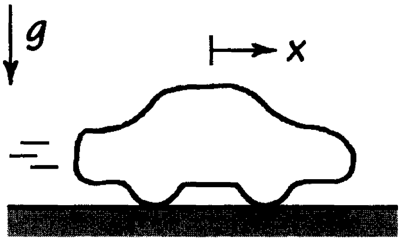
2.1.1

GOAL: Determine the distance and time needed for the car to reach its maximum speed.

GIVEN: The 2007 BMW Z4 Coupe 3.0si can accelerate from 0 to 96 km/hr in 5.6 s, and its maximum speed is $\dot{x}_{\text{max}} = 249\,\text{km/hr}$. Assume that it accelerates from 0 to 96 km/hr at a constant rate and that this acceleration is maintained as the vehicle pushes toward its maximum speed.

DRAW:



FORMULATE EQUATIONS:

Since we're assuming that the car's acceleration is constant, we can say that

$$\ddot{x} = \frac{\Delta \dot{x}}{\Delta t} \tag{1}$$

$$\dot{x}^2 - \dot{x}_0^2 = 2\ddot{x}\Delta x \tag{2}$$

SOLVE:

To go from 0 to 96 km/hr in 5.6 s at a constant rate, the car's acceleration needs to be

(1) ⇒
$$\ddot{x} = \frac{(96\,\text{km/hr})\left(\frac{\text{hr}}{3600\,\text{s}}\right)\left(\frac{10^3\,\text{m}}{\text{km}}\right)}{5.6\,\text{s}}$$
$$\ddot{x} = 4.76\,\text{m/s}^2$$

If the car continues to accelerate at this rate, then it will reach its maximum speed after traveling

(2) ⇒
$$\Delta x_{\text{max}} = \frac{(\dot{x}_{\text{max}})^2}{2\ddot{x}}$$
$$\Delta x_{\text{max}} = \frac{\left[(249\,\text{km/hr})\left(\frac{\text{hr}}{3600\,\text{s}}\right)\left(\frac{10^3\,\text{m}}{\text{km}}\right)\right]^2}{2(4.76\,\text{m/s}^2)}$$

$$\boxed{\Delta x_{\text{max}} = 502.5\text{m} = 0.5025\text{km}}$$

The time it takes for the car to attain maximum speed is

(1) $\Rightarrow$
$$\Delta t_{\max} = \frac{\dot{x}_{\max}}{\ddot{x}}$$

$$\Delta t_{\max} = \frac{(249\,\mathrm{km/hr})\left(\frac{\mathrm{hr}}{3600\,\mathrm{s}}\right)\left(\frac{10^3\,\mathrm{m}}{\mathrm{km}}\right)}{4.76\,\mathrm{m/s^2}}$$

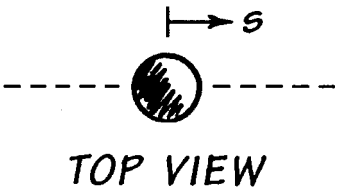
$$\boxed{\Delta t_{\max} = 14.5\,\mathrm{s}}$$

2.1.5

GOAL: Determine the ball bearing's speed v after traveling the given distance.

GIVEN: The ball bearing's acceleration is described by $\ddot{s} = as + be^{cs}$, where $a = 3\,\mathrm{s}^{-2}$, $b = 0.3\,\mathrm{m/s}^2$, and $c = 0.06\,\mathrm{m}^{-1}$. The ball bearing travels $d = 3\,\mathrm{m}$, and it starts from rest.

DRAW:



FORMULATE EQUATIONS:

Since the ball bearing's acceleration is a function of its position,

$$\ddot{s} = \frac{d\dot{s}}{dt} = \frac{d\dot{s}}{ds} \cdot \frac{ds}{dt} = \frac{d\dot{s}}{ds}\dot{s}$$

$$\ddot{s}ds = \dot{s}d\dot{s} \tag{1}$$

SOLVE:

