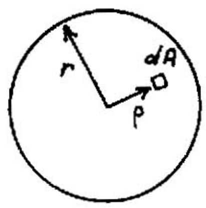
1.22	ALLOY STEEL FIG. 1.3 & 1.4	STRUCTURAL STEEL FIG. 1.5
YIELD POINT	N/A	265 MPa
YIELD STRENGTH	450 MPa	N/A
UPPER YIELD POINT	N/A	280 MPa
LOWER YIELD POINT	N/A	265 MPa
MODULUS OF RESILIENCE	N/A	0.1855 MPa
ULTIMATE TENSILE STRENGTH	715 MPa	470 MPa
STRAIN AT FRACTURE	0.23	0.26
PER CENT ELONGATION	23%	26%

1.23 ASSUME:

1. PLANE SECTIONS NORMAL TO THE AXIS OF THE ROD REMAIN PLANE UNDER APPLICATION OF THE LOAD.
2. SHEAR STRAINS VARY LINEARLY FROM THE LONGITUDINAL AXIS.
3. HOOKE'S LAW APPLIES



EQUILIBRIUM: $\Sigma M_x = 0$

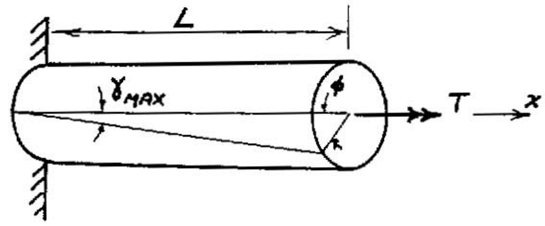
$$T = \int_A \rho \tau \, dA \qquad (a)$$

COMPATIBILITY:

$$\gamma = \frac{\gamma_{max}}{r}\rho \qquad (b)$$

HOOKE'S LAW:

$$\tau = G\gamma \qquad (c)$$



SUB (c) INTO (b) & THEN (b) INTO (a)

$$T = \frac{\tau_{max}}{r}\int_A \rho^2 dA$$

$$T = \frac{\tau_{max}}{r} J \; ; \; J = \int_A \rho^2 dA$$

$$\underline{\underline{\tau_{max} = \frac{Tr}{J}}} \quad \text{ON SURFACE}$$

$$\underline{\underline{\tau = \frac{T\rho}{J}}} \quad \text{AT ANY } \rho \qquad (d)$$

FROM GEOMETRY OF DEFORMATION:

$$\gamma_{max} = \frac{r\phi}{L} \; ; \; \gamma = \frac{\rho\phi}{L} \qquad (e)$$

SUB (d) & (e) INTO (c)

$$\frac{T\rho}{J} = G\left(\frac{\rho\phi}{L}\right)$$

$$\underline{\underline{\phi = \frac{TL}{GJ}}}$$

1.24 ASSUME: 1. PLANE SECTIONS NORMAL TO THE AXIS OF THE BAR REMAIN PLANE UNDER APPLICATION OF THE LOAD.
2. HOOKE'S LAW APPLIES.

EQUILIBRIUM:
$$P = \sigma A \qquad (a)$$

HOOKE'S LAW:
$$\sigma = E\varepsilon \qquad (c)$$

CONTINUITY:
$$\Delta L = \int_0^L \varepsilon \, dx \qquad (b)$$

GEOMETRY
$$A(x) = b\left(d_0 - \tfrac{x}{L}(d_0 - d_1)\right) \qquad (d)$$

SUB (d) INTO (a)
$$\sigma = P/\left[b\left(d_0 - \tfrac{x}{L}(d_0 - d_1)\right)\right] \qquad (e)$$

SUB. (c) & (e) INTO (b):
$$\Delta L = \int_0^L \frac{P}{Eb}\left[d_0 - \tfrac{x}{L}(d_0 - d_1)\right]^{-1} dx$$
$$\Delta L = \frac{PL}{Eb}\int_0^L \frac{dx}{d_0 L - x(d_0 - d_1)} = \frac{PL}{Eb}\left(\frac{1}{d_0 - d_1}\right)\ln\frac{d_0}{d_1}$$

1.25 RODS: $A_R = 4\left(\frac{\pi}{4}15^2\right) = 706.9\ mm^2$

PIPE: $A_P = \frac{\pi}{4}(100^2 - 90^2) = 1492\ mm^2$

AFTER ASSEMBLY: $T_R = C_P = 4(65) = 260\ kN$
$$\sigma_R = \frac{4(65000)}{706.9} = 367.8\ MPa$$
$$\sigma_P = \frac{-4(65000)}{1492} = -174.3\ MPa$$

AFTER PRESSURE IS APPLIED:

EQUILIBRIUM:
$$p\left(\frac{\pi}{4}90^2\right) = \Delta T_R + \Delta C_P \qquad (a)$$

COMPATIBILITY:
$$\Delta L_R = \Delta L_P; \quad \frac{\Delta T_R L}{A_R E} = \frac{\Delta C_P L}{A_P E} \qquad (b)$$

LEAKAGE REQUIREMENT:
$$\Delta C_P = 260\ kN \qquad (c)$$

SUB (c) INTO (b):
$$\Delta T_R = 123.2\ kN \qquad (d)$$

SUB (c) & (d) INTO (a)
$$p = 60.23\ MPa$$

$$\Delta\sigma_R = \frac{123200}{706.9} = 174.3\ MPa$$

$$\sigma_{R(FINAL)} = 542.1\ MPa$$

1.26

(a) Figure a shows the bars subjected to force P. Figure b shows the free-body diagrams of the steel bar, the aluminum bar, and point A. By the free-body diagram of point A,

$$P = P_s + P_a \qquad (a)$$

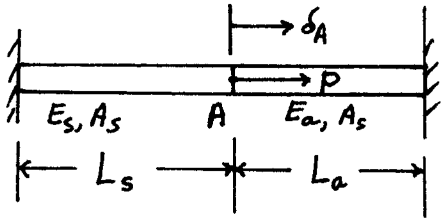


Figure a

By Eq. (1.2) and Figs. a and b,

$$\delta_a = \frac{P_s L_s}{E_s A_s} = \frac{P_a L_a}{E_a A_a} \qquad (b)$$

By Eqs. (a) and (b),

$$P_s = \frac{P_a E_s A_s L_a}{E_a A_a L_s} = \frac{(P-P_s)(E_s A_s L_a)}{E_a A_a L_s}$$

