1. (a)

$$n_i(T=300K) = 1.66\cdot 10^{15}(300)^{3/2}\cdot \exp\left[\frac{-(0.66eV)}{2(1.38\cdot 10^{-23} J/K)(300K)}\right]$$

$$= 2.5\cdot 10^{13}\ cm^{-3}$$

$$n_i(T=600K) = 1.66\cdot 10^{15}(600)^{3/2}\cdot \exp\left[\frac{-(0.66eV)}{2(1.38\cdot 10^{-23} J/K)(600K)}\right]$$

$$= 4.15\cdot 10^{16}\ cm^{-3}$$

Comparing these results with those in Example:

$$\frac{n_i(Ge @ 300K)}{n_i(Si @ 300K)} \approx 2315. \qquad \frac{n_i(Ge @ 600K)}{n_i(Si @ 600K)} \approx 27.$$

At higher temperature, the exponential terms approaches one, which implies that $n_i \sim T^{3/2}$, independent of bandgap energy, E_g.

(b) For any doped material, $n\cdot p = n_i^2$. Assuming at $T = 300K$,

$$p = 5\cdot 10^{16} cm^{-3}$$

$$n = [n_i(T=300K)]^2/p = \frac{(2.5\cdot 10^{13} cm^{-3})^2}{5\cdot 10^{16} cm^{-3}} = 1.25\cdot 10^{10} cm^{-3}$$

2 The electrons in a piece of *n*-type semiconductor take 10 ps to cross from one end to another end when a potential of 1 V is applied across it. Find the length of the semiconductor bar.

Solution

$$\frac{L}{V} = 10 \text{ ps}$$

$$\text{But} \quad V = \mu_n \mathrm{E} \quad \text{and} \quad E = \frac{V}{L}$$

$$L^2 = 10\,\text{ps} \times \mu_n V = 10\,\text{ps} \times 1350 \times 1$$

$$L = 1.16 \times 10^{-4} \text{ cm} = 1.16\ \mu\text{m}$$

3 A current of 0.05 μA flows through an n-type silicon bar of length 0.2 μm and cross section area of 0.01 μm × 0.01 μm when a voltage of 1 V is applied across it. Find the doping level at room temperature.

Solution

$$I = 0.05\mu\text{A}$$
$$L = 0.2\mu\text{m}$$
$$A = 0.01\mu\text{m} \times 0.01\mu\text{m} = 1\times 10^{-12}\,\text{cm}^2$$
$$I_{tot} = I_{\text{drift}} = q\left(n\mu_n + p\mu_p\right)AE \qquad \text{----(1)}$$
$$p = \frac{n_i^2}{n} = \frac{\left(1.08\times 10^{10}\right)^2}{n}$$
$$\mu_n = 1350cm^2/(\text{V.s}) \qquad \mu_p = 480\ \ \text{cm}^2/(\text{V.s})$$
$$E = \frac{V}{L} = \frac{1V}{0.2\mu\text{m}} = 5\times 10^4\,\text{V/cm}$$

Substituting these values in Eq. (1) we get

$$0.05\times 10^{-6}A = 1.6\times 10^{-19}\left[n\times 1350 + \frac{(1.08\times 10^{10})^2\times 480}{n}\right]10^{-12}\times 5\times 10^4$$
$$n = 4.63\times 10^{15}\ \ \text{cm}^{-3}.$$

4. Repeat Problem 2.3 for Ge. Assume μn=3900cm2/(V·s) and
μp =1900cm2/(V·s).

In order to obtain the doping level proceed as follows. We use the formula

$$\rho = \frac{1}{q(n\mu_n + p\mu_p)}$$

For n-type semiconductor *n*>>*p*, and hence

$$\rho = \frac{1}{nq\mu_n}$$

Given:-

$$I = 0.05\mu\text{A}$$
$$l = 0.2\mu\text{m}$$
$$A = (0.01\mu\text{m})^2$$
$$V = 1\text{V}$$
$$\mu_n = 3900\text{cm}^2/(\text{V}\cdot\text{s})$$
$$\mu_p = 1900\text{cm}^2/(\text{V}\cdot\text{s})$$

Resistance of the Ge bar;

$$R = \frac{V}{I} = \frac{1\text{V}}{0.05\mu\text{A}} = 20\text{M}\Omega$$

Therefore resistivity,

$$\rho = R\frac{A}{l} = 20\,\text{M}\Omega \times \frac{(0.01\mu\text{m})^2}{0.2\mu\text{m}} = 1\Omega\text{cm}$$

