

**1.1** The building in figure P.1 is used for general office space. The slab is 8 in. thick on a beam 12 in. wide by 18 in. deep, the bay dimensions are 18'-6" in the x direction and 21'-0" in the y direction and the superimposed service dead load is 12 psf. Calculate the slab service load in psf and the beam service load in klf. (**Solution:**  $q_s = 162$  psf,  $w_u = 3.12$  klf).

**SOLUTION**

- From table 1.1 Office load  $q_1 := 50$ psf
- Concrete unit weight  $\gamma_c := 150$ pcf
- Slab load  $t := 8$ in  $q_d := t \cdot \gamma_c = 100$  psf
- Superimposed dead load  $q_{sdl} := 12$ psf
- Service load  $q_s := q_1 + q_d + q_{sdl} = 162$  psf

The beam length is 21 feet and the tributary width is 18.5 ft. The beam is 12 x 18 in. of which 10 in is below the slab.

$$w_{\text{beam}} := (18\text{in} - t) \cdot 12\text{in} \cdot \gamma_c = 125 \text{ plf}$$

$$w_s := 18.5\text{ft} \cdot q_1 + 18.5\text{ft} \cdot (q_d + q_{sdl}) + w_{\text{beam}} = 3122 \text{ plf}$$

$$w_s = 3.12 \text{ klf}$$

**1.2** The building in figure P.1 is used for general office space. The slab is 8 in. thick on a 12 in. wide x 18 in. deep beam, the bay dimensions are 18'-6" in the x direction and 21'-0" in the y direction and the superimposed service dead load is 12 psf. Calculate the factored column load transferred to column C3 on the 3<sup>rd</sup> floor. (**Solution:**  $P_u = 86.4$  kips).

**SOLUTION**  $\gamma_c := 150$ pcf  $q_1 := 50$ psf  $q_{sdl} := 12$ psf

Slab load  $t := 8$ in  $q_d := t \cdot \gamma_c = 100$  psf

Tributary area  $A_t := 18.5\text{ft} \cdot 21\text{ft} = 388.5 \text{ ft}^2$

$$w_{\text{beam}} := (18\text{in} - t) \cdot 12\text{in} \cdot \gamma_c = 125 \text{ plf}$$

$$P_u := 1.6 \cdot A_t \cdot q_1 + 1.2 \cdot A_t \cdot (q_d + q_{sdl}) + 1.2 \cdot w_{\text{beam}} \cdot 21\text{ft} = 86.4 \text{ kip}$$

**1.3** The building in figure P.1 is used for general office space. The slab is 8 in. thick on beams 12 in. wide x 18 in. deep, the bay dimensions are 18'-6" in the x direction and 21'-0" in the y direction and the superimposed service dead load is 12 psf. Calculate the slab factored load in psf and the beam factored load in klf. Comment on your solution in comparison with problem 1.1.

SOLUTION  $\gamma_c := 150\text{pcf}$   $q_l := 50\text{psf}$   $q_{sdl} := 12\text{psf}$

Slab load  $t := 8\text{in}$   $q_d := t \cdot \gamma_c = 100\text{psf}$   
 $q_u := 1.6 \cdot q_l + 1.2 \cdot (q_d + q_{sdl}) = 214.4\text{psf}$

From problem 1.1  $\frac{q_u}{q_s} = 1.323$

$w_{\text{beam}} := (18\text{in} - t) \cdot 12\text{in} \cdot \gamma_c = 125\text{plf}$

The tributary width is  $l := 18.5\text{ft}$

$w_u := 1.6 \cdot q_l \cdot l + 1.2 \cdot (q_d + q_{sdl}) \cdot l + w_{\text{beam}} = 4.09\text{klf}$

Compare to problem 1.1

$\frac{w_u}{w_s} = 1.31$

This ratio is between 1.2 and 1.6 and suggests that the majority of the load comes from long term loadings.

**1.4** A slab in figure P.1 is used for lobby space. The slab is 10 in. thick on a 14 in. wide x 24 in. deep beam, the bay dimensions are 21'-0" in the x direction and 26'-0" in the y direction and the superimposed service dead load is 15 psf. Calculate the slab factored load in psf and the beam factored load in klf.