Solving this equation for P_s, we find

$$P_s = \frac{P E_s A_s L_a}{E_s A_s L_a + E_a A_a L_s}.$$ Hence, by Eq. (1.1),

the stress in the steel bar is

$$\sigma_s = \frac{P_s}{A_s} = \frac{P}{A_s}\left(\frac{E_s A_s L_a}{E_s A_s L_a + E_a A_a L_s}\right)$$

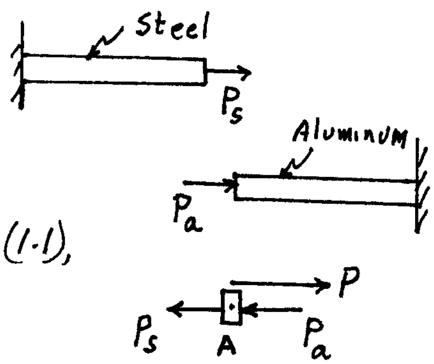


Figure b

Similarly, by Eqs. (a) and (b),

$$P_a = \frac{P_s E_a A_a L_s}{E_s A_s L_a} = \frac{(P-P_a)(E_a A_a L_s)}{E_s A_s L_a}$$

or

$$P_a = \frac{P E_a A_a L_s}{E_s A_s L_a + E_a A_a L_s}$$

Hence, the stress in the aluminum bar is

$$\sigma_a = \frac{P}{A_a}\left(\frac{E_a A_a L_s}{E_s A_s L_a + E_a A_a L_s}\right)$$

(cont.)

1.26 cont.

(b) When the left wall is displaced to the right by an amount δ, the point A is displaced to the right by an amount δ_A (Fig. c).

By Eq. (1.2) and Fig. c,

$$\delta_A = \frac{FL_a}{E_a A_a} \qquad (a)$$

and

$$\delta - \delta_A = \frac{FL_s}{E_s A_s} \qquad (b)$$

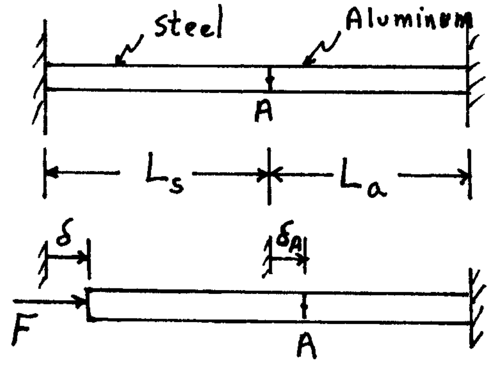


Figure C

By Eqs. (a) and (b),

$$\delta = \delta_A + \frac{FL_s}{E_s A_s} = F\left(\frac{L_a}{E_a A_a} + \frac{L_s}{E_s A_s}\right)$$

or

$$F = \frac{\delta E_a A_a E_s A_s}{E_s A_s L_a + E_a A_a L_s}$$

Hence, by Eq. (1.1), the stress in the steel bar is

$$\sigma_s = \frac{F}{A_s} = \frac{\delta E_a A_a E_s}{E_s A_s L_a + E_a A_a L_s}$$

and the stress in the aluminum bar is

$$\sigma_a = \frac{F}{A_a} = \frac{\delta E_a E_s A_s}{E_s A_s L_a + E_a A_a L_s}$$

1.27 (a) Consider the free-body diagram of a cable (Fig. a). By equilibrium,

$$\Sigma F_y = T - W = 0$$

$W = AL\rho g$, g = acceleration of gravity

$A = \pi D^2/4$

Therefore,

$$T = AL\rho g \qquad (a)$$

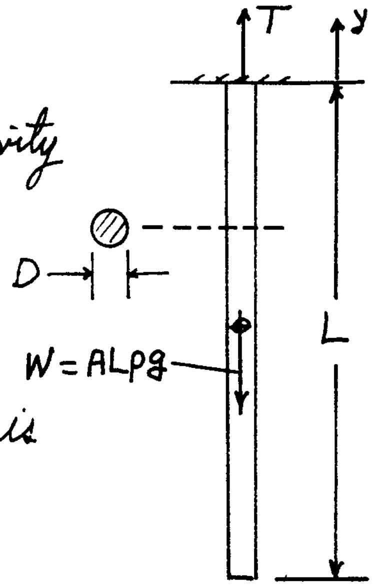


Figure a

so the maximum stress in the cable is

$$\sigma_{max} = \frac{T}{A} = L\rho g \qquad (b)$$

For the steel cable, by Eq. (b),

$$\sigma_{max} = \frac{1}{10}\sigma_u = \frac{1030}{10}\text{MPa} = L(7920)(9.81)$$

or

$$L = 1325.7 \text{ m} \qquad (c)$$

Similarly for the aluminum cable

$$\sigma_{max} = \frac{1}{10}\sigma_u = \frac{570}{10}\text{MPa} = L(2770)(9.81)$$

or

$$L = 2097.6 \text{ m} \qquad (d)$$

Consequently, the aluminum cable can be longer before exceeding a stress greater than $\frac{1}{10}\sigma_u$

(b) For a cable of length L subjected to a load P at its end, the elongation δ is [see Eq. (1.3)]

$$\delta = \frac{\sigma L}{E} \qquad (e)$$

(cont.)