Again, $\rho = \dfrac{1}{ne\mu_n}$

or, $n = \dfrac{1}{\rho e\mu_n}$

Substituting the values, we get

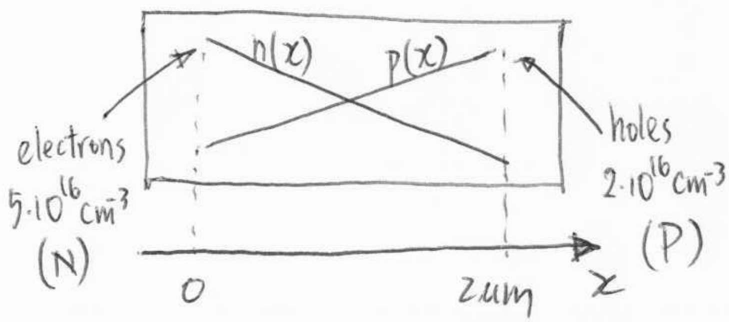
$$N_D = n = \frac{1}{1\Omega\text{cm}\times 1.6\times 10^{-19}\times 3900\,\text{cm}^2/(\text{V}\cdot\text{s})}$$

$$= \frac{1}{6.24\times 10^{-16}\,\text{cm}^3}$$

$$= \boxed{1.6\times 10^{15}\,\text{cm}^{-3}}$$

Since at room temperature all the impurities are ionized and hence carrier concentration n is equal to the doping density.

5.



Given
$D_n = 34\ cm^2/s$
$D_p = 12\ cm^2/s$
$L = 2\ \mu m$
$A = (1\ \mu m)^2$

The injected carriers diffuse from one end to the other.

$$I_{tot} = A \cdot J_{tot} = A \cdot q\left[\frac{dn}{dx}D_n - \frac{dp}{dx}D_p\right]$$

$$= A \cdot q\left[D_n\left(-\frac{N}{L}\right) - D_p\left(\frac{P}{L}\right)\right]$$

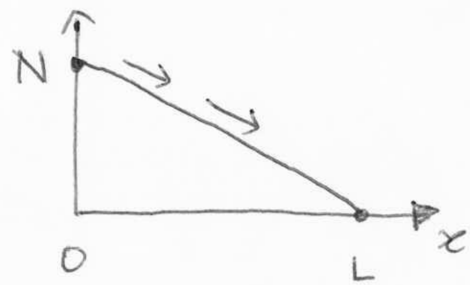
$$= (1\ \mu m)^2(1.6\cdot10^{-19}C)\left[34\frac{cm^2}{s}\left(-\frac{5\cdot10^{16}cm^{-3}}{2\ \mu m}\right) - 12\frac{cm^2}{s}\left(\frac{2\cdot10^{16}cm^{-3}}{2\ \mu m}\right)\right]$$

$$= -15.5\ \mu A.$$

6. Given Area $= a$

find total electrons stored.

$$n(x) = -\frac{N}{L}x + N$$



$\therefore$ total electrons stored

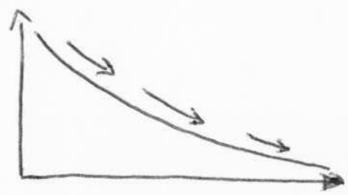
$$= \int a \cdot n(x)\, dx = \int_0^L a\left(-\frac{N}{L}x + N\right)dx$$

$$= aN\left(-\frac{x^2}{2L} + x\right)\Big|_0^L = \frac{aNL}{2}$$

7. Given Area = a

find total electrons stored.

$$n(x) = N \cdot \exp\left(\frac{-x}{L_d}\right)$$



$\therefore$ total electrons stored

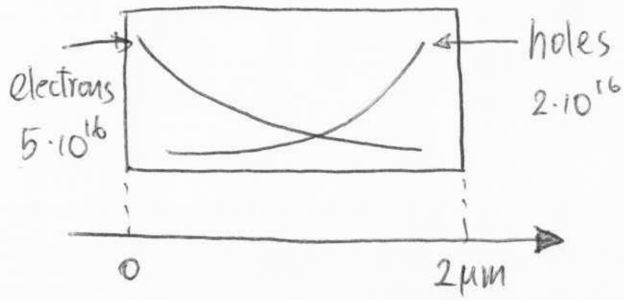
$$= \int a\, n(x)\, dx = \int_0^\infty a \cdot N \cdot \exp\left(\frac{-x}{L_d}\right) dx$$

$$= aN\left(-L_d \cdot \exp\frac{-x}{L_d}\right)\Big|_0^\infty = aNL_d.$$

For the linear profile, the result depends on the length, L.

For the exponential profile, the result is constant (since L_d is constant.)

8.



$$n(x) = N \exp(-x/L_d)$$

$$p(x) = P \exp\left(\frac{x-2}{L_d'}\right)$$

$N = 5\cdot10^{16}\,cm^{-3}$ $P = 2\cdot10^{16}\,cm^{-3}$

total number of electrons $= \int a\cdot n\, dx$

$$= \int_0^2 a\cdot n(x)\, dx = aN\left(-L_d\cdot \exp(-x/L_d)\right)\Big|_0^2$$

$$= aNL_d\left[1-\exp(-2/L_d)\right]$$

total number of holes $= \int a\cdot p\, dx$

$$= \int_0^2 a\cdot p(x)\, dx = aP\left(L_d'\cdot \exp\left(\frac{x-2}{L_d'}\right)\right)\Big|_0^2$$

$$= aPL_d'\left[1-\exp(-2/L_d')\right]$$

9 A Si semiconductor cube with side equal to 1 μm is doped with 4×10^{17} cm^{-3} phosphorous impurities. Calculate the drift current when a voltage of 5 V is applied across it.

