1-1 Derive the heat conduction equation (1-43) in cylindrical coordinates using the differential control approach beginning with the general statement of conservation of energy. Show all steps and list all assumptions. Consider Fig. 1-7.

Assume quiescent medium with no mass flow in or out of the control volume.

Assume no work by control volume.

$$\delta \dot{Q} + \delta \dot{E}_{gen} = \frac{dE_{cv}}{dt} \quad \Big\} \text{ cons. of energy}$$

per figure 1-7:

$$dV = r\,d\phi \cdot dr \cdot dz$$

$$dm = \rho\, dV = \rho\, r\, d\phi\, dr\, dz$$

$$q_r = -k A_r \frac{\partial T}{\partial r} \quad \text{with } A_r = r\,d\phi\,dz$$

$$q_z = -k A_z \frac{\partial T}{\partial z} \quad \text{with } A_z = r\,d\phi\,dr$$

$$q_\phi = -k A_\phi \left(\frac{1}{r}\right) \frac{\partial T}{\partial \phi} \quad \text{with } A_\phi = dr\,dz$$

↳ Scale factor.

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)\,dr$$

$$q_{z+dz} = q_z + \frac{\partial}{\partial z}(q_z)\,dz$$

$$q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)\,d\phi$$

1-1

$$\delta \dot{Q} = q_r - q_{r+dr} + q_z - q_{z+dz} + q_\phi - q_{\phi+d\phi}$$

using the above:

$$\rightarrow \delta \dot{Q} = \frac{\partial}{\partial r}\left(k \cdot r d\phi dz \frac{\partial T}{\partial r}\right) dr + \frac{\partial}{\partial z}\left(k \cdot r d\phi dr \frac{\partial T}{\partial z}\right) dz + \frac{\partial}{\partial \phi}\left(k \cdot \frac{dr dz}{r} \frac{\partial T}{\partial \phi}\right) d\phi$$

$\delta \dot{E}_{gen} = g \cdot dV$, with g equal to rate of internal energy per unit volume (W/m^3)

$$\rightarrow \delta \dot{E}_{gen} = g \cdot r d\phi dr dz$$

$$\frac{dE_{c.v.}}{dt} = \frac{d}{dt}(dm \cdot u) = \rho dV \cdot \frac{du}{dt}$$

with u equal to internal energy per unit mass (J/kg)

$\rightarrow u = cT + \text{constant}$

$$\rightarrow \frac{dE}{dt} = \rho r d\phi dr dz c \frac{\partial T}{\partial t}$$

with c equal to the specific heat ($J/kg\,K$)

Substituting the above three terms into the energy equation:

$$\frac{\partial}{\partial r}\left(k\cdot r\,d\phi\,dz\,\frac{\partial T}{\partial r}\right)dr + \frac{\partial}{\partial z}\left(k\cdot r\,d\phi\,dr\,\frac{\partial T}{\partial z}\right)dz$$

$$+\frac{\partial}{\partial \phi}\left(k\cdot \frac{dr\,dz}{r}\,\frac{\partial T}{\partial \phi}\right)d\phi + g\cdot r\,dr\cdot d\phi\cdot dz$$

$$= \rho c\left(r\,dr\,d\phi\,dz\right)\frac{\partial T}{\partial t}$$

→ Assume k = constant, and divide all terms by $k\cdot r\,dr\,d\phi\,dz$.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

1-2 Derive the heat conduction equation (1-46) in spherical coordinates using the differential control approach beginning with the general statement of conservation of energy. Show all steps and list all assumptions. Consider Fig. 1-8.

Assume quiescent medium with no mass flow in or out of the control volume.

Assume no work by control volume.

Cons. of energy $\left.\right\}$ $\delta \dot{Q} + \delta \dot{E}_{gen} = \dfrac{dE_{c.v.}}{dt}$

per figure 1-8:

$$dV = (r\,d\theta)(r\sin\theta\,d\phi)(dr)$$

$$dV = r^2 \sin\theta\,dr\,d\phi\,d\theta$$

$$dm = \rho\,dV$$

$$q_r = -k A_r \frac{\partial T}{\partial r} \quad \text{with} \quad A_r = r^2 \sin\theta\,d\theta\,d\phi$$

$$q_\theta = -\frac{k}{r} A_\theta \frac{\partial T}{\partial \theta} \quad \text{with} \quad A_\theta = r\sin\theta\,d\phi\,dr$$

$$q_\phi = \frac{-k}{r\sin\theta} A_\phi \frac{\partial T}{\partial \phi} \quad \text{with} \quad A_\phi = r\,d\theta\,dr$$