(1) ⇒

$$(as + be^{cs})\,ds = \dot{s}d\dot{s}$$

$$\int_0^d (as + be^{cs})\,ds = \int_0^v \dot{s}d\dot{s}$$

$$\left(\frac{1}{2}as^2 + \frac{b}{c}e^{cs}\right)\int 0d = \frac{1}{2}\dot{s}^2 \int 0v$$

$$\frac{1}{2}ad^2 + \frac{b}{c}\left(e^{cd} - 1\right) = \frac{1}{2}v^2$$

$$v = \sqrt{ad^2 + \frac{2b}{c}\left(e^{cd} - 1\right)}$$

$$v = \sqrt{(3s^{-2})(3\,\mathrm{m})^2 + \frac{2(0.3\,\mathrm{m/s}^2)}{0.06\,\mathrm{m}^{-1}}(e^{(0.06\,\mathrm{m}^{-1})(3\,\mathrm{m})} - 1)}$$

$$\boxed{v = 5.38\,\mathrm{m/s} = 19.37\mathrm{km/h}}$$

2.1.6

GOAL: Find the constant acceleration a_0 that brings a jet from 274 km/hr to 0 km/h in 73m and the elapsed time Δt.

GIVEN: Distance needed to go from landing speed to zero.

FORMULATE EQUATIONS: Because the acceleration is constant we can use

$$v_2^2 - v_1^2 = 2a_0(x_2 - x_1) \tag{1}$$

$$v_2 - v_1 = a_0 \Delta t \tag{2}$$

where a_0 is the constant acceleration, Δt denotes the elapsed time, and the subscripts 1,2 indicate initial and final conditions, respectively.

SOLVE: First we'll convert 274 km/h to m/s:

$$\frac{(274\text{km})}{(1\text{hr})} \times \frac{(10^3\text{m})}{(1\text{km})} \times \frac{(1\text{hr})}{(3600\text{s})} = 76.1\text{m/s}$$

Using this, along with the known distance traveled, in (1) gives us

$$0 - (76.1\text{m/s})^2 = 2\text{a}_0(73\text{m})$$

$$\boxed{\text{a}_0 = -39.7\text{m/s}^2}$$

Because 1 g is equal to 9.81 m/s^2 we have

$$\boxed{a_0 = -39.7\,\text{m/s}^2 \frac{(1\,g)}{(9.81\,\text{m/s}^2)} = -4.05\,g}$$

We can then use (2) to find the time Δt taken for the jet to come to a halt:

$$0 - (76.1\text{m/s}) = (-39.7\text{m/s}^2 \Delta t)$$

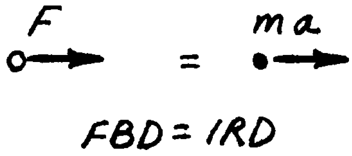
$$\boxed{\Delta t = 1.92\,\text{s}}$$

2.1.7

GOAL: Find the needed deck space to allow a fighter jet to take off from an aircraft carrier.

GIVEN: The jet is brought from 0 to 265 km/h in 2.23 s.

DRAW:



ASSUME: We'll assume that the acceleration is constant.

FORMULATE EQUATIONS: We'll use the equation relating speed and acceleration, when starting from rest under a constant acceleration:

$$v = at$$

and the expression for distance traveled under a constant acceleration:

$$x = \frac{1}{2}at^2$$

SOLVE:

First we'll convert 265 km/h to m/s:

$$\frac{(265\text{ km})}{(1\text{ hr})} \times \frac{(10^3\,\text{m})}{(1\,\text{km})} \times \frac{(1\,\text{hr})}{(3600\,\text{s})} = 73.6\,\text{m/s}$$

The acceleration is therefore

$$a = \frac{v}{t} = \frac{73.6\,\text{m/s}}{2.23\,\text{s}} = 33\,\text{m/s}^2$$

The needed space is given by

$$x = \frac{1}{2}(33\,\text{m/s}^2)(2.23\,\text{s})^2$$

$$\boxed{x = 82\,\text{m}}$$

2.1.9

GOAL: Find the speed, $\dot{x}$, and the acceleration, $\ddot{x}$, at prescribed times.

GIVEN: x as a function of time.

GOVERNING EQUATIONS: The position of the car is given by:

$$x(t) = 30\text{ m} - (27\text{ m/s})t + (3\text{ m/s}^2)t^2 \tag{1}$$

Differentiating with respect to time, we get:

$$\dot{x}(t) = -27\text{ m/s} + (6\text{ m/s}^2)t \tag{2}$$

and

$$\ddot{x}(t) = 6\text{ m/s}^2 \tag{3}$$

SOLVE:

(2) $\Rightarrow$
$$\boxed{\dot{x}(2) = -27\text{ m/s} + (6\text{ m/s}^2)(25) = -15\text{ m/s}} \tag{4}$$

and

(3) $\Rightarrow$
$$\boxed{\ddot{x}(10) = 6\text{ m/s}^2} \tag{5}$$

2.1.10

GOAL: Find the constant acceleration a_0 that brings a car from 0 to 96 km/h in 5 seconds.