Solution

$$n = 4\times10^{17}\ \text{cm}^{-3}$$
$$p = \frac{n_i^2}{n} = \frac{(1.08\times10^{10})^2}{4\times10^{17}} = 291.6$$
$$\mu_n = 1350\,\text{cm}^2/(\text{V.s}) \quad \text{and} \quad \mu_p = 480\ \ \text{cm}^2/(\text{V.s})$$
$$E = \frac{V}{L} = \frac{5\,\text{V}}{1\,\mu\text{m}} = 5\times10^4\,V/\text{cm}$$
$$A = 1\,\mu\text{m}\times1\,\mu\text{m} = 10^{-8}\,\text{cm}^2$$

Since $n\mu_n >> \mu_p$, we can write

$$I_{tot} \approx qn\mu_n AE = 0.0432\,\text{A}$$

10 One side of *pn* junction is doped with pentavalent impurities which gives a net doping of 5×10^{18} cm^{-3}. Find the doping concentration to be added to the other side to get a built-in potential of 0.6 V at room temperature.

Solution

$$V_0 = V_T \ln \frac{N_A N_D}{n_i^2}$$

$$N_D = 5 \times 10^{18} \text{ cm}^{-3},\ V_T = 26 \text{ mV},\ n_i = 1.08 \times 10^{10} \text{ cm}^{-3},\ V_0 = 0.6 \text{ V}$$

$$0.6 = 0.026 \ln\left(\frac{N_A \times 5 \times 10^{18}}{(1.08 \times 10^{10})^2}\right) \Rightarrow N_A = 2.45 \times 10^{11} \text{ cm}^{-3}$$

11 The built-in potential of an equally doped *pn* junction is 0.65 V at room temperature. When the doping level in *n*-side is doubled, keeping the doping of the *p*-side unchanged, calculate the new built-in potential and the doping level in *P* and *N* region.

Solution

$$V_0 = V_T \ln \frac{N_A N_D}{n_i^2}$$

$$N_A = N_D,\ \ V_T = 26\ \text{mV},\ \ n_i = 1.08\times10^{10}\ \ \text{cm}^{-3},\ \ V_0 = 0.65\ \text{V}$$

$$0.65 = 0.026\ \ln\left[\frac{N_A \times N_A}{(1.08\times10^{10})^2}\right] \Rightarrow N_A = 2.89\times10^{15}\ \ \text{cm}^{-3}$$

$$V_0 = 0.026\ \ln\left[\frac{2\times2.89\times10^{15}\times2.89\times10^{15}}{(1.08\times10^{10})^2}\right] = 0.668\ \text{V}$$

12 A silicon *pn* junction diode having $N_D = 10^{15}$ cm^{-3} and $N_A = 10^{17}$ cm^{-3} gives a total depletion capacitance of 0.41 pF/m^2. Determine the voltage applied across the diode.

Solution

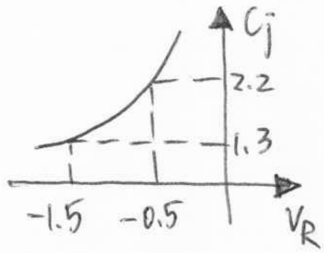
$$V_0 = V_T \ln \frac{N_A N_D}{n_i^2} = 0.026 \ln \frac{10^{15} \times 10^{17}}{(1.08 \times 10^{10})^2} = 0.7144 \text{ V}$$

$$C_{j0} = \sqrt{\frac{\varepsilon_{si} q}{2} \frac{N_A N_D}{N_A + N_D} \cdot \frac{1}{V_0}} = \sqrt{\frac{11.7 \times 8.854 \times 10^{-14} \times 10^{15} \times 10^{17}}{2 \times (10^{15} \times 10^{17}) \times 0.7144}} = 1.072 \times 10^{-8} \text{ F/cm}^2$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

$$V_R = V_0 \left[\left(\frac{C_{j0}}{C_j} \right)^2 - 1 \right] = 0.7144 \left(\frac{1.072 \times 10^{-8}}{0.41 \times 10^{-8}} \right) = 4.167 \text{ V}$$

13.