SOLUTION

$b := 14\text{in}$   $h := 24\text{in}$   $t := 10\text{in}$   $q_{sdl} := 15\text{psf}$

From Table 1.1 the lobby live load is  $q_l := 100\text{psf}$   $\gamma_c := 150\text{pcf}$

$q_{\text{slab}} := \gamma_c \cdot t = 125\text{psf}$

$w_{\text{beam}} := (h - t) \cdot b \cdot \gamma_c = 204\text{plf}$

$q_u := 1.2 \cdot (q_{\text{slab}} + q_{sdl}) + 1.6 \cdot q_l = 328\text{psf}$

The beam tributary width length is  $l := 21\text{ft}$

$w_u := q_u \cdot l + 1.2w_{\text{beam}} = 7.13\text{klf}$

**1.5** The building in figure P.1 is used for light storage space. The slab is 10 in. thick on a 16 in. wide x 20 in. deep beam, the bay dimensions are 20'-0" in the x direction and 25'-0" in the y direction and the superimposed sprinkler dead load is 4 psf. Calculate the slab factored load in psf and the beam factored load in klf.

SOLUTION

$$b := 16\text{in} \quad h := 20\text{in} \quad t := 10\text{in} \quad q_{sdl} := 4\text{psf} \quad \gamma_c := 150\text{pcf}$$

From Table 1.1 the light storage live load is  $q_l := 125\text{psf}$

$$q_{slab} := \gamma_c \cdot t = 125\text{psf}$$

$$w_{beam} := (h - t) \cdot b \cdot \gamma_c = 167\text{plf}$$

$$q_u := 1.2 \cdot (q_{slab} + q_{sdl}) + 1.6 \cdot q_l = 355\text{psf}$$

The beam tributary width length is  $l := 20\text{ft}$

$$w_u := q_u \cdot l + 1.2w_{beam} = 7.30\text{klf}$$

**1.6** The roof on the building in figure P.1 has a slab 7 in. thick on a 12 in. wide x 16 in. deep beam, the bay dimensions are 19'-0" in the x direction and 21'-0" in the y direction and the superimposed service dead load is 6 psf. Calculate the slab factored load in psf and the beam factored load in klf.

SOLUTION

$$b := 12\text{in} \quad h := 16\text{in} \quad t := 7\text{in} \quad q_{sdl} := 6\text{psf}$$

From Table 1.1 the roof live load is  $q_l := 20\text{psf}$   $\gamma_c := 150\text{pcf}$

$$q_{slab} := \gamma_c \cdot t = 87.5\text{psf}$$

$$w_{beam} := (h - t) \cdot b \cdot \gamma_c = 112\text{plf}$$

$$q_u := 1.2 \cdot (q_{slab} + q_{sdl}) + 1.6 \cdot q_l = 144\text{psf}$$

The beam tributary width length is  $l := 18.5\text{ft}$

$$w_u := q_u \cdot l + 1.2w_{beam} = 2.80\text{klf}$$

**2.1.** The specified concrete strength  $f'_c$  for a new building is 5000 psi. Calculate the required average  $f_{cr}$  for the concrete (a) if there are no prior test results for concrete with a compressive strength within 1000 psi of  $f'_c$  made with similar materials, (b) if 20 test results for concrete with  $f'_c = 5500$  psi made with similar materials produce a sample standard deviation  $s_s$  of 560 psi, and (c) if 30 tests with  $f'_c = 4500$  psi made with similar materials produce a sample standard deviation  $s_s$  of 540 psi.

**Solution:**  $f'_c = 5000$  psi

**a)** No prior results

$$f_{cr} = f'_c + 1200 \text{ psi} = 5000 + 1200 = 6200 \text{ psi}$$

**b)** 20 prior tests for concrete with  $f'_c$  within 1000 psi of  $f'_c$  of the project and  $s_s = 560$  psi. From Table 2.1,  $k = 1.08$  and  $ks_s$  is  $1.08 * 560 = 605$  psi.

Because  $f'_c = 5000$  psi, use eqs (2.1) and (2.2a)

$$f_{cr} = f'_c + 1.34 ks_s = 5000 + 1.34 * 605 = 5810 \text{ psi}$$

$$f_{cr} = f'_c + 2.33 ks_s - 500 = 5000 + 2.33 * 605 - 500 = 5910 \text{ psi}$$

USE  $f_{cr} = 5910$  psi

**c)** 30 prior tests for concrete with  $f'_c$  within 1000 psi of  $f'_c$  for the project.  $s_s = 590$  psi and  $k$  is 1.0.

$$f_{cr} = f'_c + 1.34 s_s = 5000 + 1.34 * 590 = 5790 \text{ psi}$$

$$f_{cr} = f'_c + 2.33 s_s - 500 = 5000 + 2.33 * 590 - 500 = 5870 \text{ psi}$$

USE  $f_{cr} = 5790$  psi

COMMENT: in cases b) and c) the  $f_{cr}$  would reasonably be taken as 6000 psi.

