Applied Partial Differential Equations (4th Edition)

Problem

For the following functions, sketch *f*(*x*), the Fourier series of *f*(*x*), the Fourier sine series of *f*(*x*), and the Fourier cosine series of *f*(*x*):

$$\text{(a)}\quad f(x) = 1$$

$$\text{(c)}\quad f(x) = \begin{cases} x & x < 0 \\ 1 + x & x > 0 \end{cases}$$

$$\text{(e)}\quad f(x) = \begin{cases} 2 & x < 0 \\ e^{-x} & x > 0 \end{cases}$$

Step-by-step solution

Step 1 of 23

(a)

Consider the function,

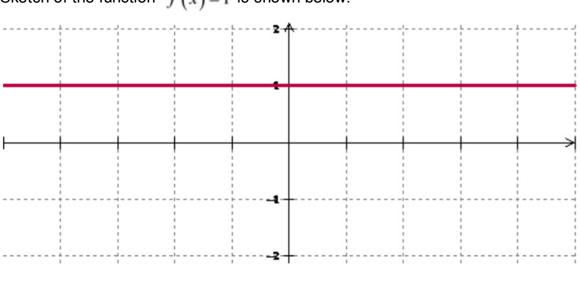
$f(x)=1$.

The objective is to sketch the function $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$.

Comment

Step 2 of 23

Sketch of the function $f(x)=1$ is shown below:

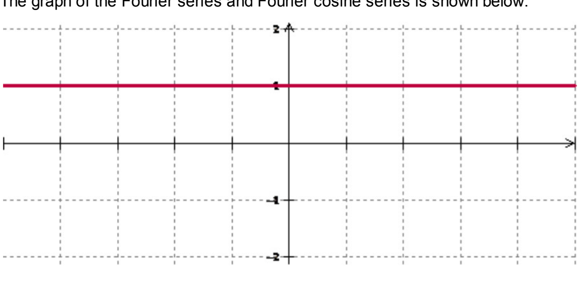


Comment

Step 3 of 23

The Fourier series and Fourier cosine series of $f(x)=1$ both converge everywhere to $f(x)=1$. Because $f(x)$ is constant, which makes it already even and periodic with no jump discontinuities.

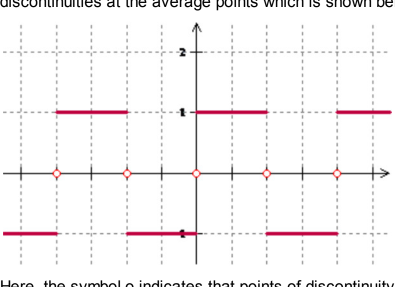
The graph of the Fourier series and Fourier cosine series is shown below:



Comment

Step 4 of 23

The Fourier sine series of $f(x)$ is the periodic odd extension of $f(x)$ with period 2*L* and jump discontinuities at the average points which is shown below:



Here, the symbol o indicates that points of discontinuity at the average.

Comment

Step 5 of 23

(b)

Consider the function,

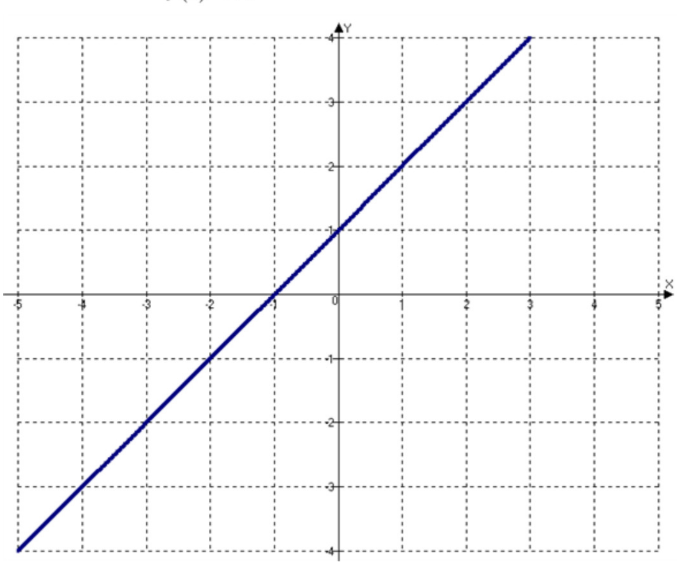
$f(x)=1+x$.

The objective is to sketch the function $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$.

Comment

Step 6 of 23

Sketch of the function $f(x)=1+x$ is shown below:

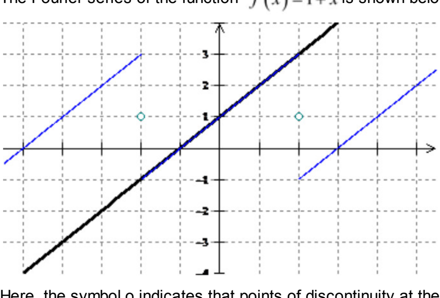


Comment

Step 7 of 23

The Fourier series converges to the periodic extension of $f(x)$, with period 2*L* and additional jump discontinuities at the endpoints.

