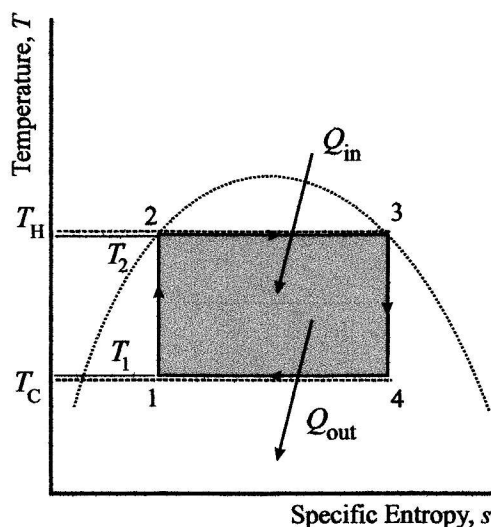


Chapter 3 Solutions

P3.1



Conditions at 1

$$p_1 = 0.5\text{bar}; t_1 = 81.3^\circ\text{C}; v_{g1} = 3.239\text{m}^3/\text{kg}$$

$$h_{f1} = 340\text{kJ/kg}; h_{g1} = 2645\text{kJ/kg}; s_{f1} = 1.091\text{kJ/kgK}; s_{g1} = 7.593\text{kJ/kgK}$$

Conditions at 2

$$p_2 = 20\text{bar}; t_2 = 212.4^\circ\text{C}; v_{g2} = 0.09957\text{m}^3/\text{kg}; h_{f2} = 909\text{kJ/kg}; s_{f2} = 2.447\text{kJ/kgK}; s_{g2} = 6.340\text{kJ/kgK}$$

Conditions at 3

$$p_3 = 20\text{bar}; s_3 = 6.340\text{kJ/kgK}; h_3 = 2799\text{kJ/kg}$$

Energy added: $q_{23} = h_3 - h_2 = 2799 - 909 = 1890\text{kJ/kg}$

Conditions at 4

$$p_4 = 0.5\text{bar}; s_4 = s_3 = 6.340\text{kJ/kgK}, \text{ thus } x_4 = \frac{6.340 - 1.091}{7.593 - 1.091} = 0.8073$$

$$h_4 = xh_g + (1-x)h_f = 0.8073 \times 2645 + (1 - 0.8073) \times 340 = 2200.8\text{kJ/kg}$$

Conditions at 1

$$p_1 = 0.5\text{bar}; s_1 = s_2 = 2.447\text{kJ/kgK}, \text{ thus } x_1 = \frac{2.447 - 1.091}{7.593 - 1.091} = 0.2086$$

$$h_1 = xh_g + (1-x)h_f = 0.2086 \times 2645 + (1 - 0.2086) \times 340 = 820.8\text{kJ/kg}$$

$$q_{41} = h_4 - h_1 = -1380\text{kJ/kg}$$

$$w_{\text{net}} = 510\text{kJ/kg}$$

$$\eta_{\text{th}} = 510/1890 = 0.2698 = 26.98\%$$

$$\text{Carnot efficiency, } \eta_{\text{th}} = 1 - \frac{T_2}{T_1} = 1 - \frac{354.3}{485.4} = 0.2700 = 27.0\%$$

Chapter 3 Solutions

P3.2

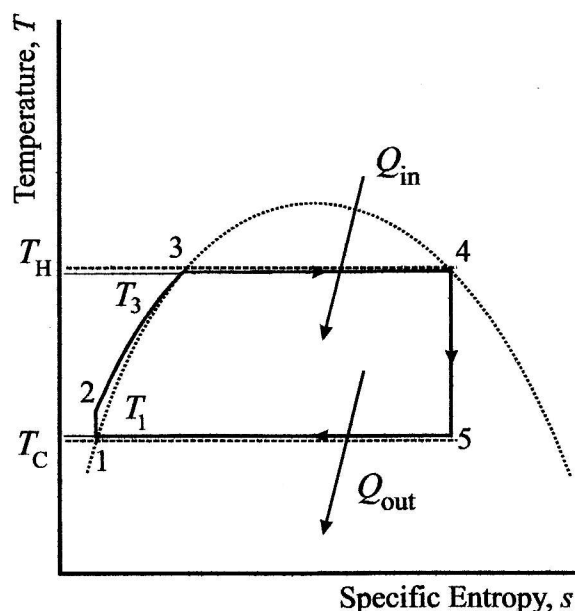
Conditions at 1

$$p_1 = 0.5 \text{ bar}; t_1 = 81.3^\circ\text{C}; v_{g1} = 3.239 \text{ m}^3/\text{kg}$$

$$h_{f1} = 340 \text{ kJ/kg}; h_{g1} = 2645 \text{ kJ/kg}; s_{f1} = 1.091 \text{ kJ/kgK}; s_{g1} = 7.593 \text{ kJ/kgK}$$