**2.2.** Ten consecutive strength tests are available for a new concrete mixture with  $f'_c = 4000$  psi: 4830, 4980, 3840, 4370, 4410, 4890, 4450, 3970, 4780, and 4040 psi.

(a) Do the strength results represent concrete of satisfactory quality? Explain your reasoning.

(b) If  $f_{cr}$  has been selected based on 30 consecutive test results from an earlier project with a sample standard deviation  $s_s$  of 570 psi, must the mixture proportions be adjusted? Explain.

**Solution:**

**a)** For  $f'_c = 4000$  psi, the strength results indicate satisfactory concrete quality because (1) no individual test is below  $f'_c - 500$  psi = 3500 psi, and (2) every arithmetic average of any three consecutive tests equals or exceeds  $f'_c$ .

**b)** For  $s_s = 570$  psi, for 30 consecutive tests calculate  $f_{cr}$  using equations 2.1 and 2.2a.

$$f_{cr} = f'_c + 1.34 ks_s = 4000 + 1.34 * 570 = 4760 \text{ psi}$$

$$f_{cr} = f'_c + 2.33 ks_s - 500 = 4000 + 2.33 * 570 - 500 = 4830 \text{ psi}$$

USE  $f_{cr} = 4830$  psi

The average of the above tests is  $(4830 + 4980 + 3840 + 4370 + 4410 + 4890 + 4450 + 3970 + 4780 + 4040) / 10 = 4460$

Because the average strength is less than the target strength, the water/cement ratio must be adjusted by adding cement or reducing water to increase the strength. If the water is reduced, a water reducer admixture would be required to maintain workability.

**2.3.** The specified concrete strength  $f'_c$  for the columns in a high-rise building is 12,000 psi. Calculate the required average  $f'_{cr}$  for the concrete ( *a* ) if there are no prior test results for concrete with a compressive strength within 1000 psi of  $f'_c$  made with similar materials, ( *b* ) if 15 test results for concrete with  $f'_c = 11,000$  psi made with similar materials produce a sample standard deviation  $s_s$  of 930 psi, and ( *c* ) if 30 tests with  $f'_c = 12,000$  made with similar materials produce a sample standard deviation  $s_s$  of 950 psi.

**Solution:**  $f'_c = 12000$  psi

**a)** No prior results

$$f'_{cr} = f'_c + 0.1 f'_c + 700 \text{ psi} = 12000 + 0.1 * 12000 + 700 = 13,900 \text{ psi}$$

**b)** 15 prior tests for concrete with  $f'_c$  within 1000 psi of  $f'_c$  of the project and  $s_s = 930$  psi. From Table 2.1,  $k = 1.16$  and  $ks_s$  is  $1.16 * 930 = 1079$  psi.

Because  $f'_c > 5000$  psi, use eqs (2.1) and (2.2b)

$$f'_{cr} = f'_c + 1.34 ks_s = 12000 + 1.34 * 1079 = 13,450 \text{ psi}$$

$$f'_{cr} = 0.9f'_c + 2.33 ks_s = 0.9 * 12000 + 2.33 * 1079 = 13,310 \text{ psi}$$

USE  $f'_{cr} = 13,450$  psi

**c)** 30 prior tests for concrete with  $f'_c$  within 1000 psi of  $f'_c$  for the project.  $s_s = 950$  psi and  $k$  is 1.0.

$$f'_{cr} = f'_c + 1.34 s_s = 12000 + 1.34 * 950 = 13,270 \text{ psi}$$

$$f'_{cr} = 0.9f'_c + 2.33 ks_s = 0.9 * 12000 + 2.33 * 950 = 13,010 \text{ psi}$$

USE  $f'_{cr} = 13,270$  psi

**3.1.** A 16 × 20 in. column is made of the same concrete and reinforced with the same six No. 9 (No. 29) bars as the column in Examples 3.1 and 3.2, except that a steel with yield strength  $f_y = 40$  ksi is used. The stress-strain diagram of this reinforcing steel is shown in Fig. 2.15 for  $f_y = 40$  ksi. For this column determine ( a ) the axial load that will stress the concrete to 1200 psi; ( b ) the load at which the steel starts yielding; ( c ) the maximum load; and ( d ) the share of the total load carried by the reinforcement at these three stages of loading. Compare results with those calculated in the examples for  $f_y = 60$  ksi, keeping in mind, in regard to relative economy, that the price per pound for reinforcing steels with 40 and 60 ksi yield points is about the same.