$$\frac{C_{j0}}{\sqrt{1+\frac{0.5}{V_0}}} = 2.2 \quad \text{—— ①}$$

$$\frac{C_{j0}}{\sqrt{1+\frac{1.5}{V_0}}} = 1.3 \quad \text{—— ②}$$

① ÷ ② : $\dfrac{1+1.5/V_0}{1+0.5/V_0} = \left(\dfrac{2.2}{1.3}\right)^2 \Rightarrow V_0 = 0.0365\ V$

Substitute V_0 into ①:

$$C_{j0} = 2.2\sqrt{1+\frac{0.5}{V_0}} \approx 8.43\ fF/\mu m^2$$

$$\Rightarrow \frac{N_A N_D}{N_A+N_D} = (C_{j0})^2 \cdot V_0 \cdot \frac{2}{\varepsilon_{si} q}$$

$$= \left(8.43\ \frac{fF}{\mu m^2}\right)^2 \times (0.0365V) \cdot \frac{2}{\varepsilon_{si} q} \approx 3.13 \cdot 10^{17}\ cm^{-3}$$

Fix a value for $N_A > \frac{N_A N_D}{N_A + N_D} \triangleq y$

$$N_A = 2 \cdot 10^{18}\, cm^{-3} \Rightarrow N_D = \frac{y N_A}{N_A - y}$$

$$= \frac{(3.13 \cdot 10^{17}\, cm^{-3})(2 \cdot 10^{18}\, cm^{-3})}{(2 \cdot 10^{18} - 3.13 \cdot 10^{17})\, cm^{-3}}$$

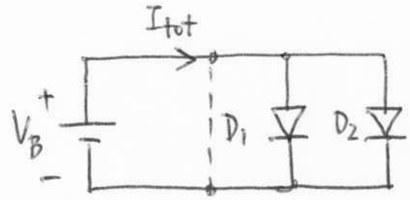
$$\approx 3.71 \cdot 10^{17}\, cm^{-3}$$

14 Two identical *pn* junction diodes are connected in series. Calculate the current flowing through each diode when a forward bias of 1 V is applied across the series combination. Assume the reverse saturation current is 1.44×10^{-17} A for each diode.

Solution

$$I_D = I_S\left(e^{\frac{V_F}{V_T}} - 1\right) = 1.44\times10^{-17}\left(e^{\frac{0.5}{0.026}} - 1\right) = 3.23\times10^{-9}\text{ A}$$

15 (a)



$$I_{tot} = I_{D_1} + I_{D_2} = I_{S_1}(e^{V_B/V_T}-1) + I_{S_2}(e^{V_B/V_T}-1)$$

$$= (I_{S_1} + I_{S_2})(e^{V_B/V_T}-1)$$

Therefore, the parallel combination operates as an exponential device, with an equivalent saturation current of $I_{S_1} + I_{S_2}$.

(b) By KVL, $V_{D_1} = V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{D_2}}{I_{S_2}}\right)$$

Also, $I_{tot} = I_{D_1} + I_{D_2} \Rightarrow I_{D_2} = I_{tot} - I_{D_1}$

$$\therefore V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{tot} - I_{D_1}}{I_{S_2}}\right)$$

$$\Rightarrow I_{D_1} = I_{tot}\left(\frac{I_{S_1}}{I_{S_1} + I_{S_2}}\right)$$

$$\Rightarrow I_{D_2} = I_{tot}\left(\frac{I_{S_2}}{I_{S_1} + I_{S_2}}\right)$$

16 Consider a *pn* junction in forward bias. Initially a current of 1 mA flows through it, and the current increases by 10 times when the forward voltage is increased by 1.5 times. Determine the initial bias applied and reverse saturation current.

Solution

$$I_D \approx I_S e^{\frac{V_F}{V_T}}$$

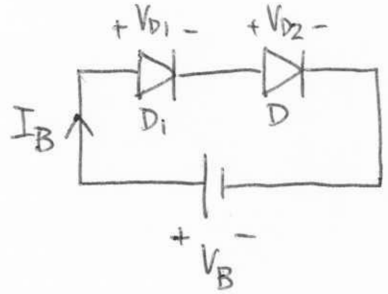
$$1 \text{ mA} = I_S e^{\frac{V_F}{V_T}} \qquad (1)$$

$$10 \text{ mA} = I_S e^{\frac{1.5V_F}{V_T}} \qquad (2)$$

$$V_F = \frac{V_T}{0.5} \ln(100) = 0.239 \text{ V}$$

From Eq. (1), we get $I_S = 10^{-7}$ A

17.