Conditions at 3

$$p_3 = 20 \text{ bar}; t_3 = 212.4^\circ\text{C}; v_{g3} = 0.09957 \text{ m}^3/\text{kg} \quad h_{f3} = 909 \text{ kJ/kg}; s_{f3} = 2.447 \text{ kJ/kgK}; s_{g3} = 6.340 \text{ kJ/kgK}$$



Many conditions remain the same as in P3.1: note change in state point numbering.

$$\text{Feed pump work, } w_p = w_{12} = -v dp. \quad v = \left\{ 0.1029 + \frac{0.5 - 0.4736}{0.578 - 0.4736} \times 0.0003 \right\} \times 10^{-2} = 0.1030 \times 10^{-2} \text{ m}^3/\text{kg}.$$

$$\text{Hence, } w_p = -0.1030 \times 10^{-2} \times (20.0 - 0.5) = -2 \text{ kJ/kg}.$$

$$h_2 = 342 \text{ kJ/kg}$$

$$\text{Energy added, } q_{in} = h_4 - h_2 = 2799 - 342 = 2457 \text{ kJ/kg}.$$

$$\text{Energy rejected, } q_{out} = h_1 - h_5 = 340 - 2200.8 = -1861 \text{ kJ/kg}.$$

$$\text{Thus efficiency, } \eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1861}{2457} = 24.25\%.$$

$$\text{Alternative calculation, } \eta_{th} = \frac{w_{net}}{q_{in}} = \frac{596}{2457} = 0.2425 \equiv 24.25\%$$

Back work ratio

$$\text{Rankine } r_{wb} = \frac{w_p}{w_T} = \frac{2}{598} = 0.0034 \equiv 0.34\%$$

$$\text{Carnot } r_{wb} = \frac{w_p}{w_T} = \frac{88}{598} = 0.147 \equiv 14.7\%$$

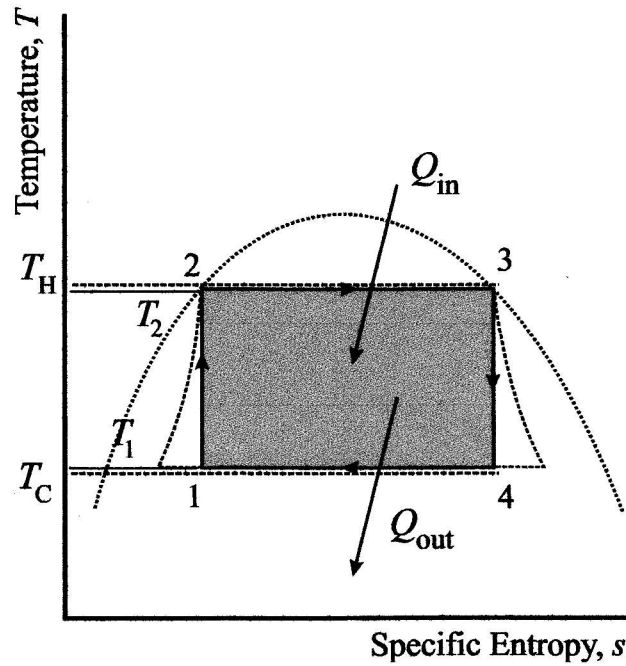
The high value of r_{wb} for the Carnot cycle shows that it is very susceptible to inefficiencies in the pump, and turbine.