GIVEN: Time needed to go from zero to 96 km/h.

FORMULATE EQUATIONS: Because the acceleration is constant we have

$$v(t) = v(0) + a_0 t \tag{1}$$

where a_0 is a constant acceleration.

SOLVE: First we'll convert 96 km/h to m/s:

$$\frac{(96 \text{ km})}{(1 \text{ hr})} \times \frac{(10^3 \text{ m})}{(1 \text{ km})} \times \frac{(1 \text{ hr})}{(3600 \text{ s})} = 26.7 \text{ m/s}$$

Using this in (1) gives us

$$26.7 \text{ m/s} = 0 + \text{ a}_0(5 \text{ s})$$

$$\boxed{a_0 = 5.3 \text{ m/s}^2}$$

Because 1 g is equal to 9.81 m/s^2 we have

$$\boxed{a_0 = 5.3 \text{ m/s}^2 \frac{(1\,g)}{(9.81 \text{ m/s}^2)} = 0.54\,g}$$

2.1.11

GOAL: Determine the time to bring a car to a stop from an initial speed along with the distance over which stopping occurs.

GIVEN: Initial speed of car and the fact that the car decelerates at 1 g.

FORMULATE EQUATIONS:

For a constant acceleration,

$$v(t) = v(0) + at$$

$$x(t) = x(0) + v(0)t + \frac{at^2}{2}$$

SOLVE:

$$113 \text{ km/h} = \frac{113 \times 10^3}{8600} \text{ m/s} = 31.4 \text{ m/s}$$

$$v(t^*) = 31.4 \text{ m/s} - (9.81 \text{ m/s}^2)t^* = 0 \Rightarrow \boxed{t^* = 3.2\,\text{s}}$$

And for the distance:

$$x(t^*) = 0 + (31.4 \text{ m/s})t^* - \frac{9.81 \text{ m/s}^2}{2} t^{*2}$$

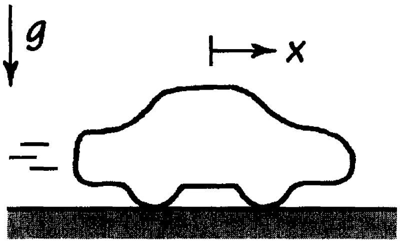
$$\boxed{\text{At } t^* = 3.2 \text{ s}, \ x = 50.3 \ \text{ m}}$$

2.1.12

GOAL: Determine the distance the car needs to reach its maximum speed and the change in its 0-to-60 time such that it and a competitor's car reach their respective maximum speed in the same distance.

GIVEN: The 2008 Audi TT Coupe can accelerate from 0 to 96 km/hr in 6.5 s and has a maximum speed of $\dot{x}_{max} = 237$ km/hr. Assume that it accelerates from 0 to 96 km/hr at a constant rate and that this acceleration is maintained as the vehicle pushes toward its maximum speed.

DRAW:



FORMULATE EQUATIONS:

Since we're assuming that the car's acceleration is constant, we can say that

$$\ddot{x} = \frac{\Delta \dot{x}}{\Delta t} \tag{1}$$

$$\dot{x}^2 - \dot{x}_0^2 = 2\ddot{x}\Delta x \tag{2}$$

SOLVE:

To go from 0 to 96 km/h in 6.5 s at a constant rate, the car's acceleration needs to be

(1) $\Rightarrow$

$$\ddot{x} = \frac{(96 \text{ km/hr}) \left(\frac{\text{hr}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{\text{km}}\right)}{6.5 \text{ s}}$$

$$\ddot{x} = 4.1 \text{ m/s}^2$$

If the car continues to accelerate at this rate, then it will reach its maximum speed after traveling

(2) $\Rightarrow$

$$\Delta x_{max} = \frac{(\dot{x}_{max})^2}{2\ddot{x}}$$

$$\frac{\left[(237 \text{ km/hr})\left(\frac{\text{hr}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{\text{km}}\right)\right]^2}{2(4.1 \text{ m/s}^2)}$$

$$\boxed{\Delta x_{\text{max}} = 529 \text{ m} = 0.529 \text{ km}}$$

We find that it takes a longer distance for the TT to reach its maximum speed as compared to its competitor. For the car to attain maximum speed in 0.50 km like its competitor, its 0-to-60 time would need to be