→ noting scale factors of $\left(\frac{1}{r}\right)$ & $\left(\frac{1}{r\sin\theta}\right)$

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)\,dr$$

$$q_{\theta+d\theta} = q_\theta + \frac{\partial}{\partial \theta}(q_\theta)\,d\theta$$

$$q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)\,d\phi$$

$$\delta \dot{Q} = q_r - q_{r+dr} + q_\theta - q_{\theta+d\theta} + q_\phi - q_{\phi+d\phi}$$

using the above expressions:

$$\rightarrow \delta \dot{Q} = \frac{\partial}{\partial r}\left(k \cdot r^2 \sin\theta \, d\theta \, d\phi \frac{\partial T}{\partial r}\right) dr$$

$$+ \frac{\partial}{\partial \theta}\left(\frac{k}{r} \cdot r \sin\theta \, d\phi \, dr \frac{\partial T}{\partial \theta}\right) d\theta$$

$$+ \frac{\partial}{\partial \phi}\left(\frac{k}{r\sin\theta} \cdot r \, d\theta \, dr \frac{\partial T}{\partial \phi}\right) d\phi$$

$\delta \dot{E}_{gen} = g \cdot dV$, with g equal to the rate of internal energy per unit volume (W/m^3).

$$\rightarrow \delta \dot{E}_{gen} = g \cdot r^2 \sin\theta \, dr \, d\phi \, d\theta$$

$$\frac{dE_{c.v.}}{dt} = \frac{d}{dt}(dm \cdot u) = \rho \, dV \cdot \frac{du}{dt}$$

with u equal to the internal energy per unit mass (J/kg)

$\rightarrow u = cT + \text{constant}$

$$\rightarrow \frac{dE}{dt} = \rho r^2 \sin\theta \, dr \, d\phi \, d\theta \cdot c \frac{\partial T}{\partial t}$$

with c equal to the specific heat $(J/kg\,K)$

Substituting the above three terms <u>into the energy equation:</u>

$$\frac{\partial}{\partial r}\left(k r^2 \sin\theta \, d\theta \, d\phi \frac{\partial T}{\partial r}\right) dr$$

$$+ \frac{\partial}{\partial \theta}\left(\frac{k}{r} \cdot r \sin\theta \, d\phi \, dr \frac{\partial T}{\partial \theta}\right) d\theta$$

$$+ \frac{\partial}{\partial \phi}\left(\frac{k}{r \sin\theta} \cdot r \, d\theta \, dr \frac{\partial T}{\partial \phi}\right) d\phi$$

$$+ g \cdot r^2 \sin\theta \, dr \, d\phi \, d\theta = \rho c r^2 \cdot \sin\theta \, dr \, d\phi \, d\theta \frac{dT}{dt}$$

<u>Assume k = constant</u>, and divide by $k \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right)$$

$$+ \frac{1}{r^2 \sin^2\theta} \underbrace{\frac{\partial}{\partial \phi}\left(\frac{\partial T}{\partial \phi}\right)}_{= \frac{\partial^2 T}{\partial \phi^2}} + \frac{g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

1/1

1-3 Show that the following two forms of the differential operator in the cylindrical coordinate system are equivalent:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}$$

LHS) $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) =$

$$\frac{1}{r}\left\{\frac{\partial r}{\partial r}\cdot\frac{\partial T}{\partial r}+r\cdot\frac{\partial^2 T}{\partial r^2}\right\}$$

$$=\frac{1}{r}\left\{1\cdot\frac{\partial T}{\partial r}+r\cdot\frac{\partial^2 T}{\partial r^2}\right\}$$

$$=\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2} = RHS$$

1/1

1-4 Show that the following three different forms of the differential operator in the spherical coordinate system are equivalent:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=\frac{1}{r}\frac{d^2}{dr^2}(rT)=\frac{d^2T}{dr^2}+\frac{2}{r}\frac{dT}{dr}$$

