CHAPTER 1

Problem 1:

(a) Trapezoidal channel with side slopes m_1 and m_2

$$A = (b + m_1 \frac{y}{2} + m_2 \frac{y}{2})y$$
$$T = b + y(m_1 + m_2)$$
$$P = b + y\sqrt{1 + m_1^2} + y\sqrt{1 + m_2^2}$$
$$D = A/T$$
$$R = A/P$$

(b) Trapezoidal channel with one vertical side
Set $m_1 = 0$ and $m_2 = m$ in the equations given in part (a).

(c) Right triangular channel
Set $m_1 = 0$, $b = 0$, and $m_2 = m$ in the equations given in part (a).

Problem 2:

(a) From the problem statement:
$\gamma = 62.4 \ lbf/ft^3$
Y_c = depth to centroid = 2 ft
$A =$ d $(1 \ ft/ft) = 4 \ ft \ (1 \ ft/ft) = 4 \ ft^2/ft$

By using Equation 1.9
$F_p = \gamma Y_c A = 62.4(2)(4) = 499 \ lbf \ / \ foot$
The e hydrostatic pressure force is normal to the vertical sidewall.

(b) From the problem statement:
$\gamma = 62.4 \ lbf/ft^3$
Y_c = depth to centroid = 2 ft
$A = (d^2 + (md)^2)^{1/2} \ (1 \ ft/ft) = (4^2 + (2x4)^2)^{1/2} \ ft \ (1 \ ft/ft) = 8.94 \ ft^2/ft$

By using Equation 1.9
$F_p = \gamma Y_c A = 62.4(2)(4) = 1116 \ lbf \ / \ foot$
The hydrostatic pressure force is normal to the inclined side.

Problem 3:

$$v = 2.5v_* \ln\left(\frac{30z}{k_s}\right)$$

Let $k = k_s/30$. Then

$$v = 2.5v_* \ln\left(\frac{z}{k}\right)$$

Define q= discharge per unit width. By definition (see Equation 1.2)

$$q = \int_k^y v\,dz = \int_k^y \left[2.5v_* \ln\left(\frac{z}{k}\right)\right]dz = \int_k^y [2.5v_* \ln z]dz - \int_k^y [2.5v_* \ln k]dz$$

$$q = 2.5v_*[z\ln z - z]_k^y - 2.5v_*[z\ln k]_k^y = 2.5v_*\left[y\ln\frac{y}{k} - (y-k)\right]$$

Problem 4:

Noting that q= discharge per unit width and $k = k_s/30$ as in Problem 3, by definition (see Equation 1.3)

$$V = \frac{q}{y-k} = 2.5v_*\left[\frac{y\ln\frac{y}{k} - (y-k)}{y-k}\right] = 2.5v_*\left[\frac{y\ln\frac{y}{k}}{y-k} - 1\right]$$

Because $y \gg k$, we have $(y-k) \approx y$. Therefore

$$V = 2.5v_*\left[\ln\frac{y}{k} - 1\right]$$

Problem 5:

For the velocity distribution given, obviously, the velocity is maximum at the free surface. Substituting $z = y$ in
$$v = 2.5v_* \ln\left(\frac{30z}{k_s}\right)$$
we obtain

$$v_{max} = 2.5v_* \ln\left(\frac{30y}{k_s}\right) = 2.5v_* \ln\left(\frac{y}{k}\right)$$

with $k = k_s/30$.

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Problem 6:

Using the results of Problems 4 and 5

$$\frac{v_{max}}{V} - 1 = \frac{2.5v_* \ln\left(\dfrac{y}{k}\right)}{2.5v_*\left[\ln\dfrac{y}{k} - 1\right]} - 1 = \frac{1}{\ln\dfrac{y}{k} - 1}$$

with $k = k_s/30$.

Problem 7:

We found an expression for V in Problem 4. Here we will determine the value of z for which $v=V$. In other words with $k = k_s/30$

$$2.5v_* \ln\left(\frac{z}{k}\right) = 2.5v_*\left[\ln\frac{y}{k} - 1\right]$$

$$\ln\left(\frac{z}{k}\right) = \left[\ln\frac{y}{k} - 1\right] = \ln\left(\frac{y}{k}\right) - \ln(2.718) = \ln\left(\frac{y/k}{2.718}\right)$$

and

$$z = \frac{y}{2.718} = 0.37y$$

The point velocity at distance $0.37y$ from the bottom or $0.63y$ from the free surface will be equal to the cross sectional average velocity. Therefore, the velocity measured at $0.6y$ from the surface will be a good approximation to the average velocity.

