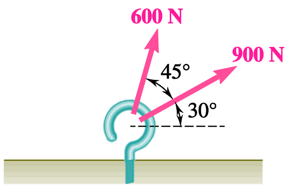
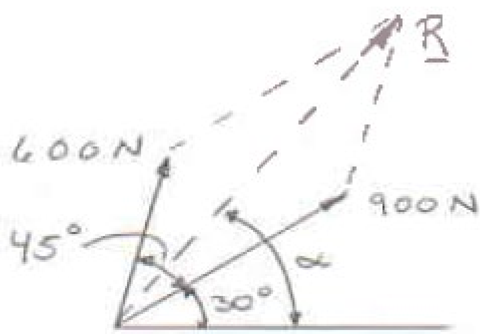


PROBLEM 2.1

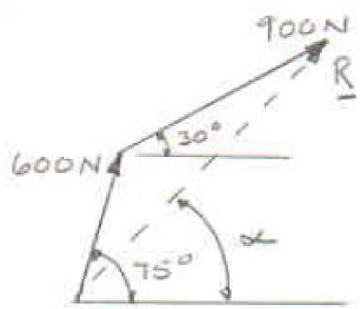
Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

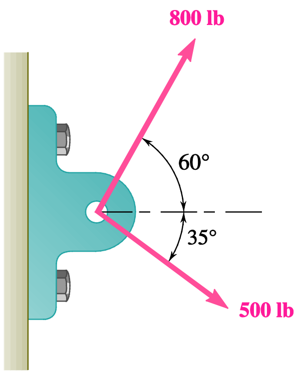
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 1391$ kN, $\alpha = 47.8°$ $\qquad$ $\mathbf{R} = 1391\text{ N} \measuredangle 47.8°$ ◀

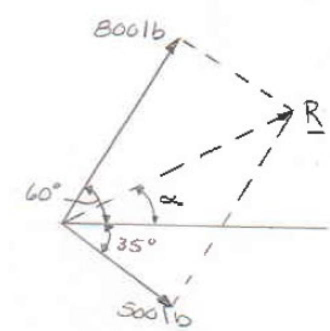


PROBLEM 2.2

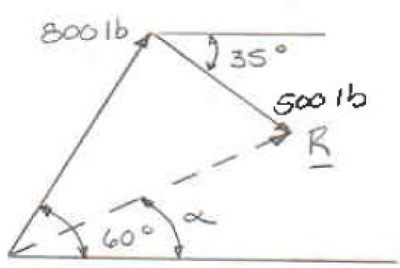
Two forces are applied as shown to a bracket support. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

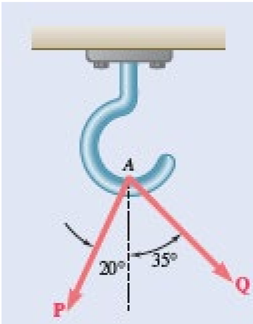
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 906 \text{ lb}, \alpha = 26.6°$ $R = 906 \text{ lb} \measuredangle 26.6°$ ◀

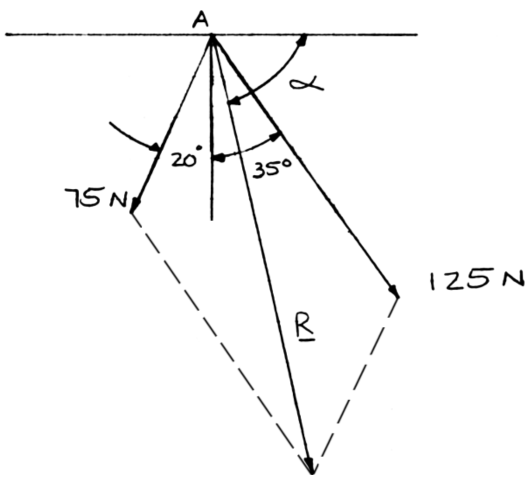


PROBLEM 2.3

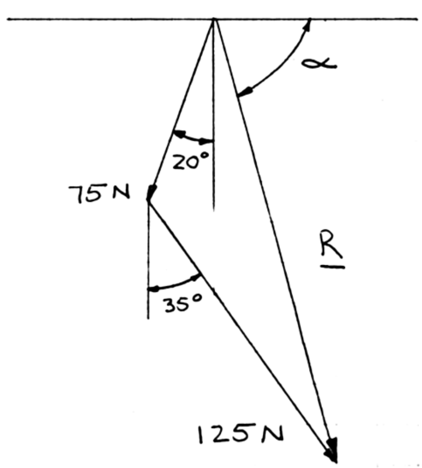
Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 75$ N and $Q = 125$ N, determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

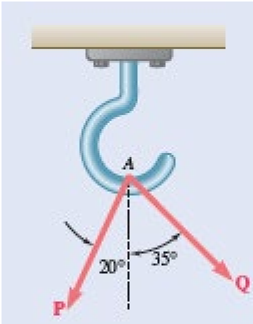
(a) Parallelogram law:



(b) Triangle rule:



We measure: $R = 179$ N, $\alpha = 75.1°$ **R** $= 179$ N ◺ $75.1°$

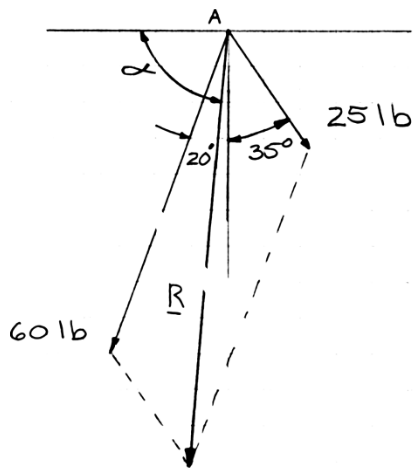


PROBLEM 2.4

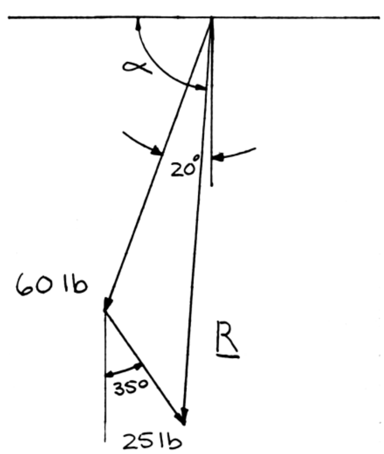
Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 60$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