Chapter 3 Solutions

P3.3

Recalculating P3.1 with $\eta_p = 0.8$ and $\eta_T = 0.9$

$$w_p = -\frac{88.2}{0.8} = -110.25 \text{ kJ/kg}; w_T = 0.9 \times 598 = 538.2 \text{ kJ/kg}$$

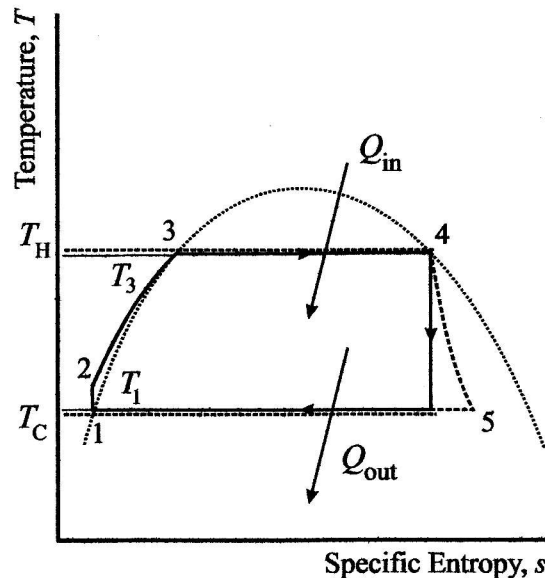


Thus

$$\eta_{th} = \frac{538.2 - 110.25}{1890} = 22.64\%$$

$$r_{wb} = \frac{110.25}{538.2} = 20.48\%$$

P3.4



Recalculating P3.2 using $\eta_p = 0.8$ and $\eta_T = 0.9$

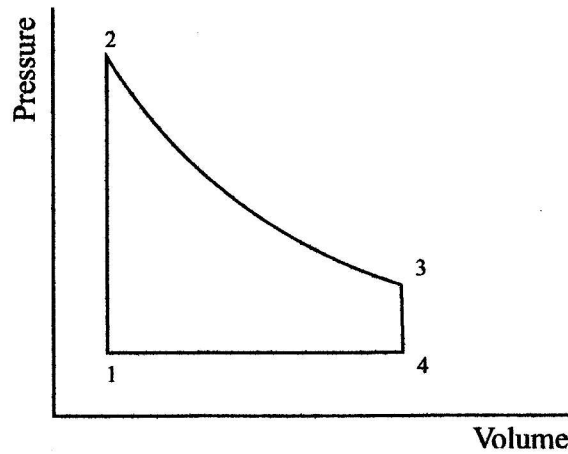
$$w_p = -\frac{2}{0.8} = -2.5 \text{ kJ/kg}; w_T = 0.9 \times 598 = 538.2 \text{ kJ/kg}$$

Chapter 3 Solutions

$$\text{Thus } \eta_{th} = \frac{538.2 - 2.5}{2799 - 342.5} = 21.81\%$$

$$r_{wb} = \frac{2.5}{538.2} = 0.4645\%$$

P3.5 Lenoir engine cycle



Conditions at 1 $p_1 = 1\text{bar}; t_1 = 15^\circ\text{C}; T_1 = 288\text{K}$

Conditions at 2 $p_2 = 10\text{bar}; T_2 = 2880\text{K}$

Volume at 1, $V_1 = (0.287 \times 10^3 \times 288) / (1 \times 10^5) = 0.82656\text{m}^3 / \text{kg}$

Expansion ratio, $r_e = V_3/V_1 = V_4/V_1 = 5; p_3 = 1.0506\text{bar}$.

$$\text{Work done in expansion 2-3 } w_{23} = \frac{p_2 v_2 - p_3 v_3}{\kappa - 1} = \left(\frac{10 \times 0.82656 - 1.0506 \times 4.1328}{0.4} \right) \frac{10^5}{10^3}$$

$$= 980.92 \text{ kJ/kg}$$

Conditions at 4 $p_4 = 1.0\text{bar}; V_4 = 5V_1$

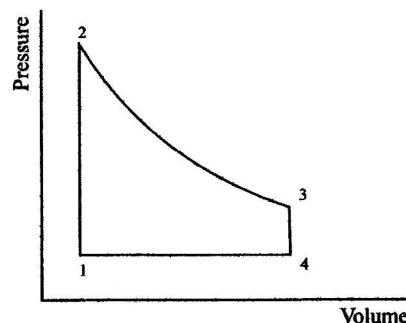
Work done in compression back to 1, $w = 1.0 \times (0.82656 - 4.1328) \times 10^2 = -330.62\text{kJ/kg}$

Total work, $w_{net} = 980.92 - 330.62 = 650.30 \text{ kJ/kg}$

Energy addition, $q_{in} = c_v (T_2 - T_1) = 0.7175 \times (2880 - 288) = 1859.76 \text{ kJ/kg}$

$$\text{Thermal efficiency, } \eta_{th} = \frac{w_{net}}{Q_{in}} = \frac{650.30}{1859.76} = 0.3497 \text{ (34.97\%)}$$