1.27 Cont. Now consider the elongation of an element of length dx cut from the cable (Fig. b). For an element of length dx, the elongation $d\delta$ is, by Eq. (e),

$$d\delta = \frac{\sigma}{E}dx = \frac{1}{E}\left(\frac{Ax\rho g}{A}\right)dx \qquad (f)$$

where A is the cross-sectional area of the cable and $Ax\rho g$ is the weight of the part of the cable below the cross section at distance x from the lower end.

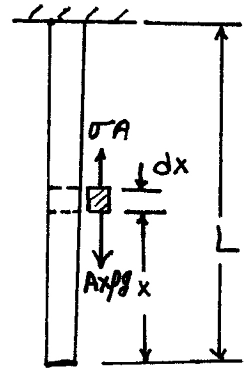


Figure b

Integration of Eq. (f) from x = 0 to x = L yields the total elongation Δ of the cable as

$$\Delta = \frac{1}{2}\frac{AL^2\rho g}{EA} = \frac{1}{2}\frac{WL}{EA} \qquad (g)$$

where $W = AL\rho g$ is the weight of the cable. Equations (g) and (1.2) show that the elongation of the cable due to its weight is equal to the elongation due to a load equal to half its weight applied at its end.

For a steel cable with $\sigma = \frac{1}{10}\sigma_u$, L = 1325.7 m, and Eq. (g) yields

$$\Delta = \frac{1}{2}(1325.7)^2(7920)(9.81)/(193\times10^9) = 0.354\ m$$

For an aluminum cable with $\sigma = \frac{1}{10}\sigma_u$, L = 2097.6 m, and

$$\Delta = \frac{1}{2}(2097.6)^2(2770)(9.81)/(72\times10^9) = 0.830\ m$$

(c) By Eq. (a), the tension at the top of the cable (Fig. a) due to the weight is $AL\rho g$. The tension at the top of the cable due to a load P applied at

(Cont.)

1.27 cont. is P. So, the total tension is $P + AL\rho g$. Therefore, the maximum stress in the cable is

$$\sigma_{max} = \frac{1}{A}(P + AL\rho g) = \frac{P}{A} + L\rho g \qquad (h)$$

For $\sigma_{max} = \frac{1}{5}\sigma_u$, $L = 1000$ m and $D = 0.075$ m, Eq.(h) yields

For a steel cable: $P = 566.8$ kN

For an aluminum cable: $P = 383.6$ kN

The steel cable can lower a cage 1.478 times as heavy as an aluminum cable.

1.28 By Eq.(1.4), with $J = \pi d^4/32$ and $\rho = d/2$, we obtain at $\tau = 30$ MPa,

$$30 \text{ MPa} = \frac{16T}{\pi d^3} \qquad (a)$$

and by Eq.(1.5), with $\psi/L = 0.005$ rad/m,

$$0.005 \text{ rad/m} = \frac{32T}{G\pi d^4} \qquad (b)$$

Assume that the maximum shear stress and the maximum angle of twist occur simultaneously. Then, by Eqs. (a) and (b)

$$T = \frac{(30\times10^6)\pi d^3}{16} = \frac{(0.005)(77\times10^9)\pi d^4}{32}$$

or $d = 0.1558$ m. For $d < 0.1558$ m, Eq.(b) yields the smallest twisting moment T; that is, the maximum angle of twist occurs before the maximum shear stress.

1.29 (a) By Fig. a, the area A of the cross section of the beam is

$$A = (6.5)(200+300) = 3250 \text{ mm}^2$$

and

$$A\bar{y} = (6.5)(300)(150) + (6.5)(200)(303.25) = 686725 \text{ mm}^3$$

or

$$\bar{y} = 211.3 \text{ mm} \qquad (a)$$

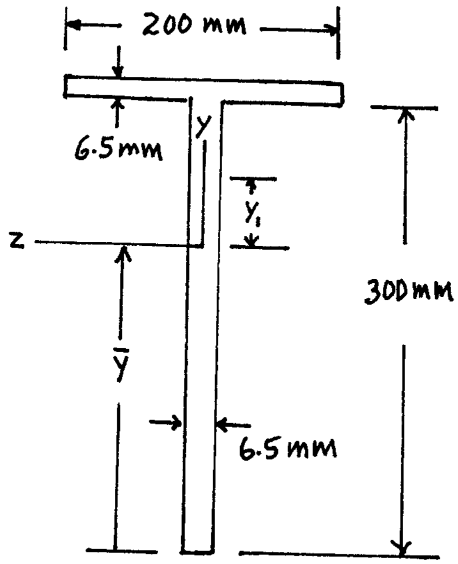


Figure a

(b) By Fig. b, the support reactions at A and B are determined by the equations

$$\Sigma F_y = A + B - 21 - (10)(3) = 0$$
$$\circlearrowleft^{+} \Sigma M_A = 3B + (21)(1) - (10)(3)(1.5) = 0 \qquad (b)$$

The solution of Eqs. (b) is

$$A = 43 \text{ kN}, \quad B = 8 \text{ kN} \qquad (c)$$

With Fig. (b) and Eqs. (c), the shear and moment diagrams may be drawn (Fig. c).

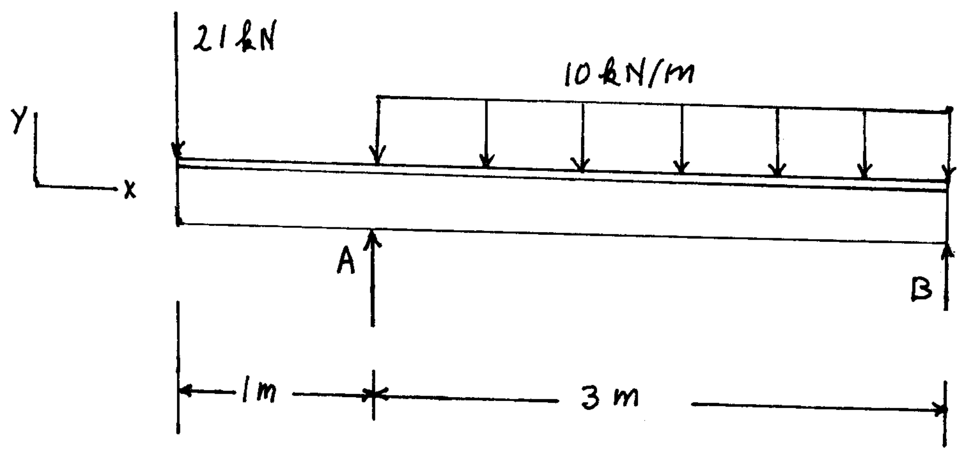


Figure b

(cont.)