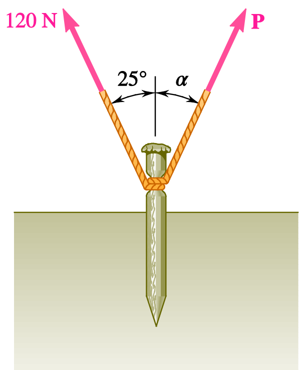


(b) Triangle rule:



We measure: $R = 77.1$ lb, $\alpha = 85.4°$

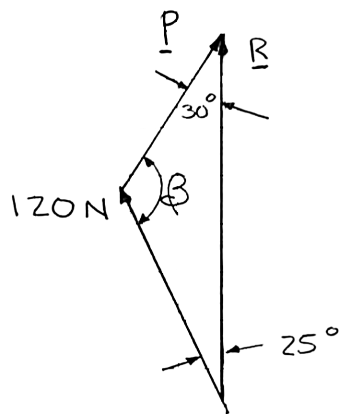
$\mathbf{R} = 77.1$ lb $\nearrow$ 85.4°



PROBLEM 2.5

A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

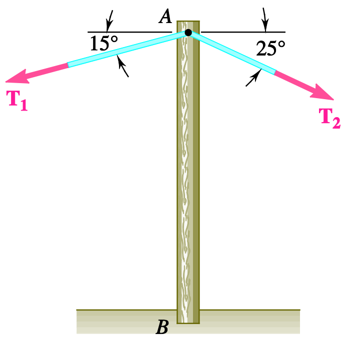


Using the triangle rule and the law of sines:

(*a*) $$\frac{120\text{ N}}{\sin 30°} = \frac{P}{\sin 25°}$$ $P = 101.4\text{ N}$ ◀

(*b*) $$30° + \beta + 25° = 180°$$
$$\beta = 180° - 25° - 30°$$
$$= 125°$$

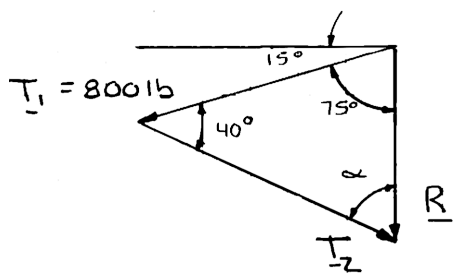
$$\frac{120\text{ N}}{\sin 30°} = \frac{R}{\sin 125°}$$ $R = 196.6\text{ N}$ ◀



PROBLEM 2.6

A telephone cable is clamped at A to the pole AB. Knowing that the tension in the left-hand portion of the cable is $T_1 = 800$ lb, determine by trigonometry (*a*) the required tension T_2 in the right-hand portion if the resultant **R** of the forces exerted by the cable at A is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

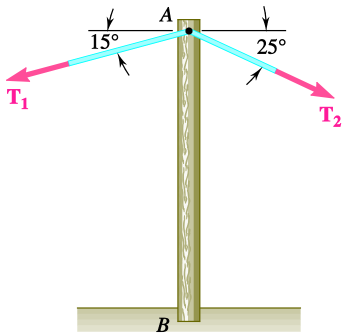
(*a*)
$$75° + 40° + \alpha = 180°$$
$$\alpha = 180° - 75° - 40°$$
$$= 65°$$

$$\frac{800 \text{ lb}}{\sin 65°} = \frac{T_2}{\sin 75°}$$

$T_2 = 853$ lb ◀

(*b*)
$$\frac{800 \text{ lb}}{\sin 65°} = \frac{R}{\sin 40°}$$

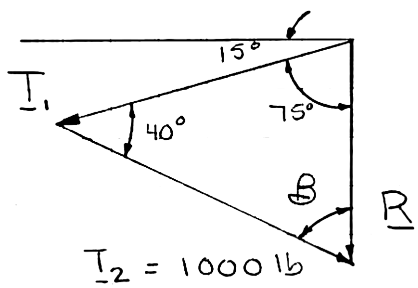
$R = 567$ lb ◀



PROBLEM 2.7

A telephone cable is clamped at A to the pole AB. Knowing that the tension in the right-hand portion of the cable is $T_2 = 1000$ lb, determine by trigonometry (*a*) the required tension T_1 in the left-hand portion if the resultant **R** of the forces exerted by the cable at A is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$75° + 40° + \beta = 180°$$
$$\beta = 180° - 75° - 40°$$
$$= 65°$$

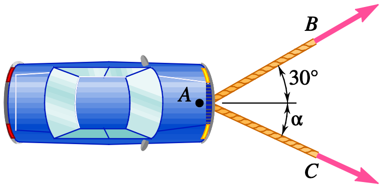
$$\frac{1000 \text{ lb}}{\sin 75°} = \frac{T_1}{\sin 65°}$$

$T_1 = 938$ lb ◀

(*b*)

$$\frac{1000 \text{ lb}}{\sin 75°} = \frac{R}{\sin 40°}$$

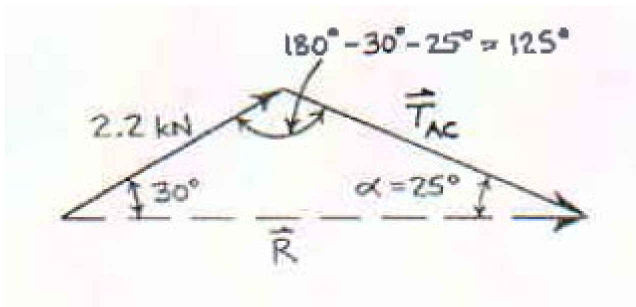
$R = 665$ lb ◀



PROBLEM 2.8

A disabled automobile is pulled by means of two ropes as shown. The tension in rope AB is 2.2 kN, and the angle α is 25°. Knowing that the resultant of the two forces applied at A is directed along the axis of the automobile, determine by trigonometry (*a*) the tension in rope AC, (*b*) the magnitude of the resultant of the two forces applied at A.