P3.6



Chapter 3 Solutions

Conditions at 1 $p_1 = 1\text{bar}; t_1 = 27^\circ\text{C}; T_1 = 300\text{K}$

Energy added, $q_{in} = q_{12} = 1000\text{ kJ/kg}$

$$T_2 = T_1 + \frac{q_{12}}{c_v} = 300 + \frac{1000}{0.717} = 1694.7\text{ K}$$

Conditions at 2 $p_2 = 5.65\text{ bar}$

Expansion ratio, $r_e = V_3/V_1 = V_4/V_1 = 3; p_3 = \frac{5.65}{3^{1.4}} = 1.214\text{bar}$.

$$w_{23} = \frac{p_2 v_2 - p_3 v_3}{\kappa - 1} = \left(\frac{5.65 \times 1.0 - 1.214 \times 3.0}{0.4} \right) \frac{10^5}{10^3} \times \frac{RT_1}{p_1}$$

Work done in expansion 2-3

$$= 5.02 \times \frac{287 \times 300}{1.0 \times 10^5} \times \frac{10^5}{10^3} = 432.22\text{ kJ/kg}$$

Conditions at 4 $p_4 = 1.0\text{bar}; V_4 = 3V_1$

Work done in compression back to 1, $w = 1.0 \times (1.0 - 3.0) \times \frac{RT_1}{p_1} \times 10^2 = -172.2\text{kJ/kg}$

Total work, $w_{net} = 432.22 - 172.2 = 260.02\text{ kJ/kg}$

Thermal efficiency, $\eta_{th} = \frac{w_{net}}{Q_{in}} = \frac{260.02}{1000.00} = 0.26002\text{ (26.00\%)}$

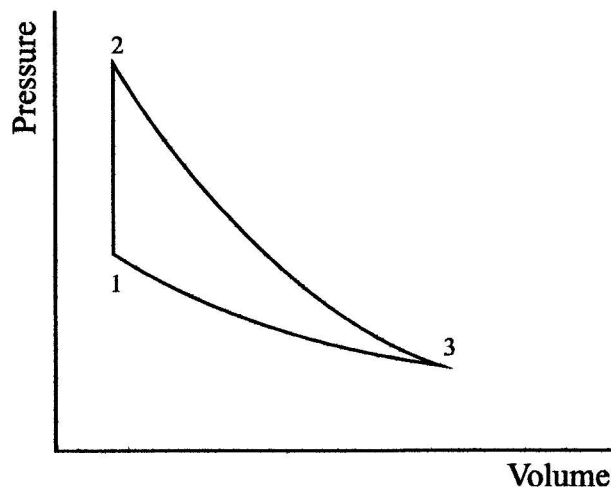
P3.7

$$w_{net} = w_{23} + w_{31}$$

$$w_{23} = \frac{p_2 v_2 - p_3 v_3}{\kappa - 1} = \frac{R(T_2 - T_3)}{\kappa - 1}$$

Net work done, $w_{31} = \int_3^1 p dv = p_1 v_1 \ln\left(\frac{v_1}{v_3}\right) = RT_1 \ln\left(\frac{v_2}{v_3}\right) = RT_1 \ln\left(\frac{T_2}{T_3}\right)^{(\kappa-1)}$

$$w_{net} = \frac{R(T_2 - T_3)}{\kappa - 1} + RT_1 (\kappa - 1) \ln\left(\frac{T_2}{T_3}\right)$$