1.29 cont.

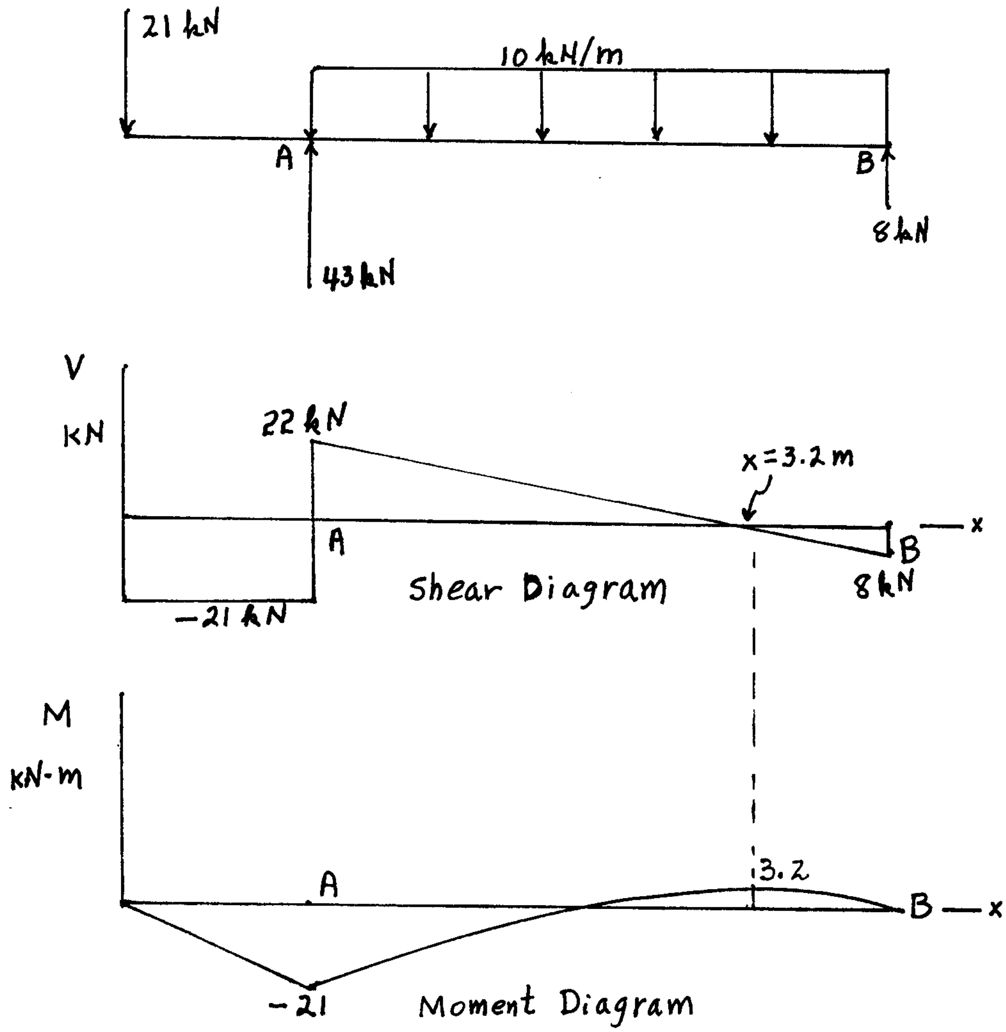


Figure c

1.30

By Fig. a, the moment of the cross-sectional area above y_1, with respect to the z axis is

$$Q = \int_{211.3}^{95.2} y\,dA = (6.5)\left[(88.7 - y_1)\left(\frac{88.7 + y_1}{2}\right)\right.$$

$$\left. + (200)(6.5)(91.95)\right)$$

or

$$Q = 3.25(7867.7 - y_1^2) + 119535 \ [\text{mm}^3] \quad (a)$$

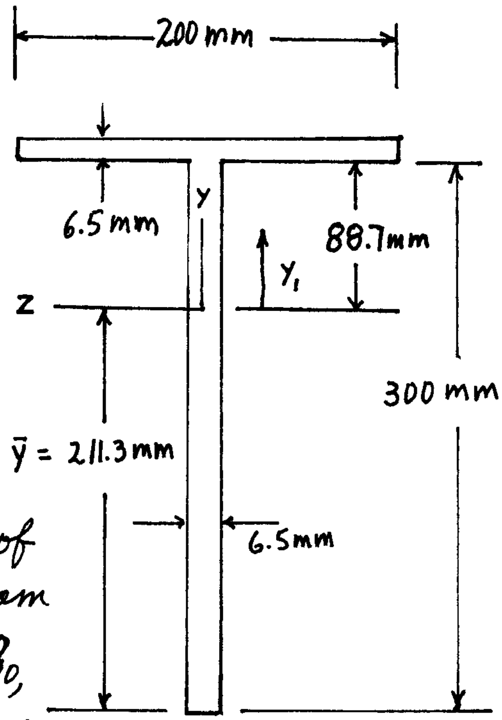


Figure a

By the shear diagram of the beam (Fig. b), the maximum shear occurs at section A and is $V = 22$ kN.