SOLUTION



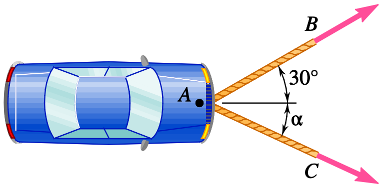
Using the law of sines:

$$\frac{T_{AC}}{\sin 30^\circ} = \frac{R}{\sin 125^\circ} = \frac{2.2 \text{ kN}}{\sin 25^\circ}$$

$$T_{AC} = 2.603 \text{ kN}$$
$$R = 4.264 \text{ kN}$$

(*a*) $T_{AC} = 2.60 \text{ kN}$ ◀

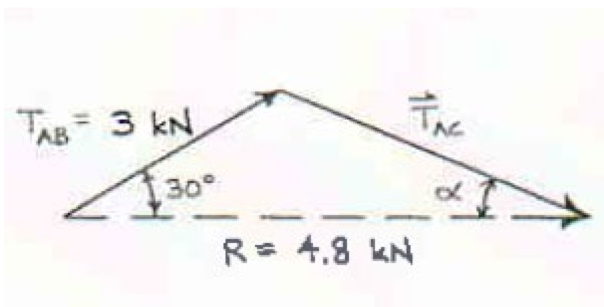
(*b*) $R = 4.26 \text{ kN}$ ◀



PROBLEM 2.9

A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in rope AB is 3 kN, determine by trigonometry the tension in rope AC and the value of α so that the resultant force exerted at A is a 4.8-kN force directed along the axis of the automobile.

SOLUTION



Using the law of cosines:

$$T_{AC}^2 = (3\text{ kN})^2 + (4.8\text{ kN})^2 - 2(3\text{ kN})(4.8\text{ kN})\cos 30°$$

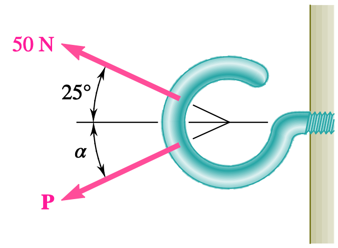
$$T_{AC} = 2.6643\text{ kN}$$

Using the law of sines:

$$\frac{\sin\alpha}{3\text{ kN}} = \frac{\sin 30°}{2.6643\text{ kN}}$$

$$\alpha = 34.3°$$

$$\mathbf{T}_{AC} = 2.66\text{ kN} \searrow 34.3° \blacktriangleleft$$

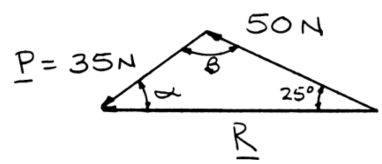


PROBLEM 2.10

Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the triangle rule and law of sines:



(*a*)

$$\frac{\sin\alpha}{50\text{ N}} = \frac{\sin 25^\circ}{35\text{ N}}$$

$$\sin\alpha = 0.60374$$

$$\alpha = 37.138^\circ$$

$\alpha = 37.1^\circ$ ◀

(*b*)

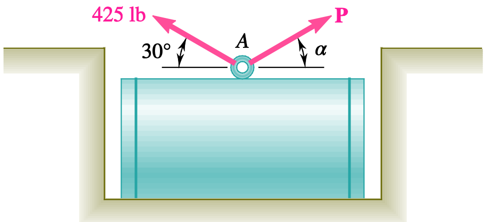
$$\alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

$$= 117.862^\circ$$

$$\frac{R}{\sin 117.862^\circ} = \frac{35\text{ N}}{\sin 25^\circ}$$

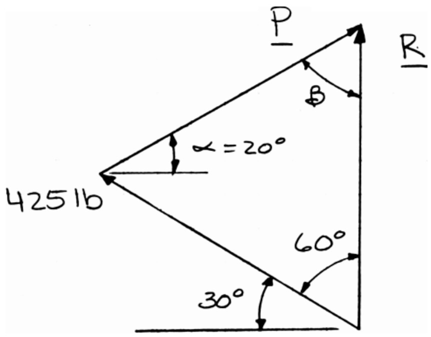
$R = 73.2\text{ N}$ ◀



PROBLEM 2.11

A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20°$, determine by trigonometry (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION

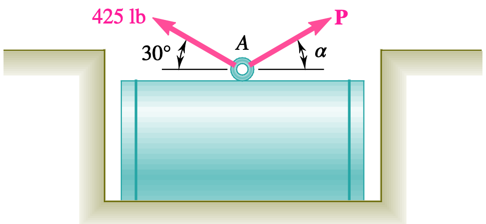


Using the triangle rule and the law of sines:

(*a*)
$$\beta + 50° + 60° = 180°$$
$$\beta = 180° - 50° - 60° = 70°$$

$$\frac{425 \text{ lb}}{\sin 70°} = \frac{P}{\sin 60°} \qquad P = 392 \text{ lb} ◀$$

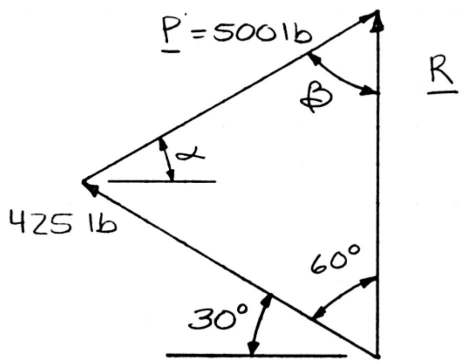
(*b*)
$$\frac{425 \text{ lb}}{\sin 70°} = \frac{R}{\sin 50°} \qquad R = 346 \text{ lb} ◀$$



PROBLEM 2.12

A steel tank is to be positioned in an excavation. Knowing that the magnitude of **P** is 500 lb, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$(\alpha + 30^\circ) + 60^\circ + \beta = 180^\circ$$

$$\beta = 180^\circ - (\alpha + 30^\circ) - 60^\circ$$

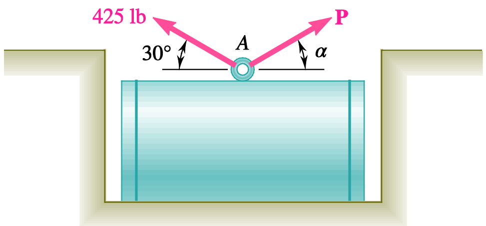
$$\beta = 90^\circ - \alpha$$

$$\frac{\sin(90^\circ - \alpha)}{425\text{ lb}} = \frac{\sin 60^\circ}{500\text{ lb}}$$

$$90^\circ - \alpha = 47.402^\circ \qquad \alpha = 42.6^\circ \blacktriangleleft$$