Problem 8:

With $k = k_s/30$, we have

$$v = 2.5v_* \ln\left(\frac{z}{k}\right)$$

By definition

$$\beta = \frac{1}{V^2 A}\int v^2 dA$$

Using the velocity distribution given, for unit width, we can write

$$\beta = \frac{(2.5v_*)^2}{V^2(y-k)}\int_k^y \ln^2\left(\frac{z}{k}\right)dz$$

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Let us first evaluate the integration

$$\int_k^y \ln^2\left(\frac{z}{k}\right)dz = \int_k^y (\ln z - \ln k)(\ln z - \ln k)dz = \int_k^y (\ln^2 z - 2\ln z \ln k + \ln^2 k)dz$$

$$\int_k^y \ln^2\left(\frac{z}{k}\right)dz = \left[z\ln^2 z\right]_k^y - 2\int_k^y \ln z dz - 2\ln k \int_k^y \ln z dz + \ln^2 k \int_k^y dz$$

$$\int_k^y \ln^2\left(\frac{z}{k}\right)dz = \left[z\ln^2 z - 2(z\ln z - z) - 2\ln k(z\ln z - z) + \ln^2 kz\right]_k^y$$

$$\int_k^y \ln^2\left(\frac{z}{k}\right)dz = 2(y-k) + y\ln^2(\frac{y}{k}) - 2y\ln(\frac{y}{k}) \qquad (*)$$

Also from Problem 4

$$V = 2.5v_*\left[\ln\frac{y}{k} - 1\right]$$

$$V^2 = (2.5v_*)^2\left[\ln\frac{y}{k} - 1\right]^2 = (2.5v_*)^2\left[\ln^2\frac{y}{k} - 2\ln\frac{y}{k} + 1\right]$$

With $y >> k$ and $(y-k) \approx y$

$$V^2(y-k) = (2.5v_*)^2 y\left[\ln^2\frac{y}{k} - 2\ln\frac{y}{k} + 1\right] \qquad (**)$$

Substituting Equations (*) and (**) into the expression for β

$$\beta = \frac{\left[(2.5v_*)^2\right]2(y-k) + y\ln^2(\frac{y}{k}) - 2y\ln(\frac{y}{k})}{(2.5v_*)^2 y\left[\ln^2\frac{y}{k} - 2\ln\frac{y}{k} + 1\right]}$$

Noting $(y-k) \approx y$ and simplifying

$$\beta = \frac{y + y\left[1 + \ln^2(\frac{y}{k}) - 2\ln(\frac{y}{k})\right]}{y\left[\ln^2\frac{y}{k} - 2\ln\frac{y}{k} + 1\right]} = 1 + \frac{1}{\left[\ln\frac{y}{k} - 1\right]^2}$$

From Problem 6

$$\frac{v_{max}}{V} - 1 = \frac{1}{\ln\frac{y}{k} - 1}$$

Substituting this into the expression for β and rearranging

$$\beta = 1 + \left[\frac{v_{max}}{V} - 1\right]^2$$

Problem 9:

$$v_* = \left(\frac{\tau_0}{\rho}\right)^{1/2} = \left(\frac{3.7}{1000}\right)^{1/2} = 0.061 m/s$$

From Problem 4

$$V = 2.5 v_* \left[\ln\frac{y}{k} - 1\right] = 2.5 v_* \left[\ln\frac{30y}{k_s} - 1\right] = (2.5)(0.061)\left[\ln\frac{30(0.94)}{(0.001)} - 1\right] = 1.41 m/s$$

$$q = \left[y - \frac{k_s}{30}\right]V \approx yV = (0.94)(1.41) = 1.33 m^2/s$$

From Problem 5

$$\frac{v_{max}}{V} - 1 = \frac{1}{\ln\frac{30y}{k_s} - 1} = \frac{1}{\ln\frac{30(0.94)}{(0.001)} - 1} = 0.11$$

$$\beta = 1 + \left[\frac{v_{max}}{V} - 1\right]^2 = 1 + (0.11)^2 = 1.01$$

Rate of momentum transfer $= \beta\rho q V = (1.01)(1000\frac{kg}{m^3})(1.33\frac{m^2}{s})(1.41\frac{m}{s}) = 1894\frac{kgm}{s^2}/m$

$$\alpha = 1 + 3\left[\frac{v_{max}}{V} - 1\right]^2 - 2\left[\frac{v_{max}}{V} - 1\right]^3 = 1 + 3(0.11)^2 - 2(0.11)^3 = 1.03$$

Rate of kinetic transfer $= \alpha\frac{\rho}{2}qV^2 = (1.03)\frac{1}{2}(1000\frac{kg}{m^3})(1.33\frac{m^2}{s})(1.41\frac{m}{s})^2 = 1362\frac{Nm}{s}/m$