Find I_B, V_{D_1}, V_{D2} in terms of V_B, $I_{\ }$, I_{S2}

By KVL, $V_B = V_{D_1} + V_{D_2} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) + V_T \ln\left(\frac{I_B}{I_{S2}}\right)$

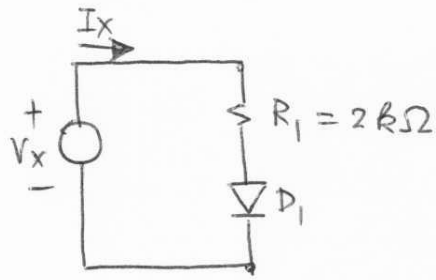
$$\Rightarrow \quad V_B = V_T \ln\left(\frac{I_B^2}{I_{S_1} I_{S_2}}\right)$$

$$\therefore \quad I_B = \sqrt{I_{S_1} I_{S2} \cdot \exp V_B/V_T} = \sqrt{I_{S_1} I_{S_2}} \cdot \exp\left(\frac{V_B}{2V_T}\right).$$

$$V_{D_1} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S_1}} \cdot \exp \frac{V_B}{2V_T}\right)$$
$$= V_T \ln\sqrt{\frac{I_{S2}}{I_{S_1}}} + \frac{V_B}{2}$$

$$V_{D_2} = V_T \ln\left(\frac{I_B}{I_{S_2}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S2}} \cdot \exp \frac{V_B}{2V_T}\right)$$
$$= V_T \ln\sqrt{\frac{I_{S_1}}{I_{S2}}} + \frac{V_B}{2}$$

18.



$$I_{D_1} = I_S\left(e^{\frac{V_{D_1}}{V_T}} - 1\right)$$

$$I_S = 2 \cdot 10^{-15} A$$

By KVL, $V_X = I_X R_1 + V_{D_1}$

$$\approx I_X R_1 + V_T \ln(I_{D_1}/I_S)$$

$$= I_X R_1 + V_T \ln(I_X/I_S)$$

This can be solved directly with special programs or graphing calculators. But this can be solved iteratively, by hand.

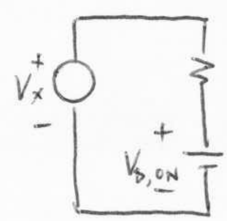
$\boxed{V_X = 0.5V}$

We suppose that D_1 is on.

$\Rightarrow$ current flows through D_1.

Assume a $V_{D,ON}$:

$\Rightarrow V_{D_1} = 0.4V$



$$\Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = \frac{(0.5-0.4)V}{2k\Omega} = 0.05\,mA$$

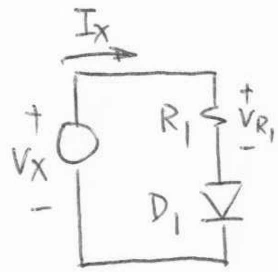
$$V_{D_1} = V_T \ln\left(\frac{I_X}{I_S}\right) = (0.026V)\ln\left(\frac{0.05mA}{2 \cdot 10^{-15}A}\right) \approx 0.62\,V$$

$\therefore$ Contradiction because V_{D_1} exceeds V_X !!

This means our assumption is incorrect

$\Rightarrow D_1$ is OFF $\Rightarrow V_{D_1} = V_X = 0.5V \quad I_X = 0$

19.



Given $V_{R_1} = V_X/2$, find V_X.
$I_S = 2 \cdot 10^{-16} A$.

By KCL,

$$\frac{V_{R_1}}{R_1} = I_S\left(e^{V_{D_1}/V_T} - 1\right)$$

Also, $V_{R_1} = V_{D_1} = V_X/2$ (KVL).

$$\therefore \frac{V_X/2}{R_1} = I_S \cdot \left(\exp\left[\frac{V_{D_1}/2}{V_T}\right] - 1\right)$$

This must be solved iteratively. From experience, suppose $V_X = 2V$.

$$V_X = 2V \Rightarrow I_X = \frac{V_X/2}{R_1} = \frac{1V}{2k\Omega} = 5mA$$

$$\Rightarrow V_X = 2 \cdot V_{D_1} = 2V_T \ln(I_X/I_S)$$

$$= 2(0.026V)\ln\left(\frac{5mA}{2 \cdot 10^{-16}A}\right) \approx 1.48V$$

$$V_X = 1.48V \Rightarrow I_X = \frac{1.48/2\,V}{2k\Omega} = 0.37\,mA$$

$$\Rightarrow V_X = 2(0.026V)\ln\left(\frac{0.37mA}{2 \cdot 10^{-16}A}\right) \approx 1.47V$$

$$V_X = 1.47V \Rightarrow I_X = \frac{(1.47)/2\,V}{2k\Omega} = 0.37\,mA$$

$$\Rightarrow V_X = 1.47\,V$$

$\boxed{V_X = 0.8\text{ V}}$ Suppose D_1 is on. (This is a reasonable assumption since most diodes turn on at around $V_D = 0.7$ V.)