LHS) $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)$

$$=\frac{1}{r^2}\left\{\frac{\partial \cdot r^2}{\partial r}\cdot\frac{\partial T}{\partial r}+r^2\cdot\frac{\partial^2 T}{\partial r^2}\right\}$$

$$=\frac{1}{r^2}\left\{2r\cdot\frac{\partial T}{\partial r}+r^2\frac{\partial^2 T}{\partial r^2}\right\}=\frac{2}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}$$

$$=RHS$$

Middle term) $\frac{1}{r}\frac{\partial^2(rT)}{\partial r^2}=\frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{\partial r}{\partial r}\cdot T+r\frac{\partial T}{\partial r}\right\}$

$$=\frac{1}{r}\frac{\partial}{\partial r}\left\{1\cdot T+r\frac{\partial T}{\partial r}\right\}$$

$$=\frac{1}{r}\left\{\frac{\partial T}{\partial r}+1\cdot\frac{\partial T}{\partial r}+r\cdot\frac{\partial^2 T}{\partial r^2}\right\}$$

$$=\frac{1}{r}\left\{2\cdot\frac{\partial T}{\partial r}+r\frac{\partial^2 T}{\partial r^2}\right\}$$

$$=\frac{2}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}=RHS$$

1-5 Set up the mathematical formulation of the following heat conduction problems. Formulation includes the simplified differential heat equation along with boundary and initial conditions. Do not solve the problems.

1. A slab in $0 \le x \le L$ is initially at a temperature $F(x)$. For times $t > 0$, the boundary at $x = 0$ is kept insulated, and the boundary at $x = L$ dissipates heat by convection into a medium at zero temperature.
2. A semi-infinite region $0 \le x \le \infty$ is initially at a temperature $F(x)$. For times $t > 0$, heat is generated in the medium at a constant, uniform rate of g_0 (W/m^3), while the boundary at $x = 0$ is kept at zero temperature.
3. A hollow cylinder $a \le r \le b$ is initially at a temperature $F(r)$. For times $t > 0$, heat is generated within the medium at a rate of $g(r)$, (W/m^3), while both the inner boundary at $r = a$ and outer boundary $r = b$ dissipate heat by convection into mediums at fluid temperature T_∞.
4. A solid sphere $0 \le r \le b$ is initially at temperature $F(r)$. For times $t > 0$, heat is generated in the medium at a rate of $g(r)$, (W/m^3), while the boundary at $r = b$ is kept at a uniform temperature T_0.

1) $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad 0 < x < L, \; t > 0$$

BC1) $$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

BC2) $$-k\left.\frac{\partial T}{\partial x}\right|_{L} = h\,T\big|_{x=L}$$

IC) $$T(t=0) = F(x)$$

2) $$\frac{\partial^2 T}{\partial x^2} + \frac{g_0}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad 0 < x < \infty,\ t > 0$$

BC1) $T(x=0) = 0$

IC) $T(t=0) = F(x)$

<u>note:</u> The I.C. is not recovered as $x \to \infty$ due to generation.

3) $$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{g(r)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad a < r < b,\ t > 0$$

BC1) $-k\frac{\partial T}{\partial r}\Big|_{r=a} = -h\left[T\big|_{r=a} - T_\infty\right]$

BC2) $-k\frac{\partial T}{\partial r}\Big|_{r=b} = +h\left[T\big|_{r=b} - T_\infty\right]$

IC) $T(t=0) = F(r)$

4) $$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{g(r)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad 0 \le r < b,\ t > 0$$

BC1) $T(r \to 0) \Rightarrow$ finite

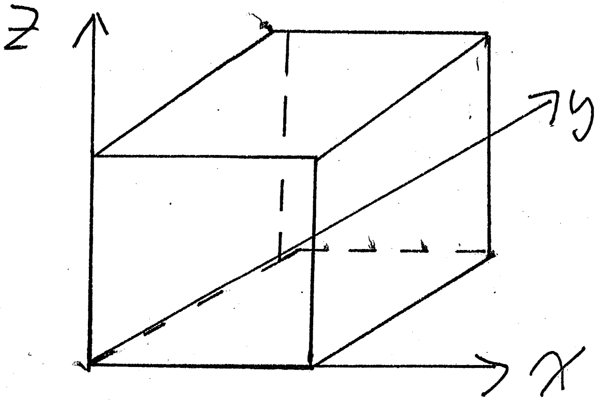
<u>or</u> $\frac{\partial T}{\partial r}\Big|_{r=0} = 0$ per symmetry

BC2) $T(r=b) = T_0$

IC) $T(t=0) = F(r)$

1-6 A solid cube of dimension L is originally at a uniform temperature T_0. The cube is then dropped into a large bath where the cube rapidly settles flat on the bottom. The fluid in the bath provides convection heat transfer with coefficient h (W/m^2 K) from the fluid at constant temperature T_∞. Formulate the heat conduction problem. Formulation includes the simplified differential heat equation along with appropriate boundary and initial conditions. Include a sketch with your coordinate axis position. Do not solve the problem.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