(*b*)

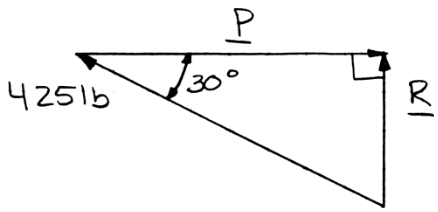
$$\frac{R}{\sin(42.598^\circ + 30^\circ)} = \frac{500\text{ lb}}{\sin 60^\circ} \qquad R = 551\text{ lb} \blacktriangleleft$$



PROBLEM 2.13

A steel tank is to be positioned in an excavation. Determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied at *A* is vertical, (*b*) the corresponding magnitude of **R**.

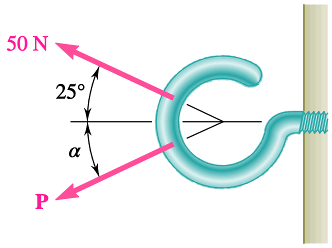
SOLUTION



The smallest force *P* will be perpendicular to *R*.

(*a*) $P = (425\text{ lb})\cos 30°$ $\mathbf{P} = 368\text{ lb} \longrightarrow$ ◀

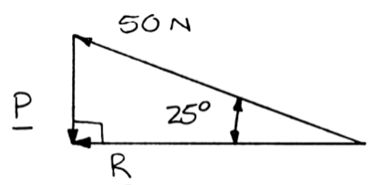
(*b*) $R = (425\text{ lb})\sin 30°$ $R = 213\text{ lb}$ ◀



PROBLEM 2.14

For the hook support of Prob. 2.10, determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied to the support is horizontal, (*b*) the corresponding magnitude of **R**.

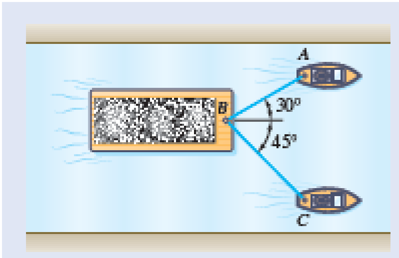
SOLUTION



The smallest force P will be perpendicular to R.

(*a*) $P = (50\text{ N})\sin 25°$ $\qquad$ $\mathbf{P} = 21.1\text{ N}\downarrow$ ◀

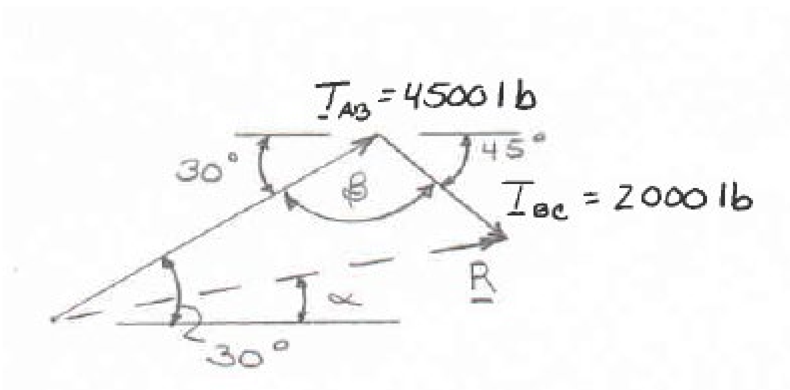
(*b*) $R = (50\text{ N})\cos 25°$ $\qquad$ $R = 45.3\text{ N}$ ◀



PROBLEM 2.15

The barge B is pulled by two tugboats A and C. At a given instant the tension in cable AB is 4500 lb and the tension in cable BC is 2000 lb. Determine by trigonometry the magnitude and direction of the resultant of the two forces applied at B at that instant.

SOLUTION



Using the law of cosines:

$$\beta = 180° - 30° - 45°$$
$$\beta = 105°$$

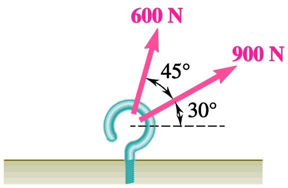
$$R^2 = (4500 \text{ lb})^2 + (2000 \text{ lb})^2 - 2(4500 \text{ lb})(2000 \text{ lb})\cos 105°$$
$$R = 5380 \text{ lb}$$

Using the law of sines:

$$\frac{R}{\sin\beta} = \frac{2000 \text{ lb}}{\sin(30° - \alpha)}$$
$$\frac{5380 \text{ lb}}{\sin 105°} = \frac{2000 \text{ lb}}{\sin(30° - \alpha)}$$

$$\alpha = 8.94°$$

$\mathbf{R} = 5380 \text{ lb} \measuredangle 8.94°$ ◀



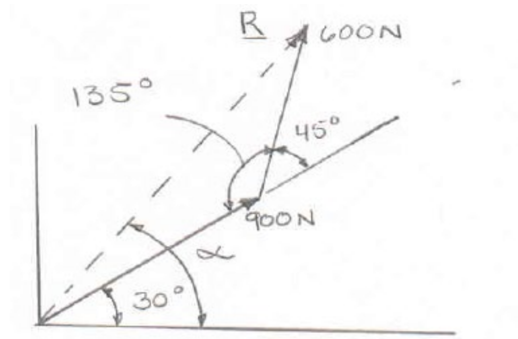
PROBLEM 2.16

Solve Prob. 2.1 by trigonometry.