The area moment of inertia of the cross section of the T-beam is $I_z = 32.948 \times 10^6$ mm^4. So, above the horizontal centroidal axis z, by Eqs. (1.9) and (a), the stress in the web is

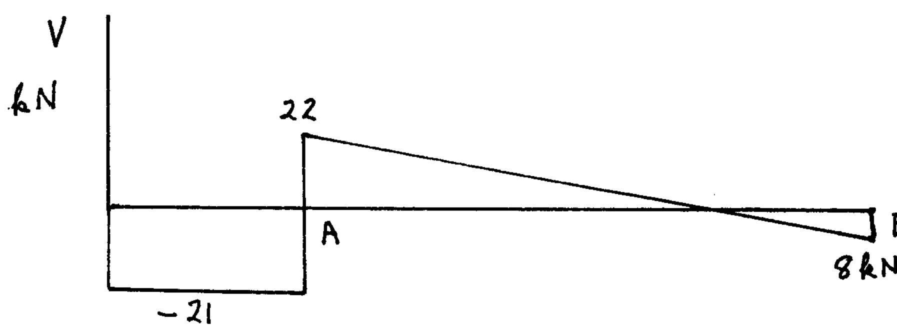


Figure b

$$\tau = \frac{VQ}{Ib} = \frac{(22)[3.25(7867.7 - y_1^2) + 119535]}{(32.948 \times 10^6)(6.5)} \quad [\text{N·mm}^{-2}] \quad (b)$$

(cont.)

1.30 cont.

By Eq. (b),

$$\tau = 14.906 \text{ MPa} \quad \text{for } y_1 = 0 \qquad \text{(c)}$$

$$\tau = 12.279 \text{ MPa} \quad \text{for } y_1 = 88.7 \text{ mm} \qquad \text{(d)}$$

Below the horizontal centroidal axis z (Fig. c),

$$Q = (6.5)(211.3 - y_1)\left(\frac{211.3 + y_1}{2}\right) = 3.25(211.3^2 - y_1^2) \qquad \text{(e)}$$

For $y_1 = 0$, Eq. (e) yields $Q = 145\,105 \text{ mm}^3$, and therefore,

$$\tau = \frac{VQ}{Ib} = \frac{(22)(145\,105)}{(32.948 \times 10^6)(6.5)} = 14.906 \text{ MPa}$$

as given previously [Eq. (c)].

For $y_1 = 211.3$, $Q = 0$ and $\tau = 0$. Hence, the maximum shear stress is $\tau_{max} = 14.906$ MPa at section A on the centroidal axis z. The minimum shear stress is $\tau_{min} = 0$ at the bottom of the web.

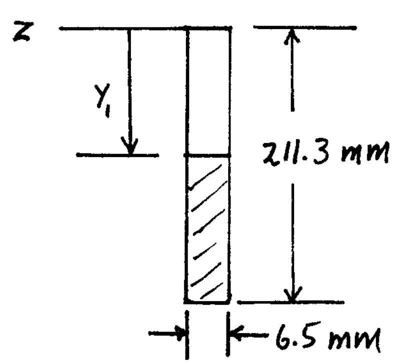


Figure c

1.31

The stress-strain curve is shown in Fig. a.
By Fig. a, the ultimate strength is $\sigma_u \approx 665$ MPa
Since each rectangular box under the stress-strain curve represents $(100)(0.05) = 5$ MN·m/m^3 of energy and we estimate that there are approximately 29.5 boxes under the stress-strain curve, we find

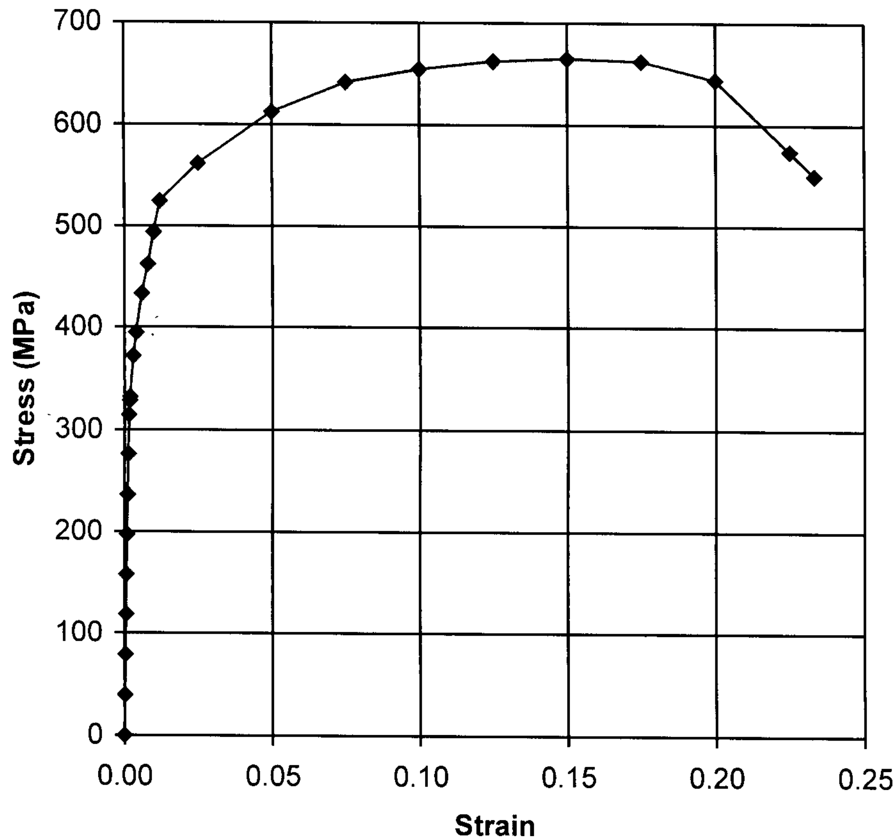


Figure a

that the modulus of toughness is

$$U_F = (5)(29.5) = 147.5 \text{ MN·m/m}^3.$$

By numerical integration,

$$U_F = 144.2 \text{ MN·m/m}^3$$

1.32

Extending the straight line portion of the stress-strain curve (Fig. a), we see that the slope of the line is

$$E = \frac{400}{.002} = 200 \text{ GPa}.$$

also, by Fig. a,

$$\sigma_{YS} = 395 \text{ MPa} \quad \text{and} \quad \sigma_{PL} = 315 \text{ MPa}$$