(1) → (2) ⇒

$$(\dot{x}_{\text{max}})^2 = \frac{2(\Delta\dot{x})(\Delta x)}{t^*}$$

$$t^* = \frac{2(\Delta\dot{x})(\Delta x)}{(\dot{x}_{\text{max}})^2}$$

$$t^* = \frac{2(96 \text{ km/hr})\left(\frac{\text{hr}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{\text{km}}\right)(0.5 \text{ km})\left(\frac{10^3 \text{ m}}{\text{km}}\right)}{\left[(237 \text{ km/hr})\left(\frac{\text{hr}}{3600s}\right)\left(\frac{10^3 \text{ m}}{\text{km}}\right)\right]^2}$$

$$t^* = 6.15 \,\text{s}$$

Thus, the car's 0-to-60 time would need to decrease by

$$\Delta t = 6.5 \,\text{s} - 6.15 \,\text{s}$$

$$\boxed{\Delta t = 0.35 \,\text{s}}$$

2.1.13

GOAL: Determine a piston's maximum speed and acceleration.

GIVEN: Position of piston as a function of time.

FORMULATE EQUATIONS: The governing equation of motion is given as

$$x(t) = \frac{8.97}{2}\sin\omega t$$

where x is given in cm and $\omega = 7000$ rpm $= 733$ rad/s.

SOLVE:

$$v(t) = \frac{d}{dt}x(t) = \left(\frac{8.97 \text{ cm}}{2}\right)\omega\cos\omega t$$

$$\boxed{v_{max} = \left(\frac{8.97 \text{ cm}}{2}\right)(733 \text{ rad/s}) = 3.29 \times 10^3 \text{ cm/s}}$$

$$a(t) = \frac{d}{dt}v(t) = -\left(\frac{8.97 \text{ cm}}{2}\right)\omega^2\sin\omega t$$

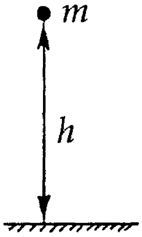
$$\boxed{a_{max} = \left(\frac{8.97 \text{ cm}}{2}\right)(733 \text{ rad/s})^2 = 2.4 \times 10^6 \text{ cm/s}^2}$$

2.1.14

GOAL: Find the height h for which a falling body will contact the ground at 56 km/hr.

GIVEN: Speed of contact.

DRAW:



ASSUME:

$$\begin{aligned} v_i &= 0 \\ v_f &= 56\,\text{km/hr} = 15.6\,\text{m/s} \\ a &= 9.81\,\text{m/s}^2 \end{aligned} \tag{1}$$

FORMULATE EQUATIONS: We'll use the formula for the difference in speed due to a constant acceleration a over a distance h:

$$v_f^2 - v_i^2 = 2ah \tag{2}$$

SOLVE:

(1)→(2)⇒

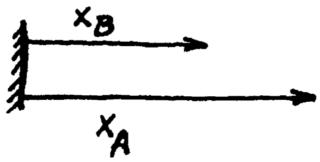
$$\begin{aligned} v_f^2 - v_i^2 &= 2gh \\ h &= \frac{v_f^2 - v_i^2}{2g} = \frac{(15.6\,\text{m/s})^2 - 0}{2(9.81\,\text{m/s}^2)} \\ h &= \boxed{12.4\text{ m}} \end{aligned}$$

2.1.16

GOAL: Find the constant speed needed for a pursuing cyclist to catch another cyclist.

GIVEN: Initial positions of the cyclists and the lead cyclist's speed.

DRAW:



FORMULATE EQUATIONS: We'll need to use the formula for position as a function of time due to a constant speed v:

$$x(t) = x(0) + vt$$

SOLVE: Bicyclist A has to travel 1610 m at the speed of 29 km/h. Thus we have

$$1610 \text{ m} = \left[\frac{(29 \times 10^3)}{3600} \text{ m/s}\right] t \Rightarrow t = 200 \text{ s}$$

Bicyclist B has to travel an additional 229 m, for a total of 1839 m and has 200 s to do so. Thus we have

$$1839 \text{ m} = v_B(200 \text{ s})$$

$$\boxed{v_B = 9.2 \text{ m/s} = 33.12 \text{ km/hr}}$$

2.1.17

GOAL: Find the terminal speed of an object with a given acceleration and determine at what time it reaches 95 percent of terminal speed.