PROBLEM 2.1

Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION



Using the law of cosines:

$$R^2 = (900\,\text{N})^2 + (600\,\text{N})^2 - 2(900\,\text{N})(600\,\text{N})\cos(135^\circ)$$

$$R = 1390.57\text{N}$$

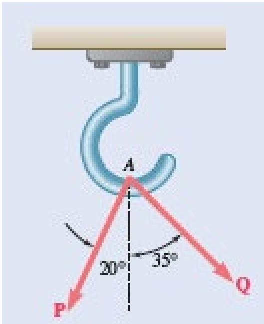
Using the law of sines:

$$\frac{\sin(\alpha - 30^\circ)}{600\text{N}} = \frac{\sin(135^\circ)}{1390.57\text{N}}$$

$$\alpha - 30^\circ = 17.7642^\circ$$

$$\alpha = 47.764^\circ$$

$\mathbf{R} = 1391\text{N} \measuredangle 47.8^\circ$ ◀◀



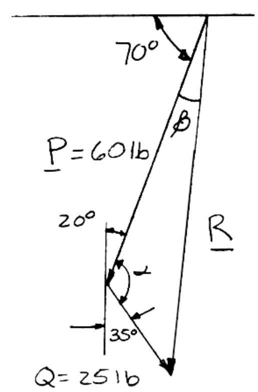
PROBLEM 2.17

Solve Problem 2.4 by trigonometry.

PROBLEM 2.4 Two forces **P** and **Q** are applied as shown at Point *A* of a hook support. Knowing that $P = 60$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

Using the triangle rule and the law of cosines:



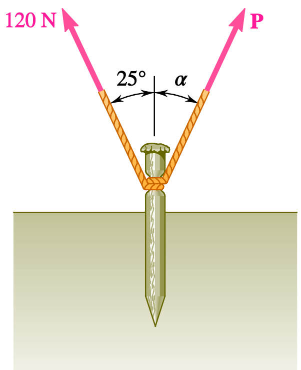
$$20° + 35° + \alpha = 180°$$
$$\alpha = 125°$$
$$R^2 = P^2 + Q^2 - 2PQ\cos\alpha$$
$$R^2 = (60 \text{ lb})^2 + (25 \text{ lb})^2 - 2(60 \text{ lb})(25 \text{ lb})\cos 125°$$
$$R^2 = 3600 + 625 + 3000(0.5736)$$
$$R = 77.108 \text{ lb}$$

Using the law of sines:

$$\frac{\sin\beta}{25 \text{ lb}} = \frac{\sin 125°}{77.108 \text{ lb}}$$
$$\beta = 15.402°$$

$$70° + \beta = 85.402°$$

$\mathbf{R} = 77.1$ lb ↗ 85.4° ◀

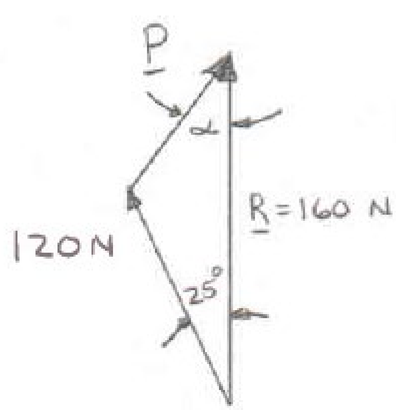


PROBLEM 2.18

For the stake of Prob. 2.5, knowing that the tension in one rope is 120 N, determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 160 N.

PROBLEM 2.5 A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION



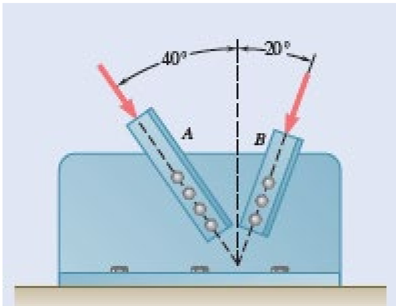
Using the laws of cosines and sines:

$$P^2 = (120\text{ N})^2 + (160\text{ N})^2 - 2(120\text{ N})(160\text{ N})\cos 25°$$
$$P = 72.096\text{ N}$$

And

$$\frac{\sin\alpha}{120\text{ N}} = \frac{\sin 25°}{72.096\text{ N}}$$
$$\sin\alpha = 0.70343$$
$$\alpha = 44.703°$$

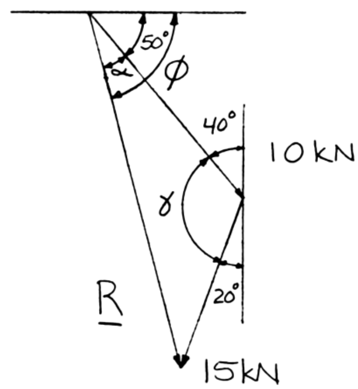
$\mathbf{P} = 72.1\text{ N} \measuredangle 44.7°$ ◀



PROBLEM 2.19

Two structural members A and B are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 10 kN in member A and 15 kN in member B, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members A and B.

SOLUTION



Using the force triangle and the laws of cosines and sines

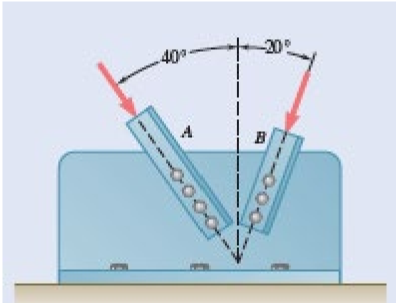
We have
$$\gamma = 180° - (40° + 20°) = 120°$$

Then
$$R^2 = (10 \text{ kN})^2 + (15 \text{ kN})^2 - 2(10 \text{ kN})(15 \text{ kN})\cos 120° = 475 \text{ kN}^2$$
$$R = 21.794 \text{ kN}$$

and
$$\frac{15 \text{ kN}}{\sin\alpha} = \frac{21.794 \text{ kN}}{\sin 120°}$$
$$\sin\alpha = \left(\frac{15 \text{ kN}}{21.794 \text{ kN}}\right)\sin 120° = 0.59605$$
$$\alpha = 36.588°$$

Hence:
$$\phi = \alpha + 50° = 86.588°$$

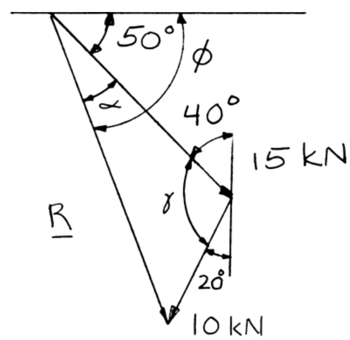
$\mathbf{R} = 21.8 \text{ kN} \measuredangle\ 86.6°$



PROBLEM 2.20

Two structural members A and B are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member A and 10 kN in member B, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members A and B.

