Fundamental Planetary Sciences, Updated Edition (2019)
Solutions to Selected Problems
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version 1.0, September 2019

Problem 1-1
(a)

Table 1: Solar System Model Parameters, 1 mm ≈ 1300 km

Solar System Object	Diameter	Distance	Model Object
Sun	1.1 m	–	very large beach ball
Mercury	3.7 mm	45 m	ball berring
Venus	9.5 mm	83 m	marble (slightly smaller)
Earth	10 mm	115 m	marble
Mars	5.2 mm	173 m	large ball berring
Asteroids	≤ 0.8 mm	330 ± 100 m	pinch of sand and dust
Jupiter	110 mm	600 m	softball
Saturn	92 mm	1.1 km	baseball, thin paper plate (rings)
Uranus	40.1 mm	2.2 km	ping-pong ball
Neptune	38.8 mm	3.5 km	20^{th} cent. ping-pong ball
Pluto, Eris	1.7 mm	4.5, 10 km	small beads
Other TNOs	≤ 1.2 mm	mostly 4 – 7 km	thimbleful of sand & dust
Proxima Cen.	160 mm	29,000 km	volleyball

One may argue that the table should include natural satellites of the planets as well, especially if Pluto and the asteroids are listed (the Galilean satellite Ganymede, for example, is slightly larger than Mercury and much larger than Pluto).

(b) The distance to Proxima Centari, 4 ly, at this scale is more than twice the diameter of the Earth! Thus, scale models are impractical.

Problem 1-4

(b) The Earth must rotate *once more than* the number of solar days since it has a prograde rotation and orbit. So the number of rotations per orbit is 366.24. The sidereal day, then is...

$$\frac{\text{Time of Orbit}}{\text{Number of Sidereal days per year}} = \frac{(365.24\text{d} \times 24\text{h})}{366.24\text{d}} = 23.9345\text{h} = 23\text{h } 56\text{m } 4\text{s}.$$

(c) A retrograde orbit adds a *solar* day to the year. For each day in its orbit, a planet with retrograde spin needs a little *less* than a full rotation for the same part of the planet to be directly sub-solar. This is the opposite of the prograde

case, where the same portion of the planet must rotate a little more following a sidereal rotation to reach the sub-solar position. Hence, a retrograde planet that rotates once per year sidereally has 2 solar days per year.

(d) The angular velocity of the Sun with respect to a fixed point on the planet is determined by the sidereal rotation velocity plus a factor involving the movement of the planet with respect to the Sun. If the planet is moving in the prograde direction, the Sun appears to move in the opposite angular direction, so that if the planet was sitting still,

$$\omega_\odot = -\omega_{\text{sidereal}}$$

while for prograde orbits, the apparent motion of the planet slows the motion of the Sun for a fixed point on the planet, so that

$$\omega_\odot = \omega_{orbit} - \omega_{\text{sidereal}}.$$

From here, using the relation $\omega = \frac{2\pi}{P}$, where P is the period of sidereal or solar rotation or orbit,

$$\frac{2\pi}{P_\odot} = \frac{2\pi}{P_{\text{sid}}} - \frac{2\pi}{P_{\text{orb}}}$$

and

$$P_\odot = \frac{1}{\frac{1}{P_{\text{sid}}} - \frac{1}{P_{\text{orb}}}} = \frac{P_{\text{sid}} P_{\text{orb}}}{P_{\text{sid}} - P_{\text{orb}}}.$$

For Mercury, $P_{\text{sid}} = 58.65\text{d}$, $P_{\text{orb}} = 87.969\text{d}$, so $P_\odot = 176\text{d}$.

Likewise...

Venus:

$$P_\odot = 116.7\text{d}$$

Note that Venus is in a retrograde rotation, and so has a negative value for the sidereal day in the formula.

Mars:

$$P_\odot = 24.67\text{h}$$

Jupiter:

$$P_\odot = 0.410080\text{d},\ P_{\text{sid}} = 0.410042\text{d}$$

(e) To first order, the Earth, or any planet, spins at a fixed rate with respect to the background stars. The sidereal day, then, is of fixed length (for non-geological time scales, and neglecting tidal effects). The orbit of a planet, as we've seen, affects $\omega_\odot$, so that the longest days of the year for a prograde orbit and rotation occur when ω_{orbit} is greatest, i.e. at the Earth's perihelion (neglecting inclination and hemispherical effects).

Using Kepler's laws... (here the ω's are obital)

$$a(1-e) = \text{closest distance} \approx 0.983\text{AU}$$

$$\omega_{peri}\ r_{peri} = \bar{a}\ \bar{\omega}$$

$$\omega_{peri} = \frac{\bar{a}}{r_{peri}}\ \bar{\omega} = 1.017\ \bar{\omega}$$

$$(P_{sid} - \bar{P}_\odot) \cdot 1.017 = 0.0666\text{h}$$

$$0.0666\text{h} + 23.9345\text{h} = 24.0011\text{h}$$