$0 < x < L$

$0 < y < L$

$0 < z < L$

$t > 0$

BC1) $-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = -h\left[T|_{x=0} - T_\infty\right]$

BC2) $-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = +h\left[T|_{x=L} - T_\infty\right]$

BC3) $-k\left.\frac{\partial T}{\partial y}\right|_{y=0} = -h\left[T|_{y=0} - T_\infty\right]$

BC4) $-k\left.\frac{\partial T}{\partial y}\right|_{y=L} = +h\left[T|_{y=L} - T_\infty\right]$

BC5) $-k\left.\frac{\partial T}{\partial z}\right|_{z=L} = +h\left[T|_{z=L} - T_\infty\right]$

BC6) $T(z=0) = T_\infty$ <u>or</u> $\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0$

→ Convection B.C. not reasonable on the bottom.

IC) $T(t=0) = T_0$

1-7 For an anisotropic solid, the three components of the heat conduction vector q_x, q_y and q_z are given by equations (1-80). Write the similar expressions in the cylindrical coordinates for q_r, q_ϕ, q_z and in the spherical coordinates for q_r, q_ϕ, q_θ.

Cylinder

$$q''_r = -\left(k_{11}\frac{\partial T}{\partial r} + k_{12}\frac{1}{r}\frac{\partial T}{\partial \phi} + k_{13}\frac{\partial T}{\partial z}\right)$$

$$q''_\phi = -\left(k_{21}\frac{\partial T}{\partial r} + k_{22}\frac{1}{r}\frac{\partial T}{\partial \phi} + k_{23}\frac{\partial T}{\partial z}\right)$$

$$q''_z = -\left(k_{31}\frac{\partial T}{\partial r} + k_{32}\frac{1}{r}\frac{\partial T}{\partial \phi} + k_{33}\frac{\partial T}{\partial z}\right)$$

Sphere

$$q''_r = -\left(k_{11}\frac{\partial T}{\partial r} + k_{12}\frac{1}{r\cdot\sin\theta}\frac{\partial T}{\partial \phi} + k_{13}\frac{1}{r}\frac{\partial T}{\partial \theta}\right)$$

$$q''_\phi = -\left(k_{21}\frac{\partial T}{\partial r} + k_{22}\frac{1}{r\cdot\sin\theta}\frac{\partial T}{\partial \phi} + k_{23}\frac{1}{r}\frac{\partial T}{\partial \theta}\right)$$

$$q''_\theta = -\left(k_{31}\frac{\partial T}{\partial r} + k_{32}\frac{1}{r\cdot\sin\theta}\frac{\partial T}{\partial \phi} + k_{33}\frac{1}{r}\frac{\partial T}{\partial \theta}\right)$$

1-8 An infinitely long, solid cylinder (D = diameter) has the ability for uniform internal energy generation given by the rate g_o (W/m^3) by passing a current through the cylinder. Initially (t=0), the cylinder is at a uniform temperature T_o. The internal energy generation is then turned on (i.e. current passed) and maintained at a constant rate g_o, and at the same moment the cylinder is exposed to convection heat transfer with coefficient h (W/m^2 K) from a fluid at constant temperature T_∞, noting that $T_\infty > T_o$. The cylinder has uniform and constant thermal conductivity k (W/m K). The Biot number $hD/k \ll 1$. Solve for time t at which point the surface heat flux is exactly zero. Present your answer in variable form.

For $\frac{hD}{k} << 1$, use lumped analysis: $T = T(t)$.

$$\cancelto{0}{Q_{in}} + E_{gen} - Q_{out} = \rho V c \frac{\partial T}{\partial t}$$

$$\left(\frac{\pi}{4}D^2 L\right) g_o - (\pi D L) h (T - T_\infty) = \rho\left(\frac{\pi}{4}D^2 L\right) c \frac{\partial T}{\partial t}$$

$$\text{IC)} \quad T(t=0) = T_o$$

<u>now cancel (πL) & let $\theta(t) = T - T_\infty$</u>

$$\left.\begin{aligned} &\frac{\partial \theta}{\partial t} + \frac{4h}{\rho D c}\cdot\theta = \frac{g_o}{\rho c} \\ &\theta(t=0) = T_o - T_\infty \end{aligned}\right\} \quad \begin{gathered}\text{O.D.E.} \\ + \\ \text{I.C.}\end{gathered}$$