GIVEN: $a_0 = 24,384\,\text{m/s}^2$, $a_1 = 0.108\,\text{s/ m}^2$

FORMULATE EQUATIONS: The acceleration is given by

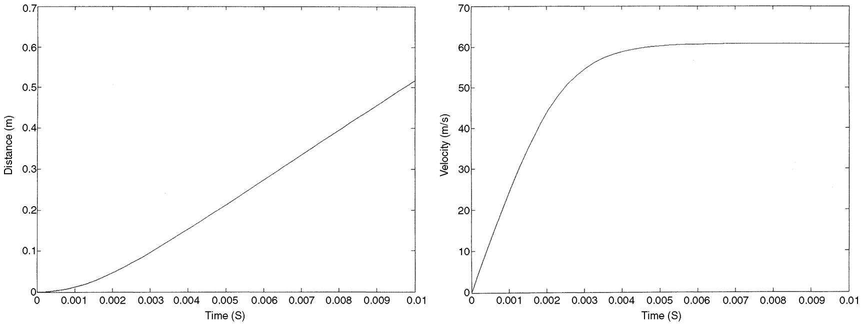
$$\ddot{s} = a_0 - a_1\dot{s}^3$$

SOLVE:

We can numerically integrate using MATLAB with initial conditions of $s(0) = 0$, $\dot{s}(0) = 0$. Using a time interval from $t = 0$ to $t = 0.01\,\text{s}$ yields the following plot:

It can be seen from the plot that the terminal speed is

$$\boxed{v_{term} = 60.9\,\text{m/s}}$$



(a) (b)

This result can be seen analytically as well. When v_{term} is reached, the acceleration $\ddot{s}$ is zero. Using this in our acceleration equation gives

$$0 = a_0 - a_1 v_{term}^3$$

which, when solved, returns the result $v_{term} = 60.9\,\text{m/s}$.

An examination of the output data allows the time at which the speed reaches 95 percent of its terminal value $(0.95(60.9\,\text{m/s}) = 57.9\,\text{m/s})$ to be determined as

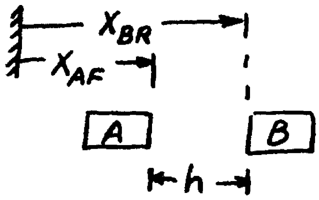
$$\boxed{t_{term} = 0.00367\,\text{s}}$$

2.1.22

GOAL: Find the time of collision between Car A to hit Car B and their relative speed at collision.

GIVEN: At $t = 0$ s Car A is traveling at a constant speed of 30 m/s and Car B is 6.9 m in front of Car A, traveling at 24 m/s and decelerating at 6 m/s^2. At $t = 0.5$ s Car A decelerates at a constant 9 m/s^2. The separation of the two cars is given by h.

DRAW:



FORMULATE EQUATIONS: We'll use the general expressions for position and speed, given a constant acceleration and initial position and speed x_0, v_0:

$$x(t) = x_0 + v_0 t + \frac{at^2}{2}$$

$$v(t) = v_0 + at$$

SOLVE:

At $t = 0$ s let the position of Car A be $x_{AF} = 0$ m and the position of Car B be $x_{BR} = 6.9$ m. After 0.5 s have elapsed the positions and speeds are found from

$$x_{AF}(0.5\,\text{s}) = (30\,\text{m/s})(0.5\,\text{s}) = 15\,\text{m}$$

$$x_{BR}(0.5\,\text{s}) = 6.9\,\text{m} + (24\,\text{m/s})(0.5\,\text{s}) - 0.5(6\,\text{m/s}^2)(0.5\,\text{s})^2 = 18.9\,\text{m} - 0.75\,\text{m} = 18.15\,\text{m}$$

$$v_{AF}(0.5\,\text{s}) = 30\,\text{m/s}$$

$$v_{BR}(0.5\,\text{s}) = 24\,\text{m/s} - (6\,\text{m/s}^2)(0.5s) = 21\,\text{m/s}$$

At this point Car A begins to decelerate at $-9\,\text{m/s}^2$. For convenience we'll reset time to zero (t indicates time beyond the 0.5 s needed for Car A's braking to begin.)

$$x_{AF}(t) = 15\,\text{m} + (30\,\text{m/s})t + 0.5(-9\,\text{m/s}^2)t^2$$

$$x_{BR}(t) = 18.15\,\text{m} + (21\,\text{m/s})t + 0.5(-6\,\text{m/s}^2)t^2$$

The separation h is given by

$$h = x_{BR} - x_{AF} = 3.15\,\text{m} - (9\,\text{m/s})t + (1.5\,\text{m/s}^2)t^2$$