For startup, use $V_{D_1} = 0.7$ V.

$$V_{D_1} = 0.7\text{ V} \Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = 0.05\text{ mA}$$

$$\Rightarrow V_{D_1} = V_T \ln(I_X / I_{S_1}) \approx 0.622\text{ V}$$

$$V_{D_1} = 0.622\text{ V} \Rightarrow I_X = \frac{(0.8 - 0.622)\text{V}}{2\text{ k}\Omega} = 0.089\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{ V}) \ln\left(\frac{0.089\text{ mA}}{2 \cdot 10^{-15}\text{ A}}\right) \approx 0.637\text{ V}$$

$$V_{D_1} = 0.637\text{ V} \Rightarrow I_X = \frac{(0.8 - 0.637)\text{V}}{2\text{ k}\Omega} = 0.082\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V}) \ln\left(\frac{0.082\text{ mA}}{2 \cdot 10^{-15}\text{ A}}\right) \approx 0.635\text{V}$$

$$V_{D_1} = 0.635\text{V} \Rightarrow I_X = \frac{(0.8 - 0.635)\text{V}}{2\text{ k}\Omega} = 0.083\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V}) \ln\left(\frac{0.083\text{ mA}}{2 \cdot 10^{-15}\text{ A}}\right) \approx 0.635\text{V}$$

$\therefore$ With an accuracy of three decimal points,

$V_{D_1} \approx 0.635$ V (of course, more iterations give a more accurate result.)

$I_X \approx 0.082$ mA.

$\boxed{V_X = 1\text{ V}}$ Suppose, again, that D_1 is on. Use V_{D_1} from previous calculations as starting point.

$$V_{D_1} = 0.635\text{ V} \Rightarrow I_X = \frac{(1-0.635)\text{V}}{2\text{ k}\Omega} = 0.18\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.18\text{mA}}{2\cdot 10^{-15}\text{A}}\right) \approx 0.656\text{ V}$$

$$V_{D_1} = 0.656\text{V} \Rightarrow I_X = \frac{(1-0.656)\text{V}}{2\text{ k}\Omega} = 0.17\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.17\text{mA}}{2\cdot 10^{-15}\text{A}}\right) \approx 0.655\text{ V}$$

$$V_{D_1} = 0.655\text{ V} \Rightarrow I_X = \frac{(1-0.655)\text{V}}{2\text{ k}\Omega} = 0.17\text{ mA}$$

$$\Rightarrow V_{D_1} = 0.655\text{ V}$$

$$\therefore\ V_{D_1} \approx 0.655\text{ V}$$
$$I_X \approx 0.17\text{ mA}.$$

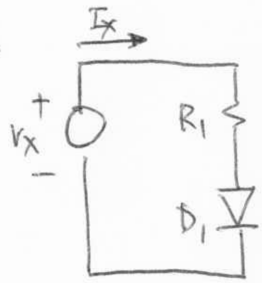
$\boxed{V_X = 1.2\text{ V}}$ Using similar assumptions as those in previous calculations,

$$V_{D_1} = 0.655\text{ V} \Rightarrow I_X = 0.27\text{ mA} \Rightarrow V_{D_1} \approx 0.667\text{V}$$
$$V_{D_1} = 0.667\text{ V} \Rightarrow I_X = 0.27\text{ mA} \Rightarrow V_{D_1} \approx 0.666\text{V}$$
$$V_{D_1} = 0.666\text{V} \Rightarrow I_X = 0.27\text{mA} \Rightarrow V_{D_1} \approx 0.666\text{V}$$

$$\therefore\ I_X \approx 0.27\text{mA} \qquad V_{D_1} = 0.666\text{ V}$$

For more than 3x increase in I_X, V_{D1} only increases by ~30mV, which is less than 10% of the turn-on voltage of the diode. In other words, once the diode conducts current, its voltage varies marginally (expected due to its exponential characteristic). This also implies that the diode, once on, can allow any amount of current to flow through (until $V_{D1} \times I_{D1}$ becomes so large that the diode simply "breaks down".)

20.



Given $V_X = 1V \Rightarrow I_X = 0.2mA$

$V_X = 2V \Rightarrow I_X = 0.5mA$

Find R_1 and I_S.

By KVL, $V_{D_1} = V_X - I_X R_1 = V_T \ln\left(\frac{I_X}{I_S}\right)$

$$\Rightarrow 1 - (0.2mA)R_1 = (0.026V)\ln\left(\frac{0.2mA}{I_S}\right) \quad \text{—— ①}$$

$$2 - (0.5mA)R_1 = (0.026V)\ln\left(\frac{0.5mA}{I_S}\right) \quad \text{—— ②}$$

$$② - ① : \quad 1 - (0.3mA)R_1 = (0.026V)\ln\left(\frac{0.5}{0.2}\right)$$

$$\Rightarrow R_1 = \frac{1-(0.026)\ V}{0.3m\ A} = 3.25\ k\Omega$$

Substitute R_1 into ①:

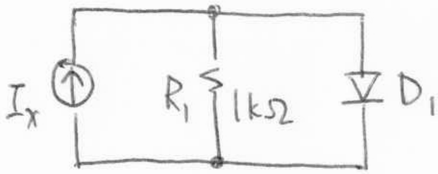
$$I_S = I_X \cdot \exp\left[-\frac{V_X - I_X R_1}{V_T}\right]$$

$$= (0.2mA)\exp\left[-\frac{1-(0.2m)(3.25k)}{0.026}\right] \approx 2.94 \cdot 10^{-10} A$$

$\therefore R_1 \approx 3.25\ k\Omega$

$I_S \approx 2.94 \cdot 10^{-10} A.$

21.