<u>now $\theta(t) = \theta_H + \theta_P$</u>

$$\Rightarrow \theta(t) = C_1 \cdot e^{-\frac{4h}{\rho D c}t} + \frac{g_o D}{4h}$$

$$\text{per IC)} \quad C_1 = (T_o - T_\infty) - \frac{g_o D}{4h}$$

1-8 2/2

$$\theta(t) = \left[T_0 - T_\infty - \frac{g_0 D}{4h}\right] e^{-\frac{4h}{\rho D c} t} + \frac{g_0 D}{4h}$$

then
$$T(t) = \left[T_0 - T_\infty - \frac{g_0 D}{4h}\right] e^{-\frac{4h}{\rho D c} t} + \frac{g_0 D}{4h} + T_\infty$$

Surface flux is zero at $t = t_0$ if $T(t_0) = T_\infty$:

$$T_\infty = \left(T_0 - T_\infty - \frac{g_0 D}{4h}\right) e^{-\frac{4h}{\rho D c} t_0} + \frac{g_0 D}{4h} + T_\infty$$

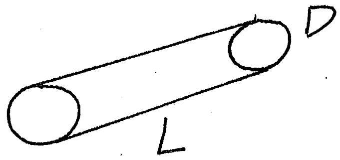
$$e^{-\frac{4h}{\rho D c} t_0} = \left(\frac{-g_0 D}{4h}\right) \frac{1}{\left(T_0 - T_\infty - \frac{g D}{4h}\right)}$$

Yields:
$$t_0 = \left(\frac{-\rho D c}{4h}\right) \ln\left[\frac{\left(\frac{-g_0 D}{4h}\right)}{\left(T_0 - T_\infty - \frac{g_0 D}{4h}\right)}\right]$$

where the term in the brackets is positive for $T_0 > T_\infty$.

1-9 A long cylindrical iron bar of diameter $D = 5$ cm, initially at temperature $T_0 = 650°C$, is exposed to an air stream at $T_\infty = 50°C$. The heat transfer coefficient between the air stream and the surface of the bar is $h = 80$ W/(m^2·K). Thermophysical properties are constant: $\rho = 7800$ kg/m^3, $c_v = 460$ J/(kg·K), and $k = 60$ W/(m·K). Determine the time required for the temperature of the bar to reach 250°C by using the lumped system analysis.

long cylinder



neglect ends

$$Bi = \frac{hD}{k} = \frac{(80)(.05)}{60} = 0.06 \ll 1$$

→ lumped analysis.

$$\cancelto{0}{Q_{in}} + \cancelto{0}{E_{gen}} - Q_{out} = \rho V c \frac{\partial T}{\partial t}$$

$$-hA(T - T_\infty) = \rho V c \frac{\partial T}{\partial t}$$

let $\theta(t) = T(t) - T_\infty$

$$\left.\begin{aligned} &\frac{d\theta}{dt} + \frac{hA}{\rho V c}\theta = 0 \\ &\theta(t=0) = T_0 - T_\infty \end{aligned}\right\} \begin{aligned} &\text{O.D.E.} \\ &+ \\ &\text{I.C.} \end{aligned}$$

$$\theta(t) = C_1 \, e^{\frac{-hA}{\rho V c} t}$$

$$C_1 = T_0 - T_\infty$$

$$\Rightarrow \quad \theta(t) = (T_0 - T_\infty)\, e^{\frac{-hA}{\rho V c} t}$$

1-9 E/2

$$\frac{T(t)-T_\infty}{T_0-T_\infty} = e^{-\frac{hA}{\rho V c}t}$$

where: $A = \pi D L$

$V = \frac{\pi}{4} D^2 L$

$$\Rightarrow \quad \frac{T(t)-T_\infty}{T_0-T_\infty} = e^{-\frac{4h}{\rho D c}t}$$

$$\frac{250-50}{650-50} = e^{\frac{-4(80)}{(7800)(.05)(460)}t}$$

$$\underline{t = 616\ s.}$$

1-10 A thermocouple is to be used to measure the temperature in a gas stream. The junction may be approximated as a sphere having thermal conductivity $k = 25$ W/(m·K), $\rho = 8400$ kg/m^3, and $c_v = 0.4$ kJ/(kg·K). The heat transfer coefficient between the junction and the gas stream is $h = 560$ W/(m^2·K). Calculate the diameter of the junction if the thermocouple should measure 95% of the applied temperature difference in 3 s.