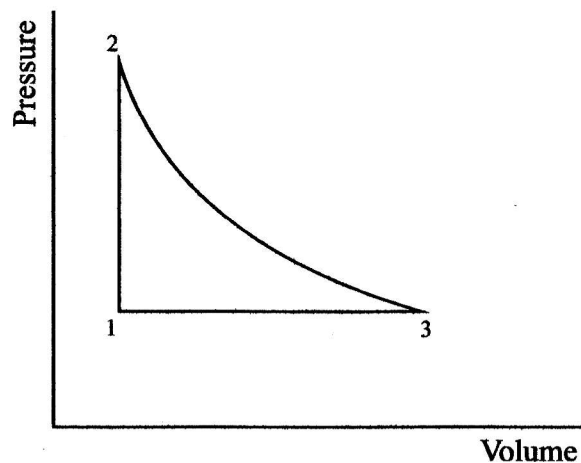


Energy addition, $q_{in} = c_v(T_2 - T_1) = \frac{R(T_2 - T_1)}{\kappa - 1}$

Chapter 3 Solutions

Hence, $\eta_{th} = 1 - \frac{T_1}{T_2 - T_1} \ln \frac{T_2}{T_1}$

P3.8



Energy added: $q_{12} = c_v(T_2 - T_1)$

Energy rejected: $q_{31} = c_p(T_1 - T_3)$

Net work: $w_{net} = c_v(T_2 - T_1) + c_p(T_1 - T_3)$

Thermal efficiency: $\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{c_v(T_2 - T_1) + c_p(T_1 - T_3)}{c_v(T_2 - T_1)} = 1 - \frac{\kappa(T_3 - T_1)}{(T_2 - T_1)}$

$w_{23} = \frac{p_2 v_2 - p_3 v_3}{\kappa - 1} = \frac{R(T_2 - T_3)}{\kappa - 1}$

$w_{31} = p_1(v_1 - v_3) = RT_1(1 - T_3/T_1)$

Calculate around cycle:

$p_1 = 1 \text{ bar}; t_1 = 27^\circ\text{C} \cong 300\text{K}$

Energy added, $q_{in} = 2000 \text{ kJ / kg}$

$T_2 = T_1 + q_{in}/c_v = 300 + 2000/0.718 = 3085\text{K}$

$p_2 = p_1(T_2/T_1) = 1 \times (3085/300) = 10.28 \text{ bar}$

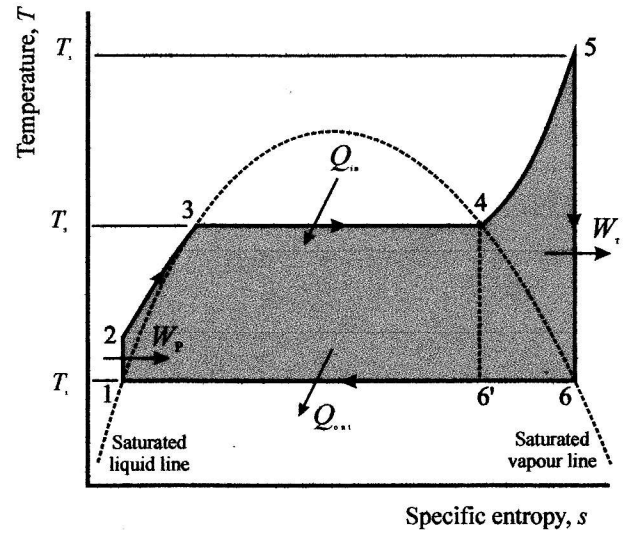
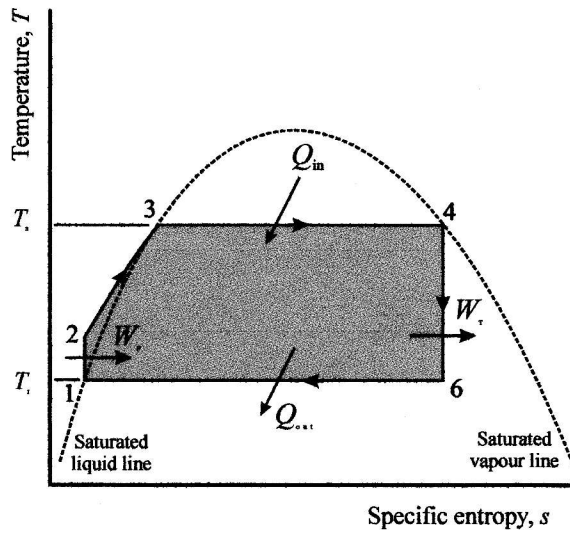
$T_3 = T_2 \left(\frac{p_3}{p_2} \right)^{(\kappa-1)/\kappa} = 3085 \times (1/10.28)^{0.286} = 1584\text{K}$

Hence, $\eta_{th} = 1 - \frac{1.4 \times (1584 - 300)}{(3085 - 300)} = 35.45\%$

$w_{net} = \eta_{th} q_{in} = 0.3545 \times 2000 = 709 \text{ kJ / kg}$

The following solutions will just give the values at the salient points: no calculations will be shown. The Rankine cycles applicable to the early solutions are shown below.