The Fourier series of the function $f(x)=1+x$ is shown below:



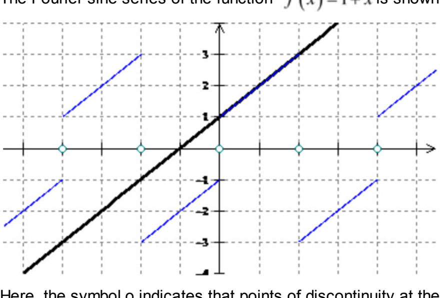
Here, the symbol o indicates that points of discontinuity at the end and $f(x)$ is shown in black.

Comment

Step 8 of 23

The Fourier sine series converges to the periodic odd extension of $f(x)$, with period 2*L* and jump discontinuities at the average points.

The Fourier sine series of the function $f(x)=1+x$ is shown below:



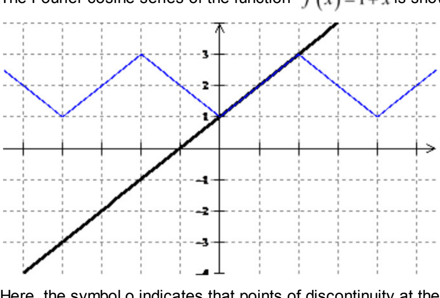
Here, the symbol o indicates that points of discontinuity at the average end points and $f(x)$ is shown in black.

Comment

Step 9 of 23

The Fourier cosine series converges to the periodic even extension of $f(x)$, with period 2*L* and jump discontinuities at the average points.

The Fourier cosine series of the function $f(x)=1+x$ is shown below:



Here, the symbol o indicates that points of discontinuity at the average end points and $f(x)$ is shown in black.

Comment

Step 10 of 23

(c)

Consider the function,

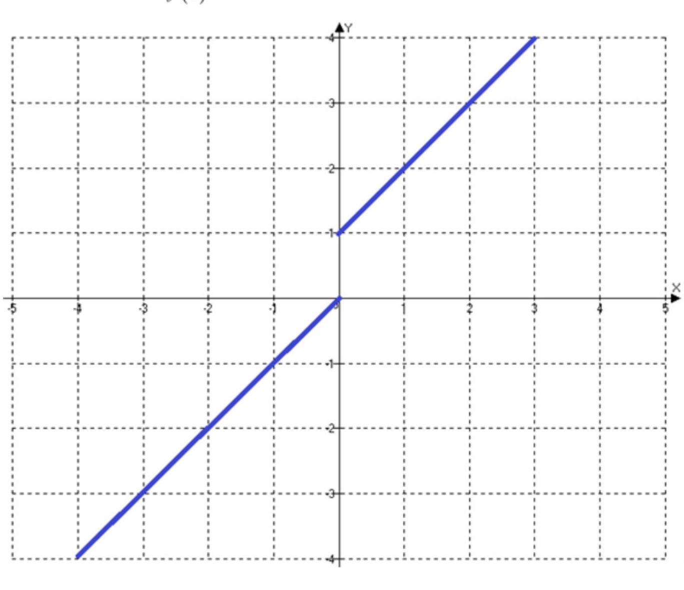
$$f(x)=\begin{cases} x, & x<0 \\ 1+x, & x>0 \end{cases}$$

The objective is to sketch the function $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$.

Comment

Step 11 of 23

Sketch of the function $f(x)$ is shown below:

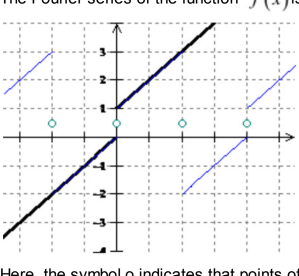


Comment

Step 12 of 23

The Fourier series converges to the periodic extension of $f(x)$, with period 2*L* and additional jump discontinuities at the endpoints.

The Fourier series of the function $f(x)$ is shown below:



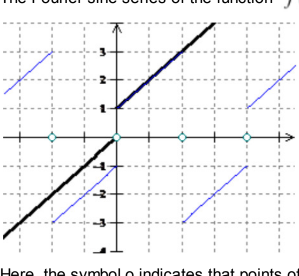
Here, the symbol o indicates that points of discontinuity at the end and $f(x)$ is shown in black.

Comment

Step 13 of 23

The Fourier sine series converges to the periodic odd extension of $f(x)$, with period 2*L* and jump discontinuities at the average points.

The Fourier sine series of the function $f(x)$ is shown below:



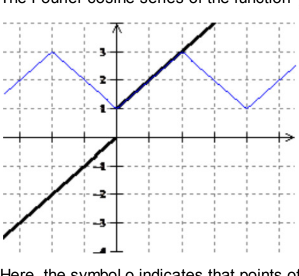
Here, the symbol o indicates that points of discontinuity at the average end points and $f(x)$ is shown in black.

Comment

Step 14 of 23

The Fourier cosine series converges to the periodic even extension of $f(x)$, with period 2*L* and jump discontinuities at the average points.

The Fourier cosine series of the function $f(x)$ is shown below:



Here, the symbol o indicates that points of discontinuity at the average end points and $f(x)$ is shown in black.