$$A_s := 6.0 \text{ in}^2$$

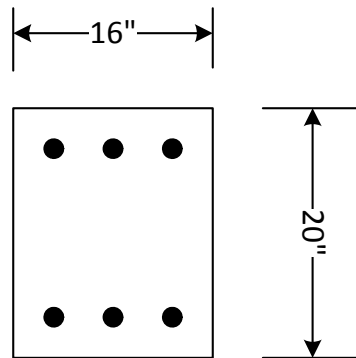
$$A_g := 16 \text{ in} \cdot 20 \text{ in} = 320 \cdot \text{in}^2$$

$$A_c := A_g - A_s = 314 \cdot \text{in}^2$$

$$f_c := 4000 \text{ psi} \quad f_y := 40000 \text{ psi} \quad f_{y1} := 60000 \text{ psi}$$

$$E_c := 3600000 \text{ psi} \quad E_s := 29000000 \text{ psi}$$

$$n := \frac{E_s}{E_c} = 8.1$$



**Part a The solution is identical for grade 40 and grade 60 reinforcement**

$$f_c := 1200 \text{ psi}$$

$$P := f_c \cdot (A_c + n \cdot A_s) = 434800 \text{ lbf}$$

$$P_s := f_c \cdot n \cdot A_s = 58000 \text{ lbf}$$

$$\frac{P_s}{P} = 0.133 \quad \text{The steel carries 13.3 percent of the load}$$

**Part b**

$$\epsilon_y := \frac{f_y}{E_s} = 0.00138$$

$$\epsilon_{y1} := \frac{f_{y1}}{E_s} = 0.00207$$

$$\text{For slow loading} \quad f_{c1} := 3000 \text{ psi}$$

$$f_{c1} := 3300 \text{ psi}$$

$$P := A_c \cdot f_c + A_s \cdot f_y = 1182000 \text{ lbf}$$

$$P_1 := A_c \cdot f_{c1} + A_s \cdot f_{y1} = 1396200 \text{ lbf}$$

$$P_s := A_s \cdot f_y = 240000 \text{ lbf}$$

$$P_{s1} := A_s \cdot f_{y1} = 360000 \text{ lbf}$$

$$\frac{P_s}{P} = 0.203$$

$$\frac{P_{s1}}{P_1} = 0.258$$

Problem 3.1

Part c

$$f_c := 3400 \text{ psi}$$

$$P_u := A_c \cdot f_c + A_s \cdot f_y = 1307600 \text{ lbf}$$

$$P_s := A_s \cdot f_y = 240000 \text{ lbf}$$

$$\frac{P_s}{P_u} = 0.184$$

$$P_{u1} := A_c \cdot f_c + A_s \cdot f_{y1} = 1427600 \text{ lbf}$$

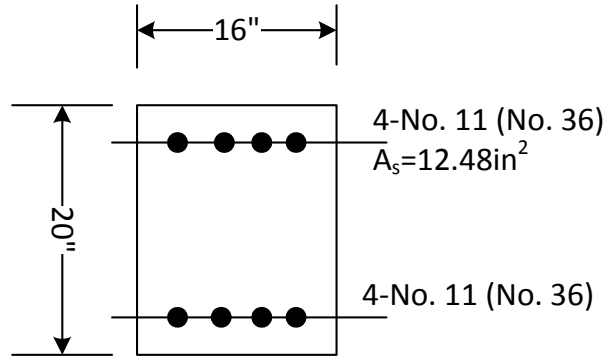
$$P_{s1} := A_s \cdot f_{y1} = 360000 \text{ lbf}$$

$$\frac{P_{s1}}{P_u} = 0.275$$

Comments

1. There is no difference at  $f_c = 1200$  psi and elastic assumptions are used
2. As the strain increases, the steel with  $f_y = 60,000$  psi contributes more to the total load and the column has a higher total capacity
3. Grade 40 and Grade 60 have the same cost, therefore Grade 60 provides a 9% increase in capacity for no increase in cost.