SOLUTION



Using the force triangle and the laws of cosines and sines:

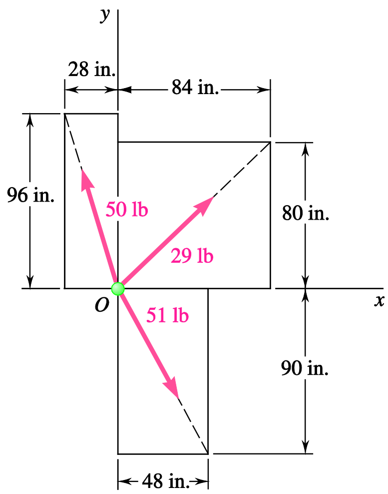
We have
$$\gamma = 180° - (40° + 20°)$$
$$= 120°$$

Then
$$R^2 = (15 \text{ kN})^2 + (10 \text{ kN})^2 - 2(15 \text{ kN})(10 \text{ kN})\cos 120°$$
$$= 475 \text{ kN}^2$$
$$R = 21.794 \text{ kN}$$

and
$$\frac{10 \text{ kN}}{\sin\alpha} = \frac{21.794 \text{ kN}}{\sin 120°}$$
$$\sin\alpha = \left(\frac{10 \text{ kN}}{21.794 \text{ kN}}\right)\sin 120°$$
$$= 0.39737$$
$$\alpha = 23.414$$

Hence:
$$\phi = \alpha + 50° = 73.414$$

$\mathbf{R} = 21.8 \text{ kN} \measuredangle 73.4°$

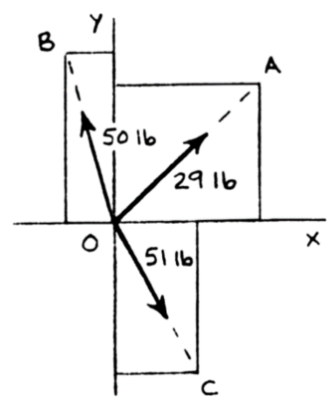


PROBLEM 2.21

Determine the x and y components of each of the forces shown.

SOLUTION

Compute the following distances:



$$OA = \sqrt{(84)^2 + (80)^2} = 116 \text{ in.}$$

$$OB = \sqrt{(28)^2 + (96)^2} = 100 \text{ in.}$$

$$OC = \sqrt{(48)^2 + (90)^2} = 102 \text{ in.}$$

29-lb Force:

$$F_x = +(29 \text{ lb})\frac{84}{116} \qquad F_x = +21.0 \text{ lb} \blacktriangleleft\blacktriangleleft$$

$$F_y = +(29 \text{ lb})\frac{80}{116} \qquad F_y = +20.0 \text{ lb} \blacktriangleleft\blacktriangleleft$$

50-lb Force:

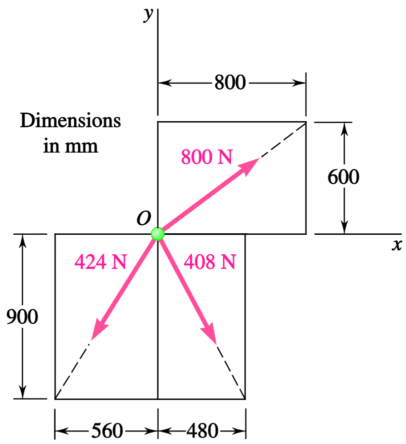
$$F_x = -(50 \text{ lb})\frac{28}{100} \qquad F_x = -14.00 \text{ lb} \blacktriangleleft\blacktriangleleft$$

$$F_y = +(50 \text{ lb})\frac{96}{100} \qquad F_y = +48.0 \text{ lb} \blacktriangleleft\blacktriangleleft$$

51-lb Force:

$$F_x = +(51 \text{ lb})\frac{48}{102} \qquad F_x = +24.0 \text{ lb} \blacktriangleleft\blacktriangleleft$$

$$F_y = -(51 \text{ lb})\frac{90}{102} \qquad F_y = -45.0 \text{ lb} \blacktriangleleft\blacktriangleleft$$

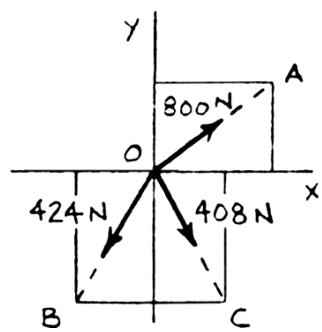


PROBLEM 2.22

Determine the x and y components of each of the forces shown.

SOLUTION

Compute the following distances:



$$OA = \sqrt{(600)^2 + (800)^2}$$
$$= 1000 \text{ mm}$$
$$OB = \sqrt{(560)^2 + (900)^2}$$
$$= 1060 \text{ mm}$$
$$OC = \sqrt{(480)^2 + (900)^2}$$
$$= 1020 \text{ mm}$$

800-N Force:

$$F_x = +(800 \text{ N})\frac{800}{1000} \qquad F_x = +640 \text{ N} ◀◀$$

$$F_y = +(800 \text{ N})\frac{600}{1000} \qquad F_y = +480 \text{ N} ◀◀$$

424-N Force:

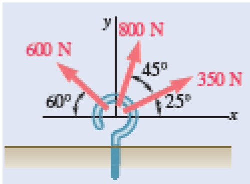
$$F_x = -(424 \text{ N})\frac{560}{1060} \qquad F_x = -224 \text{ N} ◀◀$$

$$F_y = -(424 \text{ N})\frac{900}{1060} \qquad F_y = -360 \text{ N} ◀◀$$

408-N Force:

$$F_x = +(408 \text{ N})\frac{480}{1020} \qquad F_x = +192.0 \text{ N} ◀◀$$

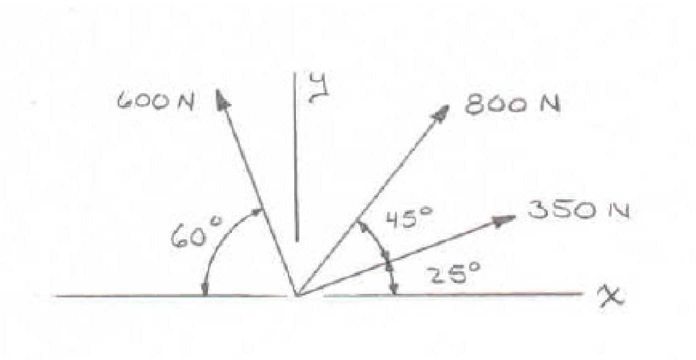
$$F_y = -(408 \text{ N})\frac{900}{1020} \qquad F_y = -360 \text{ N} ◀◀$$



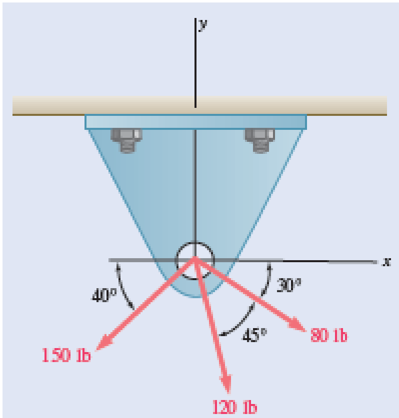
PROBLEM 2.23

Determine the x and y components of each of the forces shown.

SOLUTION



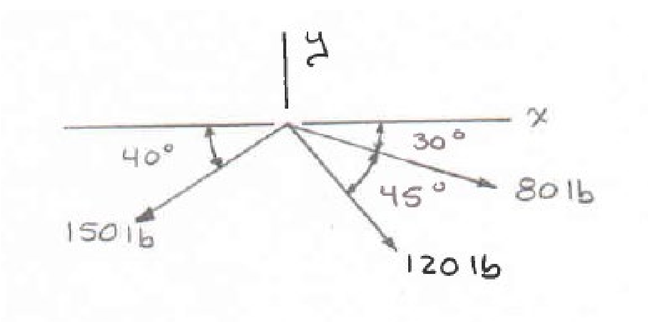
350-N Force:	$F_x = +(350\text{ N})\cos 25°$	$F_x = +317\text{ N}$ ◀◀
	$F_y = +(350\text{ N})\sin 25°$	$F_y = +147.9\text{ N}$ ◀◀
800-N Force:	$F_x = +(800\text{ N})\cos 70°$	$F_x = +274\text{ N}$ ◀◀
	$F_y = +(800\text{ N})\sin 70°$	$F_y = +752\text{ N}$ ◀◀
600-N Force:	$F_x = -(600\text{ N})\cos 60°$	$F_x = -300\text{ N}$ ◀◀
	$F_y = +(600\text{ N})\sin 60°$	$F_y = +520\text{ N}$ ◀◀



PROBLEM 2.24

Determine the x and y components of each of the forces shown.

SOLUTION



80-lb Force: $F_x = +(80 \text{ lb})\cos 30°$ $\quad F_x = +69.3 \text{ lb}$ ◀

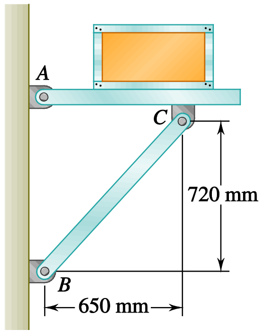
$F_y = -(80 \text{ lb})\sin 30°$ $\quad F_y = -40.0 \text{ lb}$ ◀

120-lb Force: $F_x = +(120 \text{ lb})\cos 75°$ $\quad F_x = +31.1 \text{ lb}$ ◀

$F_y = -(120 \text{ lb})\sin 75°$ $\quad F_y = -115.9 \text{ lb}$ ◀

150-lb Force: $F_x = -(150 \text{ lb})\cos 40°$ $\quad F_x = -114.9 \text{ lb}$ ◀

$F_y = -(150 \text{ lb})\sin 40°$ $\quad F_y = -96.4 \text{ lb}$ ◀

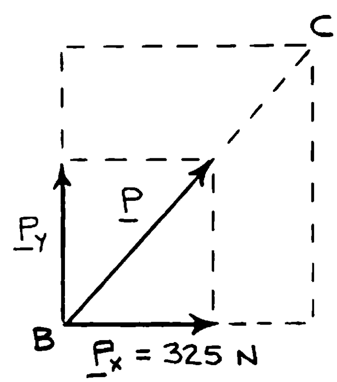


PROBLEM 2.25

Member *BC* exerts on member *AC* a force **P** directed along line *BC*. Knowing that **P** must have a 325-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION

$$BC = \sqrt{(650\text{ mm})^2 + (720\text{ mm})^2}$$
$$= 970\text{ mm}$$



(*a*)

$$P_x = P\left(\frac{650}{970}\right)$$

or

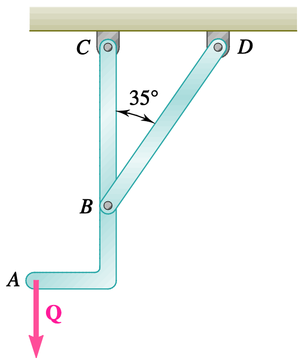
$$P = P_x\left(\frac{970}{650}\right)$$
$$= 325\text{ N}\left(\frac{970}{650}\right)$$
$$= 485\text{ N}$$

$P = 485\text{ N}$ ◀◀

(*b*)

$$P_y = P\left(\frac{720}{970}\right)$$
$$= 485\text{ N}\left(\frac{720}{970}\right)$$
$$= 360\text{ N}$$

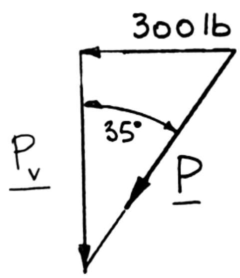
$P_y = 970\text{ N}$ ◀◀



PROBLEM 2.26

Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 300-lb horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION

