, determine (*a*) the acceleration during the street-start test, (*b*) the deceleration during the braking test.

SOLUTION

$$10 \text{ km/h} = 2.7778 \text{ m/s} \qquad 100 \text{ km/h} = 27.7778 \text{ m/s}$$

(a) Acceleration during start test.

$$a = \frac{dv}{dt}$$

$$\int_0^{8.2} a\,dt = \int_{2.7778}^{27.7778} v\,dt$$

$$8.2\,a = 27.7778 - 2.7778 \qquad a = 3.05 \text{ m/s}^2 \blacktriangleleft$$

(b) Deceleration during braking.

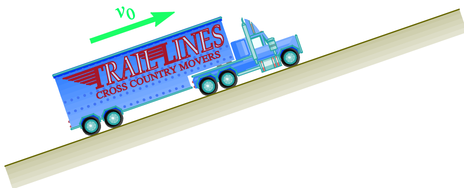
$$a = v\frac{dv}{dx} =$$

$$\int_0^{44} a\ dx = \int_{27.7778}^{0} v\ dv =$$

$$a(x)\Big|_0^{44} = \frac{1}{2}(v^2)\Big|_{27.7778}^{0}$$

$$44\ a = -\frac{1}{2}(27.7778)^2$$

$$a = -8.77 \text{ m/s}^2 \qquad \text{deceleration} = -a = 8.77 \text{ m/s}^2 \blacktriangleleft$$



Problem 11.35

Steep safety ramps are built beside mountain highways to enable vehicles with defective brakes to stop safely. A truck enters a 750-ft ramp at a high speed v_0 and travels 540 ft in 6 s at constant deceleration before its speed is reduced to $v_0/2$. Assuming the same constant deceleration, determine (*a*) the additional time required for the truck to stop, (*b*) the additional distance traveled by the truck.

SOLUTION

Given: $x_o = 0,\ x_A = 540\text{ m},\ t_A = 6\text{ s},\ v_A = \frac{1}{2}v_o,\ L_{ramp} = 750\,ft$

Uniform Acceleration: $v_A = v_o + at_A$

Substitute known values: $\frac{1}{2}v_o = v_o + (a)(6)$

$$a = -\frac{1}{12}v_o$$

Uniform Acceleration: $x_A = x_o + v_o t_A + \frac{1}{2}at_A^2$

Substitute known values: $540 = 0 + v_o(6) - \frac{1}{24}v_o(6)^2$

$$v_o = 120\text{ ft/s and } a = -10\text{ ft/s}^2$$

(*a*) $v_B = v_o + at_B$

Substitute known values: $0 = 120 - 10t_B$

Solve for t_B: $t_B = 12\text{ s}$

Additional time to stop $\quad t_B - t_A = 6.0\text{ s}$ ◀

(*b*) $x_B = x_o + v_o t_B + \frac{1}{2}at_B^2$

Substitute known values: $x_B = 0 + 120(12) - \frac{1}{2}10(12)^2$

Solve for x_B: $x_B = 720\text{ ft}$

Additional distance to stop $\quad x_B - x_A = 180.0\text{ ft}$ ◀