Chapter 3 Solutions



P3.9. Conditions at 1.

$$p_1 = 0.15 \text{ bar}; t_1 = 53.98^\circ\text{C}; v_{g1} = 10.02 \text{ m}^3/\text{kg}; v_{f1} = 0.1014 \times 10^{-2} \text{ m}^3/\text{kg}$$

$$h_{f1} = 226 \text{ kJ/kg}; h_{g1} = 2598 \text{ kJ/kg}; s_{f1} = 0.7551 \text{ kJ/kgK}$$

$$s_{g1} = 8.01 \text{ kJ/kgK}$$

Conditions at 3.

$$p_3 = 20 \text{ bar}; t_3 = 212.4^\circ\text{C}; v_{g3} = 0.09957 \text{ m}^3/\text{kg}; h_{f3} = 909 \text{ kJ/kg}$$

$$s_{f3} = 2.447 \text{ kJ/kgK}; s_{g3} = 6.340 \text{ kJ/kgK}$$

Feedpump work $w_p = w_{12} = -\int v dp$

$$v = 0.1014 + \frac{0.15 - 0.1233}{0.1574 - 0.1233} = 0.10143 \times 10^{-2} \text{ m}^3/\text{kg}$$

$$\text{Thus } w_p = w_{12} = -\int v dp = -0.001013 \times (20 - 0.15) = 2.01 \text{ kJ/kg}$$

$$\text{Hence, } h_2 = 226 + 2 = 228 \text{ kJ/kg}$$

Conditions at 4

$$p_4 = 20 \text{ bar}; t_4 = 212.4^\circ\text{C}; s_4 = 6.340 \text{ kJ/kgK}; h_4 = 2799 \text{ kJ/kg}$$

Conditions at 6

$$p_6 = 0.15 \text{ bar}; s_{g6} = 8.01 \text{ kJ/kgK}; s_{f6} = 0.7551 \text{ kJ/kgK}$$

$$t_6 = 53.98^\circ\text{C}; h_{f6} = 226 \text{ kJ/kg}; h_{g6} = 2598 \text{ kJ/kg}$$

Thus

$$x_6 = \frac{6.340 - 0.7551}{8.01 - 0.7551} = 0.7698$$

$$\text{giving } h_6 = h_{f6} + x_6(h_{g6} - h_{f6}) = 226 + 0.7698(2598 - 226)$$

$$= 2052 \text{ kJ/kg}$$

Net work done by turbine plant

$$w_{\text{net}} = (h_4 - h_6) + (h_1 - h_2)$$

$$= (2799 - 2052) - 2 = 744 \text{ kJ/kg}$$

$$\text{Energy added, } q_{\text{in}} = (h_4 - h_2) = 2799 - 228 = 2571 \text{ kJ/kg}$$

$$\text{Hence } \eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{744}{2571} = 28.94\%$$

Could also be calculated as $\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}}$

$$\text{then } \eta_{\text{th}} = 1 - \frac{h_6 - h_1}{q_{\text{in}}} = 1 - \frac{1826}{2571} = 28.97\%$$

The Carnot cycle efficiency between these two temperatures would be $\eta_{\text{Carnot}} = \frac{212.4 - 53.9}{(212.4 + 273)} = 32.65\%$

P3.10. In this question the steam is superheated to 400°C in an effort to increase the dryness at point 6. Conditions at state points 1, 2, and 3 are same as in P3.9

Conditions at 5 are superheated

$$p_5 = 2 \text{ bar}; t_5 = 400^\circ\text{C}; h_5 = 3248 \text{ kJ/kg}; s_5 = 7.126 \text{ kJ/kg}$$

Conditions at 6 $p_6 = 0.15 \text{ bar}; t_6 = 53.9^\circ\text{C}$

$$x_6 = \frac{7.126 - 0.755}{8.01 - 0.755} = 0.8782 \text{ (dryness has been improved)}$$

$$h_6 = h_{f6} + x_6 (h_{g6} - h_{f6}) = 226 + 0.8782 \times (2598 - 226) = 2309 \text{ kJ/kg}$$

$$\text{Hence net work output, } w_{\text{net}} = (h_5 - h_6) + (h_1 - h_2) = (3248 - 2309) - 2 = 937 \text{ kJ/kg.}$$