→ Assume lumped analysis $\left(\frac{hD}{k} << 1\right)$

$$\cancelto{0}{Q_{in}} + \cancelto{0}{E_{gen}} - Q_{out} = \rho V c \frac{\partial T}{\partial t}$$

$$-hA(T - T_\infty) = \rho V c \frac{\partial T}{\partial t}$$

let $\theta(t) = T(t) - T_\infty$

$$\left.\begin{aligned} &\frac{d\theta}{dt} + \frac{hA}{\rho V c}\theta = 0 \\ &\theta(t=0) = T_0 - T_\infty \end{aligned}\right\} \quad \begin{gathered} \text{O.D.E.} \\ + \\ \text{I.C.} \end{gathered}$$

$$\theta(t) = C_1 \cdot e^{-\frac{hA}{\rho V c} t}$$

$$C_1 = T_0 - T_\infty$$

$$\Rightarrow \theta(t) = (T_0 - T_\infty)\, e^{-\frac{hA}{\rho V c} t}$$

$$A = \pi D^2$$

$$V = \frac{\pi}{6} D^3 \quad \Rightarrow \quad \frac{A}{V} = \frac{6}{D}$$

$$\Rightarrow \frac{T(t)-T_\infty}{T_0-T_\infty} = e^{-\frac{6h}{\rho D c}t}$$

To measure 95% of applied temp. difference,

$$\frac{T(t)-T_\infty}{T_0-T_\infty} = \frac{5}{100}$$

$$\Rightarrow \frac{5}{100} = e^{-\frac{6(560)}{(8400)(D)(400)}(3)}$$

$$\rightarrow D = 0.0010\,m$$
$$= 1.0\,mm$$

Check Bi #)

$$\frac{hD}{k} = \frac{(560)(.001)}{(25)}$$

$$= 0.022 \ll 1$$

$\rightarrow$ lumped analysis valid.

1/2

1-11 Determine the constants C_1 and C_2 for the constant-area fin solution of equations (1-100) for the case of the prescribed base temperature of equation (1-102), and the following tip conditions:

1. Convective tip per equation (1-103).
2. Insulated or symmetric tip per equation (1-104).
3. Prescribed temperature tip per equation (1-105).
4. Infinitely long fin per equation (1-106).

1) $\theta(x) = C_1 \cdot \cosh(mx) + C_2 \cdot \sinh(mx)$

BC1) $\theta(x=0) = \theta_b = C_1 \cdot 1 + C_2 \cdot 0$

$$\rightarrow \underline{C_1 = \theta_b}$$

BC2) $-k \dfrac{\partial \theta}{\partial x}\Big|_L = h\theta\Big|_L$

$$-k\left[\theta_b \cdot \sinh(mL) + C_2 \cdot \cosh(mL)\right](m)$$

$$= h\left[\theta_b \cdot \cosh(mL) + C_2 \cdot \sinh(mL)\right]$$

$$\rightarrow C_2 = -\frac{\theta_b\left[mk \cdot \sinh(mL) + h \cdot \cosh(mL)\right]}{\left[h \cdot \sinh(mL) + mk \cdot \cosh(mL)\right]}$$

2) $\theta(x) = C_1 \cdot \cosh(mx) + C_2 \cdot \sinh(mx)$

$\rightarrow$ $\underline{C_1 = \theta_b \text{ per above}}$

BC2) $\dfrac{\partial \theta}{\partial x}\Big|_{x=L} = 0$

$$0 = \left(\theta_b \cdot \sinh(mL) + C_2 \cdot \cosh(mL)\right)(m)$$

$$\rightarrow C_2 = \frac{-\theta_b \cdot \sinh(mL)}{\cosh(mL)} = -\theta_b \cdot \tanh(mL)$$

3) $\theta(x) = C_1 \cdot \cosh(mx) + C_2 \cdot \sinh(mx)$

$\rightarrow \underline{C_1 = \theta_b \text{ per above}}$

BC2) $\theta(x = L) = \theta_{tip}$

$$\theta_{tip} = \theta_b \cdot \cosh(mL) + C_2 \cdot \sinh(mL)$$

$$\rightarrow C_2 = \frac{\theta_{tip} - \theta_b \cdot \cosh(mL)}{\sinh(mL)}$$

4) $\theta(x) = C_1 e^{-mx} + C_2 e^{mx}$

BC1) $\theta(x=0) = \theta_b = C_1 + C_2$

BC2) $\theta(x \to \infty) = 0 = C_1(0) + C_2(\infty)$

$\rightarrow \underline{\underline{C_2 = 0}}$

then $\underline{\underline{C_1 = \theta_b}}$