**3.2** The area of steel, expressed as a percentage of gross concrete area, for the column of Problem 3.1 is lower than would often be used in practice. Recalculate the comparisons of Problem 3.1, using  $f_y$  of 40 ksi and 60 ksi as before, but for a 16 × 20 in. column reinforced with eight No. 11 (No. 36) bars. Compare your results with those of Problem 3.1.



$$A_{s11} := 1.56 \text{ in}^2$$

$$A_s := 8 \cdot A_{s11} = 12.48 \cdot \text{in}^2$$

$$A_g := 16 \text{ in} \cdot 20 \text{ in} = 320 \cdot \text{in}^2$$

$$A_c := A_g - A_s = 307.52 \cdot \text{in}^2$$

$$f_c := 4000 \text{ psi} \quad f_y := 40000 \text{ psi} \quad f_{y1} := 60000 \text{ psi}$$

$$E_c := 3600000 \text{ psi} \quad E_s := 29000000 \text{ psi}$$

$$n := \frac{E_s}{E_c} = 8.1$$

**Part a The solution is identical for grade 40 and grade 60 reinforcement**

$$f_c := 1200 \text{ psi}$$

$$P := f_c \cdot (A_c + n \cdot A_s) = 489664 \text{ lbf} \quad P_s := f_c \cdot n \cdot A_s = 120640 \text{ lbf}$$

$$\frac{P_s}{P} = 0.246 \quad \text{The steel carries 25 percent of the load}$$

**Part b**

$$\epsilon_y := \frac{f_y}{E_s} = 0.00138$$

$$\epsilon_{y1} := \frac{f_{y1}}{E_s} = 0.00207$$

For slow loading  $f_c := 3000 \text{ psi}$

$$f_{c1} := 3300 \text{ psi}$$

$$P := A_c \cdot f_c + A_s \cdot f_y = 1421760 \text{ lbf}$$

$$P_1 := A_c \cdot f_{c1} + A_s \cdot f_{y1} = 1763616 \text{ lbf}$$

$$P_s := A_s \cdot f_y = 499200 \text{ lbf}$$

$$P_{s1} := A_s \cdot f_{y1} = 748800 \text{ lbf}$$

$$\frac{P_s}{P} = 0.351$$

$$\frac{P_{s1}}{P_1} = 0.425$$



Problem 3.2

Part c

$$f_c := 3400 \text{ psi} \quad \text{both cases}$$

$$P_u := A_c \cdot f_c + A_s \cdot f_y = 1544768 \text{ lbf}$$

$$P_s := A_s \cdot f_y = 499200 \text{ lbf}$$

$$\frac{P_s}{P_u} = 0.323$$

$$P_{u1} := A_c \cdot f_c + A_s \cdot f_{y1} = 1794368 \text{ lbf}$$

$$P_{s1} := A_s \cdot f_{y1} = 748800 \text{ lbf}$$

$$\frac{P_{s1}}{P_u} = 0.485$$

Comments

1. There is no difference at  $f_c = 1200$  psi and elastic assumptions are used
2. There is a 16% capacity increase at nominal using Grade 60 reinforcement
3. The higher steel ratio produces a higher overall capacity compared to problem 3.1

3.3. A square concrete column with dimensions 22 × 22 in. is reinforced with a total of eight No. 10 (No. 32) bars arranged uniformly around the column perimeter. Material strengths are  $f_y = 60 \text{ ksi}$  and  $f_c = 4000 \text{ psi}$ , with stress-strain curves as given by curves a and c of Fig. 3.3 . Calculate the percentages of total load carried by the concrete and by the steel as load is gradually increased from 0 to failure, which is assumed to occur when the concrete strain reaches a limit value of 0.0030. Determine the loads at strain increments of 0.0005 up to the failure strain, and graph your results, plotting load percentages vs. strain. The modular ratio may be assumed at  $n = 8$  for these materials.

Using Concrete data from Figure 3.3

$$A_s = 10.12 \text{ in}^2$$

$$A_c = 474 \text{ in}^2$$

$$f_y = 60000 \text{ psi}$$

$$f'_c = 4000 \text{ psi}$$

Strain	$f_c$ (psi)	$P_c$ (kips)	$f_s$ (psi)	$P_s$ (kips)	$P_{total}$ (kips)	$P_c/P_{total}$	$P_s/P_{total}$
0.0000	0	0	0	0	0	0.00%	0.00%
0.0005	1600	758	14500	147	905	83.8%	16.2%
0.0010	2600	1232	29000	293	1526	80.8%	19.2%
0.0015	3100	1469	43500	440	1910	76.9%	23.1%
0.0020	3300	1564	58000	587	2151	72.7%	27.3%
0.0025	3400	1612	60000	607	2219	72.6%	27.4%
0.0030	3400	1612	60000	607	2219	72.6%	27.4%