Energy added to working fluid

$$q_{\text{in}} = h_5 - h_2 = 3428 - 228 = 3020 \text{ kJ/kg}$$

$$\text{Hence } \eta_{\text{th}} = \frac{937}{3020} = 0.3102 = 31.02\%$$

Carnot efficiency for these temperatures

$$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_H} = 1 - \frac{326.9}{673} = 0.5142 = 51.42\%$$

Mean temperature of heat (energy) addition

$$\bar{T}_H = \frac{h_5 - h_2}{s_5 - s_2} = \frac{3248 - 228}{7.126 - 0.7551} = 474 \text{ K.}$$

Mean temperature of heat (energy) rejection

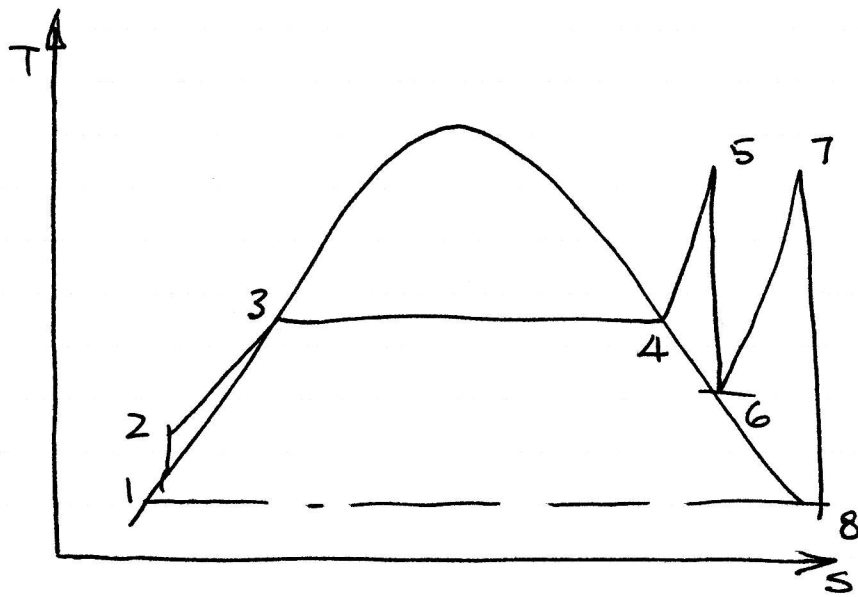
$$\bar{T}_C = 273 + 53.9 = 326.9 \text{ K}$$

Thus Carnot efficiency based on \bar{T}_H and \bar{T}_C is

$$\eta = 1 - \frac{\bar{T}_C}{\bar{T}_H} = 1 - \frac{326.9}{474} = 31.79\%$$

Hence, Carnot efficiency based on mean temperatures is same as actual cycle efficiency

P3.11 Steam turbine with reheat



The front end of the cycle to state pt 4 and 5 is same as in P3.10.

State points 6 to 8 are different.

Expansion 5 to 6 is from 20bar to 10bar.

Hence state point 6 is $p_6 = 10\text{bar}$; $s_6 = 7.126 \text{ kJ/kg K}$.

Thus state point 6 is still superheated and $t_6 = 300^\circ\text{C}$

Hence, $h_6 = 3052 \text{ kJ/kg}$

and $w_{56} = 3248 - 3052 = 196 \text{ kJ/kg}$

Reheat along isobar from 6 to 7 at 10bar.

state point 7, $p_7 = 10\text{bar}$; $t_7 = 400^\circ\text{C}$

$h_7 = 3264 \text{ kJ/kg}$; $s_7 = 7.464 \text{ kJ/kg K}$.

Hence, $x_8 = \frac{7.464 - 0.755}{8.01 - 0.755} = 0.9247$

and $h_8 = 226 + 0.9247 \times (2598 - 226) = 2419 \text{ kJ/kg}$

Work done, $w_{78} = h_7 - h_8 = 3264 - 2419 = 845 \text{ kJ/kg}$

Thus total work output, $w_{net} = w_{56} + w_{78} + w_{12}$

$$= 196 + 845 - 2 = 1039 \text{ kJ/kg}$$

Total energy added, $q_{in} = (h_5 - h_2) + (h_7 - h_6)$

$$= (3248 - 228) + (3264 - 3052)$$

$$= 3232 \text{ kJ/kg}$$

Thus thermal efficiency, $\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{1039}{3232} = 32.14\%$

Reheat has improved the dryness of the steam and the thermal efficiency