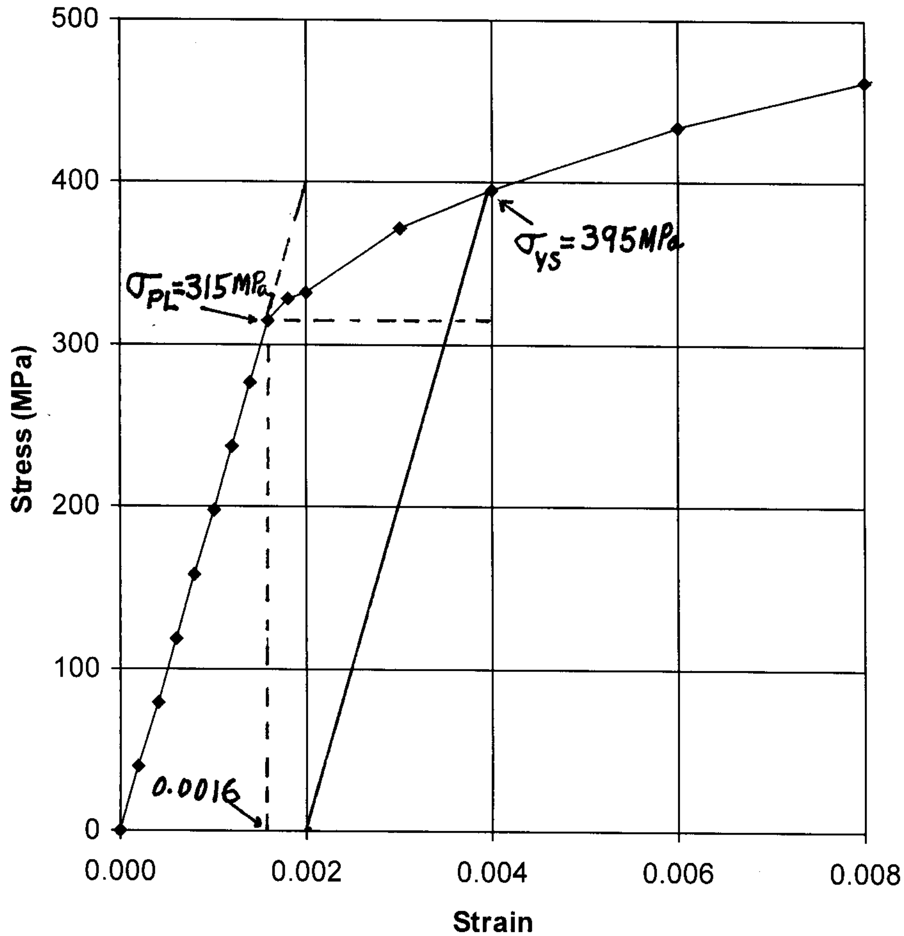


Figure a

The area under the stress-strain curve to the yield strength is approximately (see Fig. a)

$$\text{Area} = \frac{1}{2}(315)(0.0016) + (315)(0.004-0.016) + \frac{1}{2}(395-315)(0.004-0.0016)$$

or,

$$\text{Modulus of Resilience} = 1.104 \ \text{MN}\cdot\text{m/m}^3$$

1.33 The area of the test specimen is

$$A = \pi D^2/4 = \pi (20)^2/4 = 314.16 \text{ mm}^2$$

The stress at a load $P = 75.4$ kN is

$$\sigma = \frac{P}{A} = 240 \text{ MPa} \qquad (a)$$

The strain of the specimen is

$$\epsilon = \frac{\text{elongation}}{\text{gage length}} = \frac{0.330}{100} = 0.0033 \qquad (b)$$

By Eqs. (a) and (b), the modulus of elasticity is

$$E = \frac{\sigma}{\epsilon} = 72.73 \text{ GPa}$$

The radial strain in the bar at a load of 75.4 kN is

$$\epsilon_r = \frac{19.978 - 20.0}{20} = -0.0011 \qquad (c)$$

By Eqs. (b) and (c), the Poisson ratio is

$$\nu = -\frac{\epsilon_r}{\epsilon} = 0.33$$

Since the load and elongation are proportional up to $P = 75.4$ kN, the proportional limit is

$$\sigma_{PL} = \frac{P}{A} = \frac{75.4}{314.16} = 240 \text{ MPa}$$

1.34

The initial area of the test specimen was

$$A_0 = \pi D^2/4 = \pi(10)^2/4 = 78.54 \text{ mm}^2 \quad (a)$$

Since the reduction of area of the specimen is 55%, the area at fracture is, with Eq. (a),

$$A_F = (0.45)A_0 = 35.34 \text{ mm}^2 \quad (b)$$

Therefore, the true fracture stress is, with $P = 43.2$ kN at fracture (see Table P1.31),

$$\sigma_t = \frac{43.2}{35.34} = 1220 \text{ MPa}$$

The engineering fracture stress is

$$\sigma_E = \frac{43.2}{78.54} = 550 \text{ MPa}$$

The ratio $\sigma_E/\sigma_t = \left(\frac{550}{1220}\right)(100\%) = 45\%$.