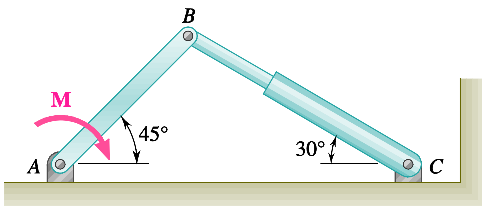


(*a*) $$P \sin 35° = 300 \text{ lb}$$

$$P = \frac{300 \text{ lb}}{\sin 35°}$$ $P = 523$ lb ◀

(*b*) Vertical component $$P_v = P \cos 35°$$

$$= (523 \text{ lb}) \cos 35°$$ $P_v = 428$ lb ◀

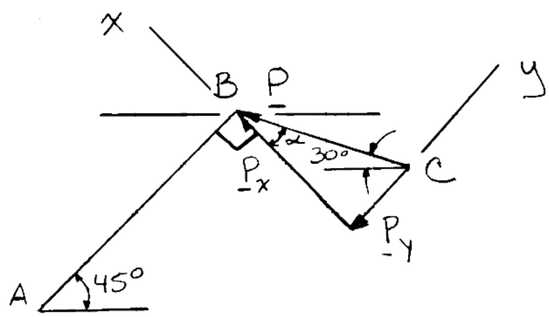


PROBLEM 2.27

The hydraulic cylinder *BC* exerts on member *AB* a force **P** directed along line *BC*. Knowing that **P** must have a 600-N component perpendicular to member *AB*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AB*.

SOLUTION

$$180° = 45° + \alpha + 90° + 30°$$
$$\alpha = 180° - 45° - 90° - 30°$$
$$= 15°$$



(*a*)

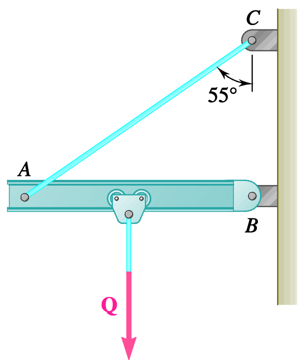
$$\cos\alpha = \frac{P_x}{P}$$
$$P = \frac{P_x}{\cos\alpha}$$
$$= \frac{600\text{ N}}{\cos 15°}$$
$$= 621.17\text{ N}$$

$P = 621$ N ◀◀

(*b*)

$$\tan\alpha = \frac{P_y}{P_x}$$
$$P_y = P_x \tan\alpha$$
$$= (600\text{ N})\tan 15°$$
$$= 160.770\text{ N}$$

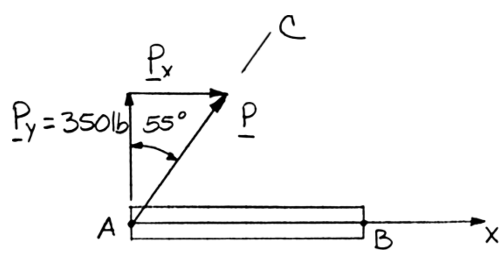
$P_y = 160.8$ N ◀◀



PROBLEM 2.28

Cable *AC* exerts on beam *AB* a force **P** directed along line *AC*. Knowing that **P** must have a 350-lb vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION



(*a*)

$$P = \frac{P_y}{\cos 55°}$$

$$= \frac{350 \text{ lb}}{\cos 55°}$$

$$= 610.21 \text{ lb}$$

$P = 610$ lb ◀

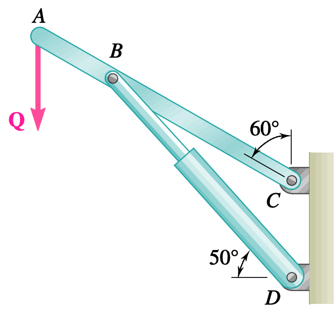
(*b*)

$$P_x = P \sin 55°$$

$$= (610.21 \text{ lb}) \sin 55°$$

$$= 499.85 \text{ lb}$$

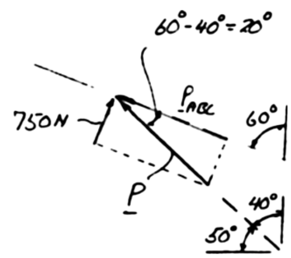
$P_x = 500$ lb ◀



PROBLEM 2.29

The hydraulic cylinder BD exerts on member ABC a force **P** directed along line BD. Knowing that **P** must have a 750-N component perpendicular to member ABC, determine (*a*) the magnitude of the force **P**, (*b*) its component parallel to ABC.

SOLUTION

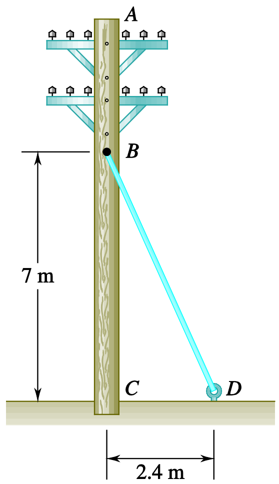


(*a*) $$750 \text{ N} = P \sin 20°$$

$$P = 2192.9 \text{ N}$$ $P = 2190$ N ◀

(*b*) $$P_{ABC} = P \cos 20°$$

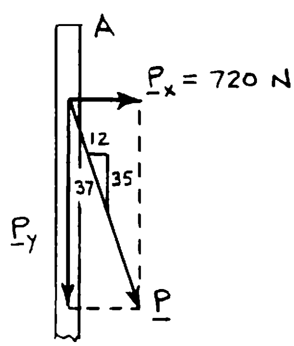
$$= (2192.9 \text{ N}) \cos 20°$$ $P_{ABC} = 2060$ N ◀



PROBLEM 2.30

The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** must have a 720-N component perpendicular to the pole *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AC*.

SOLUTION



(*a*)

$$\begin{aligned} P &= \frac{37}{12}P_x \\ &= \frac{37}{12}(720\text{ N}) \\ &= 2220\text{ N} \end{aligned}$$

$P = 2.22$ kN ◀

(*b*)

$$\begin{aligned} P_y &= \frac{35}{12}P_x \\ &= \frac{35}{12}(720\text{ N}) \\ &= 2100\text{ N} \end{aligned}$$

$P_y = 2.10$ kN ◀