Given $I_S = 3 \cdot 10^{-16} A$, find V_{D_1}.

By KCL, $I_X = \dfrac{V_{D_1}}{R_1} + I_{D_1} = \dfrac{V_T}{R_1} \ln\left(\dfrac{I_{D_1}}{I_S}\right) + I_{D_1}$

Since I_X, V_T, R_1 and I_S are known, this can be solved directly with special programs or graphing calculators. However, this can be also solved by iterations: Assume a V_{D_1}, calculate I_{D_1}, and re-iterate on V_{D_1}.

Assume $V_{D_1} = 0.7\ V$ as starting point.

$\boxed{I_X = 1mA}$

$V_{D_1} = 0.7\ V \Rightarrow I_{D_1} = I_X - V_{D_1}/R_1 = 1mA - \dfrac{0.7V}{1k\Omega} = 0.3mA$

$\Rightarrow V_{D_1} = V_T \ln\left(I_X/I_S\right)$

$= (0.026V) \ln\left(\dfrac{0.3mA}{3 \cdot 10^{-16}A}\right) \approx 0.718\,V$

$V_{D_1} = 0.718V \Rightarrow I_{D_1} = 1\,mA - \dfrac{0.718\,V}{1\,k\Omega} = 0.28\,mA$

$\Rightarrow V_{D_1} = (0.026V) \ln\left(\dfrac{0.28mA}{3 \cdot 10^{-16}A}\right) \approx 0.717\,V$

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 1mA - \dfrac{0.717V}{1k\Omega} = 0.28mA$

$\Rightarrow V_{D_1} = 0.717\,V$

$\therefore V_{D_1} \approx 0.717\,V.$

$\boxed{I_X = 2mA}$ Assume $V_{D_1} = 0.717\,V$ from previous result.

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 2mA - \dfrac{0.717V}{1k\Omega} = 1.28\,mA$

$\Rightarrow V_{D_1} = (0.026V)\ln\left(\dfrac{1.28mA}{3\cdot10^{-16}A}\right) \approx 0.756V$

$V_{D_1} = 0.756\,V \Rightarrow I_{D_1} = 2mA - \dfrac{0.756V}{1k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\dfrac{1.24mA}{3\cdot10^{-16}A}\right) \approx 0.755V$

$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 2mA - \dfrac{0.755\,V}{1\,k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = 0.755V$

$\therefore V_{D_1} = 0.755V$

$\boxed{I_X = 4\,mA}$ Assume $V_{D_1} = 0.755\,V$ from previous result.

$$V_{D_1} = 0.755\,V \Rightarrow I_{D_1} = 4\,mA - \frac{0.755\,V}{1\,k\Omega} = 3.25\,mA$$

$$\Rightarrow V_{D_1} = (0.026)V \ln\left(\frac{3.25\,mA}{3\cdot 10^{-16}\,A}\right) \approx 0.780\,V$$

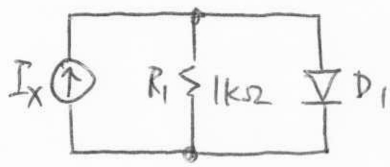
$$V_{D_1} = 0.780\,V \Rightarrow I_{D_1} = 4\,mA - \frac{0.780\,V}{1\,k\Omega} = 3.22\,mA$$

$$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\frac{3.22\,mA}{3\cdot 10^{-16}\,A}\right) \approx 0.780\,V$$

$\therefore V_{D_1} \approx 0.780\,V.$

Note: As I_X increases, I_{D_1} increases, while (V_{D1}/R_1) stays relatively the same. Because of the exponential characteristic, the diode, once on, will <u>absorb</u> as much current as necessary to satisfy KCL.

22.



Given $I_{D_1} = 0.5mA$ when $I_X = 1.3mA$, find I_S.

This means $V_{D_1} = (I_X - I_{D_1})R_1$

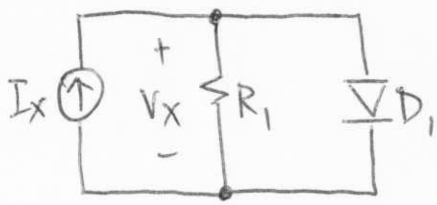
$$= (0.8mA)1k\Omega = 0.8V$$

$$\Rightarrow I_S = I_{D_1} \cdot \exp\left[-V_{D_1}/V_T\right]$$

$$= (0.5mA)\exp\left[-0.8V/0.026V\right]$$

$$\approx 2.17 \cdot 10^{-17} A$$

23.