Comment

Step 15 of 23

(d)

Consider the function,

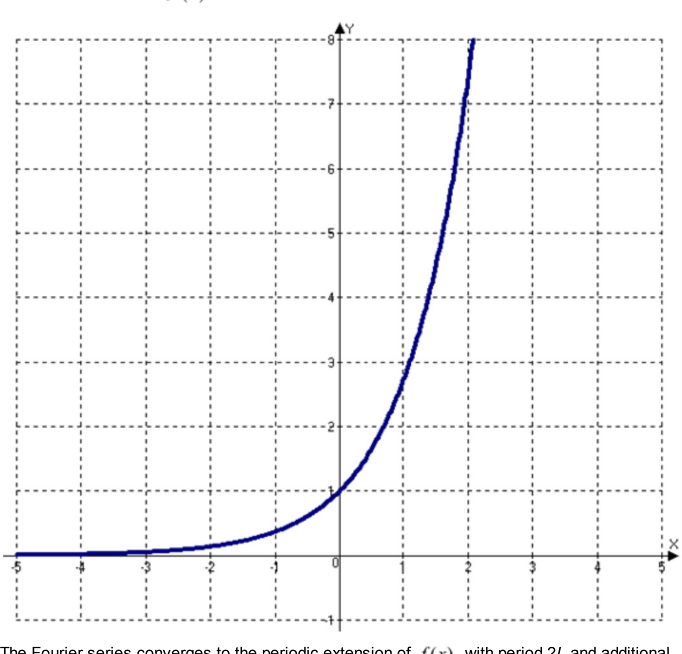
$f(x)=e^x$.

The objective is to sketch the function $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$.

Comment

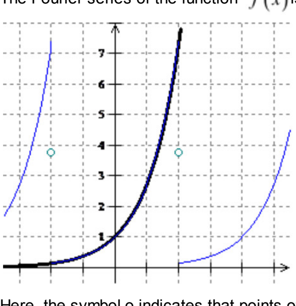
Step 16 of 23

Sketch of the function $f(x)=e^x$ is shown below:



The Fourier series converges to the periodic extension of $f(x)$, with period 2*L* and additional jump discontinuities at the endpoints.

The Fourier series of the function $f(x)$ is shown below:



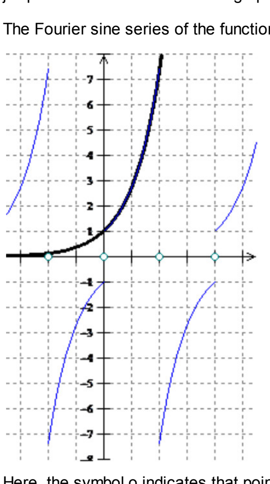
Here, the symbol o indicates that points of discontinuity at the end and $f(x)$ is shown in black.

Comment

Step 17 of 23

The Fourier sine series converges to the periodic odd extension of $f(x)$, with period 2*L* and jump discontinuities at the average points.

The Fourier sine series of the function $f(x)$ is shown below:



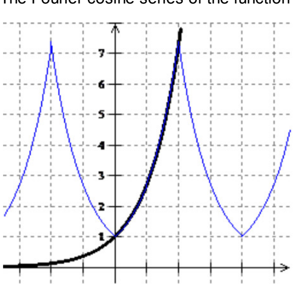
Here, the symbol o indicates that points of discontinuity at the average end points and $f(x)$ is shown in black.

Comment

Step 18 of 23

The Fourier cosine series converges to the periodic even extension of $f(x)$, with period 2*L* and jump discontinuities at the average points.

The Fourier cosine series of the function $f(x)$ is shown below:



Comment

Step 19 of 23

(e)

Consider the function,

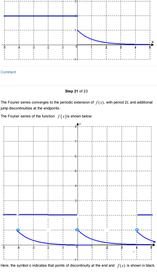
$$f(x) = \begin{cases} \frac{1}{2}, & x < 0 \\ e^{-x}, & x > 0 \end{cases}$$

The objective is to sketch the function $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$.

Comment

Step 20 of 23

Sketch of the function $f(x)$ is shown below:

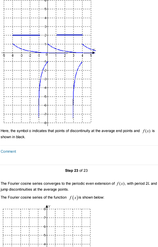


Comment

Step 21 of 23

The Fourier series converges to the periodic extension of $f(x)$, with period 2L and additional jump discontinuities at the endpoints.

The Fourier series of the function $f(x)$ is shown below:



Here, the symbol o indicates that points of discontinuity at the end and $f(x)$ is shown in black.

Comment

Step 22 of 23

The Fourier sine series converges to the periodic odd extension of $f(x)$, with period 2L and jump discontinuities at the average points.

The Fourier sine series of the function $f(x)$ is shown below:

Here, the symbol o indicates that points of discontinuity at the average and points and $f(x)$ is shown in black.

Comment

Step 23 of 23

The Fourier cosine series converges to the periodic even extension of $f(x)$, with period 2L and jump discontinuities at the average points.

The Fourier cosine series of the function $f(x)$ is shown below:

Here, the symbol o indicates that points of discontinuity at the average and points and $f(x)$ is shown in black.

Comment

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