Problem 10:

$A_1 = 1\ m$ x $20\ m = 20\ m^2$
$A_2 = 4\ m$ x $4\ m = 16\ m^2$
$A_3 = 0.5(1\ m$ x $22\ m) = 11\ m^2$

$$Q = V_1 A_1 + V_2 A_2 + V_3 A_3 = 0.5(20) + 1.5(16) + 0.3(11) = 37.3 \ m^3/s$$

$$V = \frac{Q}{A} = \frac{Q}{A_1 + A_2 + A_3} = \frac{37.3}{20 + 16 + 11} = 0.79 \ m/s$$

Problem 11:

From Equation 1.15

$$\beta = \frac{V_1^2 A_1 + V_2^2 A_2 + V_3^2 A_3}{V^2 A} = \frac{0.5^2(20) + 1.5^2(16) + 0.3^2(11)}{0.79^2(47)} = 1.43$$

Then by using Equation 1.11 with $\rho = 1000 \ kg/m^3$

Rate of momentum transfer $= \beta \rho Q V = 1.43(1000)(37.3)(0.79) = 42,138 \ kg \cdot m/s^2$

Likewise, from Equation 1.21

$$\alpha = \frac{V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3}{V^3 A} = \frac{0.5^3(20) + 1.5^3(16) + 0.3^3(11)}{0.79^3(47)} = 2.45$$

Then by using Equation 1.19

Rate of kinetic energy transfer $= \alpha \frac{\rho}{2} Q V^2 = 2.45 \left(\frac{1000}{2} \right)(37.3)(0.79)^2 = 28,517 \ kg \cdot m/s^3$

Problem 12:

$$V = \frac{Q}{A} = \frac{Q}{y(b + my)} = \frac{100}{3.15(5 + 2 \times 3.15)} = \frac{100}{35.6} = 2.81 \ fps$$

$$R = \frac{A}{P} = \frac{y(b + my)}{b + 2y\sqrt{1 + m^2}} = \frac{3.15(5 + 2 \times 3.15)}{5 + 2(3.15)\sqrt{1 + 2^2}} = \frac{35.6}{19.1} = 1.86 \ ft$$

$$D = \frac{y(b + my)}{b + 2my} = \frac{3.15(5 + 2 \times 3.15)}{5 + 2 \times 2 \times 3.15} = \frac{35.6}{17.6} = 2.02 \ ft$$

(a) Using Equation 1.23

$$R_e = \frac{4VR}{\nu} = \frac{4VR}{\nu} = \frac{4(2.81)(1.86)}{1.217 \times 10^{-5}} = 1,925,000$$

The flow is turbulent.

(b) Using Equation 1.24

$$F_r = \frac{V}{\sqrt{gD}} = \frac{2.81}{\sqrt{32.2 \times 2.02}} = 0.35$$

The flow is subcritical.

Problem 13:

(a) Nonuniform
(b) Nonuniform
(c) Uniform
(d) Nonuniform

Problem 14:

(a) Unsteady
(b) Steady
(c) Steady

Problem 15:

Verify that

$$\frac{\partial (AY_C)}{\partial x} = A\frac{\partial y}{\partial x}$$

Consider a horizontal strip of flow area having a length of b and thickness of dz and extending from one side of a flow section to the other side. Suppose the vertical distance between the centroid of this strip and the channel bottom is z. Then by definition

$$\frac{\partial (AY_C)}{\partial x} = \frac{\partial}{\partial x}\int_0^y (y - z)b\,dz$$

Using the Leibnitz rule, we can write this as

$$\frac{\partial (AY_C)}{\partial x} = \int_0^y \frac{\partial}{\partial x}(y - z)b\,dz + (y - y)b\,dz\frac{\partial y}{\partial x} - (0 - z)b\,dz\frac{\partial 0}{\partial x}$$

The last two terms on the right hand side are zero. Then

$$\frac{\partial (AY_C)}{\partial x} = \int_0^y \frac{\partial}{\partial x}(y - z)b\,dz = \int_0^y \frac{\partial}{\partial x}(yb\,dz) - \int_0^y \frac{\partial}{\partial x}(zb\,dz)$$

Since z and b are not functions of x at a given section, the last term on the right hand side is zero. Also, since y is not a function of z and b and dz are not functions of x

$$\frac{\partial (AY_C)}{\partial x} = \int_0^y \frac{\partial}{\partial x}(yb\,dz) = \frac{\partial y}{\partial x}\int_0^y b\,dz = A\frac{\partial y}{\partial x}$$

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