This result was to be expected, since the area at fracture is 45% of the original area

2.1

With $\sigma_{xx} = 50$ MPa, $\sigma_{yy} = -30$ MPa, $\sigma_{zz} = 20$ MPa, $\sigma_{xy} = 5$ MPa, $\sigma_{xz} = -30$ MPa, $\sigma_{yz} = 0$, and $l = m = n = 1/\sqrt{\ }$, Eq. (2.10) yields

$$\sigma_{Px} = l\sigma_{xx} + m\sigma_{yx} + n\sigma_{zx} = \frac{1}{\sqrt{3}}(50 + 5 - 30) = 14.434 \text{ MPa}$$

$$\sigma_{Py} = l\sigma_{xy} + m\sigma_{yy} + n\sigma_{zy} = \frac{1}{\sqrt{3}}(5 - 30 + 0) = -14.434 \text{ MPa}$$

$$\sigma_{Pz} = l\sigma_{xz} + m\sigma_{yz} + n\sigma_{zz} = \frac{1}{\sqrt{3}}(-30 + 0 + 20) = -5.774 \text{ MPa}$$

then, by Eq. (2.11),

$$\sigma_{PN} = l^2\sigma_{xx} + m^2\sigma_{yy} + n^2\sigma_{zz} + 2mn\sigma_{yz} + 2ln\sigma_{xz} + 2lm\sigma_{xy}$$
$$= \frac{1}{3}[50 - 30 + 20 + 2(0) + 2(-30) + 2(5)] = -3.333 \text{ MPa}$$

Therefore, by Eq. (2.12),

$$\sigma_{PS} = \sqrt{\sigma_{Px}^2 + \sigma_{Py}^2 + \sigma_{Pz}^2 - \sigma_{PN}^2} = \sqrt{(14.434)^2 + (-14.434)^2 + (-5.774)^2 - (-3.333)^2}$$

or

$$\sigma_{PS} = 20.950 \text{ MPa}$$

2.2

By Figure P2.2, the section cut off by plane A-A is shown in Fig. b. By equilibrium of forces,

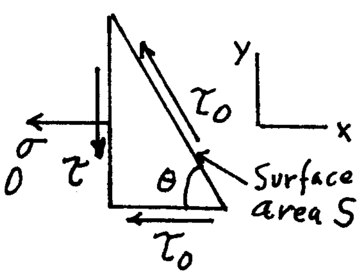


Figure a

$$\Sigma F_x = -\sigma(S\sin\theta) - \tau_0(S\cos\theta) - \tau_0 S(\cos\theta) = 0$$

$$\Sigma F_y = -\tau(S\sin\theta) + \tau_0 S(\sin\theta) = 0$$

Therefore,

$$\sigma = -2\tau_0\cot\theta \quad \text{(compression)}$$

$$\tau = \tau_0$$

2.3 (a) By Eqs. (2.21), with the given stress components,

$$I_1 = \sigma_{xx} = 20, \quad I_2 = -\sigma_{xz}^2 = -300, \quad I_3 = 0 \qquad \text{(a)}$$

By Eqs. (a) and (2.20), $\sigma^3 - 20\sigma^2 - 300\sigma = 0$, or

$\sigma(\sigma - 30)(\sigma + 10) = 0$. Hence, $\sigma = 0$, $\sigma = 30$, $\sigma = -10$.

Ordering the stresses, we have

$$\sigma_1 = 30 \text{ MPa}, \quad \sigma_2 = 0, \quad \sigma_3 = -10 \text{ MPa}$$

(b) With $\sigma = \sigma_1$, Eqs. (2.18) yield

$$(20-30)l + (0)m + 10\sqrt{3}\,n = 0 \Rightarrow l = \sqrt{3}\,n \qquad \text{(b)}$$

$$-30m \qquad = 0 \Rightarrow m = 0 \qquad \text{(c)}$$

$$10\sqrt{3}\,l + (0)m - 30n = 0 \qquad \text{(d)}$$

where

$$l^2 + m^2 + n^2 = 1 \qquad \text{(e)}$$

The solution of Eqs. (b), (c), and (e) is

$$l = \pm\sqrt{3}/2, \quad m = 0, \quad n = \pm 1/2 \qquad \text{(f)}$$

Note that Eqs. (f) satisfy Eq. (d) identically.

2.4 (a) By Eq. (2.19) and the given stress components, the principal stresses are the roots of

$$\begin{vmatrix} (-100-\sigma) & 0 & -80 \\ 0 & (20-\sigma) & 0 \\ -80 & 0 & (20-\sigma) \end{vmatrix} = 0$$

or

$$(20-\sigma)(\sigma-60)(\sigma+140) = 0$$

Hence, $\sigma_1 = 60 \text{ MPa}$, $\sigma_2 = 20 \text{ MPa}$, $\sigma_3 = -140 \text{ MPa}$ (a)

(Cont.)

2.4 cont.

(b) By Eqs.(2.22) and (a),

$$9\tau_{oct}^2 = (\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2 = 67200$$

or

$$\tau_{oct} = 86.41 \text{ MPa} \qquad (b)$$

(c) By Eqs.(2.39) and (a),

$$\tau_{max} = \frac{1}{2}(\sigma_1-\sigma_3) = 100 \text{ MPa} \qquad (c)$$

By Eqs.(b) and (c),

$$\tau_{max} = 1.16\,\tau_{oct}$$

(d) By Mohr's circle in the x-z plane (Fig. a), the maximum shear stress occurs on planes for which $2\theta_1 = 90^\circ$ (points P and P'); that is, on planes for which the direction cosines are $(\pm\frac{1}{\sqrt{2}}, 0, \pm\frac{1}{\sqrt{2}})$, relative to principal stress axes, and planes perpendicular to these planes (Fig. b).