Given $I_X = 1mA \rightarrow V_X = 1.2V$
$I_X = 2mA \rightarrow V_X = 1.8V$

find R_1 and I_S.

$$I_{D_1} = I_X - V_X/R_1 \quad (KCL)$$

By KVL, $V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X - V_X/R_1}{I_S}\right)$

$$\Rightarrow (1.2\,V) = (0.026\,V)\ln\left[\frac{(1mA) - (1.2V)/R_1}{I_S}\right] \text{ —— } ①$$

$$(1.8\,V) = (0.026\,V)\ln\left[\frac{(2mA) - (1.8V)/R_1}{I_S}\right] \text{ —— } ②$$

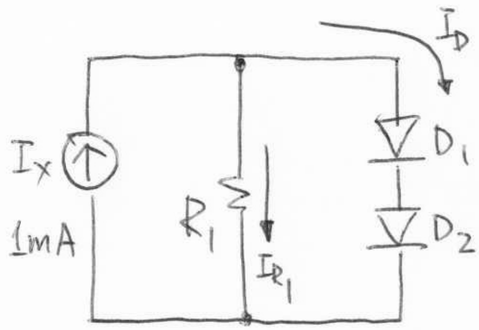
$$② - ①: \quad 0.6V = (0.026V)\ln\left(\frac{2mA - 1.8V/R_1}{1mA - 1.2V/R_1}\right)$$

$$\Rightarrow R_1 = \frac{1.2 \cdot \exp[0.6/0.026] - 1.8}{1mA \cdot \exp[0.6/0.026] - 2mA} \approx 1.2\,k\Omega$$

$$I_S = I_D \exp\left[-V_X/V_T\right] = \left(2mA - \frac{1.8V}{1.2k\Omega}\right)\exp\left[\frac{-1.8V}{0.026V}\right]$$

$$\approx 4.29 \cdot 10^{-34}\,A.$$

24.



Given $I_{R_1} = 0.5\text{ mA}$, $I_S = 5\cdot 10^{-16}\text{ A}$ for each diode.

Find R_1.

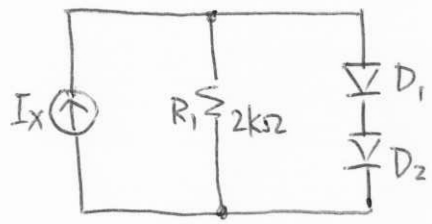
By KCL, $I_D = I_X - I_{R_1} = 0.5\text{ mA}$

$$\Rightarrow V_{D_1} = V_{D_2} = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.026 \ln\left(\frac{0.5\text{mA}}{5\cdot 10^{-16}\text{A}}\right)$$

$$\approx 0.718\text{ V}$$

$$\therefore R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{2V_{D_1}}{I_{R_1}} = \frac{2(0.718\text{V})}{0.5\text{mA}} = 2.87\text{ k}\Omega$$

25.



Given $D_1 = D_2$ with $I_S = 5 \cdot 10^{-16} A$

Find V_{R_1} for $I_X = 2mA$.

Current through the diodes $= I_D$
$= I_X - \frac{V_{R_1}}{R_1}$ where V_{R_1} = voltage across R_1

$$\Rightarrow V_{R_1} = 2 \cdot V_T \ln\left(\frac{I_D}{I_S}\right) = 2\left[V_T \ln\left(\frac{I_X}{I_S} - \frac{V_{R_1}}{I_S R_1}\right)\right]$$

This can be solved directly with special programs or graphing calculators or by hand iteratively.

Assume a V_{R_1}, calculate I_D, and re-iterate on new $V_{R_1} = (2 \times V_{D_1})$. From experience, most diodes conduct at $V_D \approx 0.7V$. Assume $V_{R_1} = 1.4V$.

$$V_{R_1} = 1.4 V \Rightarrow I_D = I_X - \frac{V_{R_1}}{R_1} = 2mA - \frac{1.4V}{2k\Omega} = 1.3mA$$

$$\Rightarrow V_{R_1} = 2 V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$= 2(0.026V)\ln\left(\frac{1.3mA}{5 \times 10^{-16}A}\right) \approx 1.49 V$$

$$V_{R_1} = 1.49\,V \Rightarrow I_D = 2mA - \frac{1.49}{2k\Omega} = 1.26\ mA$$

$$\Rightarrow V_{R_1} = 2(0.026V)\ln\left(\frac{1.26\,mA}{5\cdot 10^{-16}A}\right) \approx 1.48\,V$$

$$V_{R_1} = 1.48\,V \Rightarrow I_D = 2mA - \frac{1.48\,V}{2k\Omega} = 1.26\ mA$$

$$\Rightarrow V_{R_1} = 1.48\,V$$

$\therefore$ voltage across $R_1 = 1.48\,V$