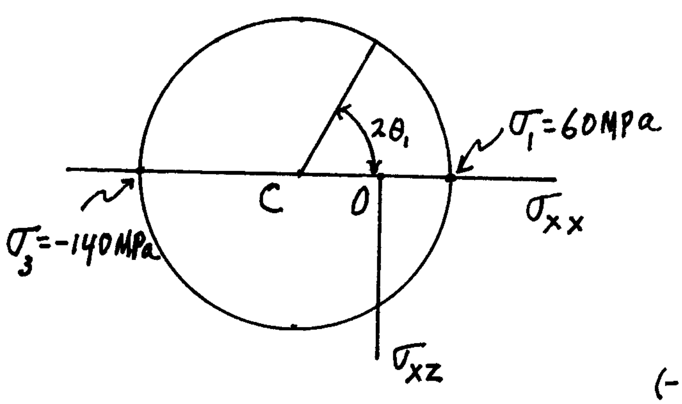


Figure a

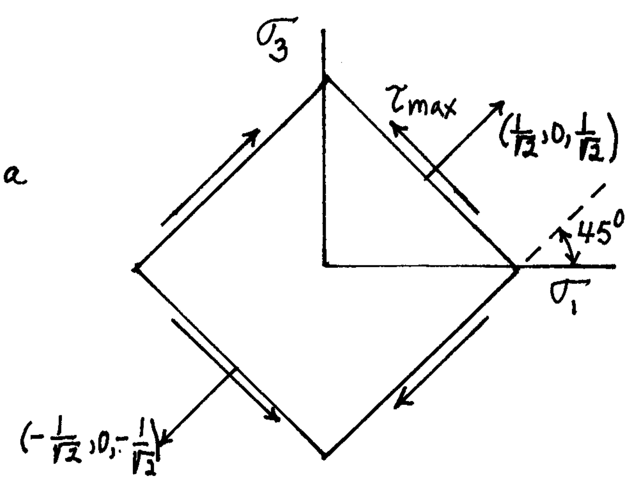


Figure b

2.5

(a) a unit vector in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ has direction cosines

$$l = \frac{1}{\sqrt{6}}, \quad m = \frac{2}{\sqrt{6}}, \quad n = \frac{1}{\sqrt{6}} \qquad (a)$$

Hence, by Eqs. (a) and (2.10) and the given stress components, the stress vector on the plane normal to the vector $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ has components

$$\sigma_{Px} = l\sigma_{xx} + m\sigma_{xy} + n\sigma_{xz} = \frac{1}{\sqrt{6}}[80 + 2(20) + 40] = \frac{160}{\sqrt{6}}$$

$$\sigma_{Py} = l\sigma_{xy} + m\sigma_{yy} + n\sigma_{zy} = \frac{1}{\sqrt{6}}[20 + 2(60) + 10] = \frac{150}{\sqrt{6}} \qquad (b)$$

$$\sigma_{Pz} = l\sigma_{xz} + m\sigma_{yz} + n\sigma_{zz} = \frac{1}{\sqrt{6}}[40 + 2(10) + 20] = \frac{80}{\sqrt{6}}$$

By Eqs. (b) and (2.9), the stress vector is

$$\underset{\sim}{\sigma_P} = 65.32\hat{\imath} + 61.24\hat{\jmath} + 32.66\hat{k}$$

(b) With the given stress components and Eqs. (2.21), the stress invariants are

$$I_1 = 80 + 60 + 20 = 160, \quad I_2 = 5500, \quad I_3 = 0 \qquad (c)$$

By Eqs. (c) and (2.20),

$$\sigma^3 - 160\sigma^2 + 5500\sigma + 0 = 0$$

or

$$\sigma(\sigma^2 - 160\sigma + 5500) = 0 \qquad (d)$$

The roots of Eq. (d) are

$$\sigma_1 = 110\text{ MPa}, \quad \sigma_2 = 50\text{ MPa}, \quad \sigma_3 = 0$$

(c) The maximum shear stress is

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) = 55\text{ MPa}$$

(cont.)

2.5 cont. (d) The octahedral shear stress is given by

$$\tau_{oct}^2 = \frac{1}{9}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right] = \frac{18200}{9}$$

or

$$\tau_{oct} = 44.97 \text{ MPa}$$

2.6 (a) Given

$$T = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 6 & 0 \\ 2 & 0 & 8 \end{bmatrix} \qquad (a)$$

By Eqs. (a) and (2.21), the stress invariants of T are

$$I_1 = 4+6+8 = 18$$

$$I_2 = \begin{vmatrix} 4 & 1 \\ 1 & 6 \end{vmatrix} + \begin{vmatrix} 4 & 2 \\ 2 & 8 \end{vmatrix} + \begin{vmatrix} 6 & 0 \\ 0 & 8 \end{vmatrix} = 99 \qquad (b)$$

$$I_3 = \begin{vmatrix} 4 & 1 & 2 \\ 1 & 6 & 0 \\ 2 & 0 & 8 \end{vmatrix} = 160$$

(b) The 45° rotation of axes (x, y) is shown in Fig. a. The direction cosines are given in Table a. Then, by Eqs. (a) and (2.15), with Table a,

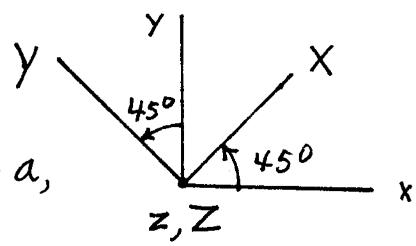


Figure a

$$\sigma_{XX} = \left(\tfrac{1}{\sqrt{2}}\right)^2(4) + \left(\tfrac{1}{\sqrt{2}}\right)^2(6) + (0)(8)$$
$$+2\left(\tfrac{1}{\sqrt{2}}\right)(0)(0) + 2(0)\left(\tfrac{1}{\sqrt{2}}\right)(2)$$
$$+2\left(\tfrac{1}{\sqrt{2}}\right)\left(\tfrac{1}{\sqrt{2}}\right)(1) = 6 \text{ MPa}$$

Similarly,

$$\sigma_{YY} = 4 \text{ MPa}, \quad \sigma_{ZZ} = 8 \text{ MPa}$$

Table a	x	y	z
X	$1/\sqrt{2}$	$1/\sqrt{2}$	0
Y	$-1/\sqrt{2}$	$1/\sqrt{2}$	0
Z	0	0	1

(cont.)