CHAPTER 1 *INTRODUCTION AND MATHEMATICAL CONCEPTS*

1. ***REASONING*** We use the fact that 1 km = 1000 m and the formula for the surface area of a sphere to calculate the answer.

SOLUTION Converting 3.476 x 10^3 m to 3.476 km and using the surface area formula for a sphere gives:

$$\text{Surface Area} = 4\pi r^2 = 4\pi(3.476\ \text{km})^2 = \boxed{151.8\ \text{km}^2}$$

2. ***REASONING*** We will need to use conversion factors for both time and distance to find the solution. Using dimensional analysis, we can convert seconds into years and centimeters into nanometers to arrive at our answer.

SOLUTION

$$\left(2\ \frac{\cancel{\text{cm}}}{\cancel{s}}\right)\left(\frac{1\ \cancel{m}}{100\ \cancel{\text{cm}}}\right)\left(\frac{1.00\times10^9\ \text{nm}}{1\ \cancel{m}}\right)\left(\frac{60\ \cancel{s}}{1\ \cancel{\text{min}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\ \cancel{hr}}\right)\left(\frac{24\ \cancel{hr}}{1\ \cancel{day}}\right)\left(\frac{365\ \cancel{day}}{1\ \text{yr}}\right) = \boxed{6\times10^{14}\ \text{nm/yr}}$$

3. ***REASONING*** The dimensions of the variables *v*, *x*, and *t* are known, and the numerical factor 4 is dimensionless. Therefore, we can solve the equation for *z* and then substitute the known dimensions. The dimensions $[\text{L}]$ and $[\text{T}]$ can be treated as algebraic quantities to determine the dimensions of the variable *z*.

SOLUTION Since $v = \frac{1}{4}zxt^2$, it follows that $z = \frac{4v}{xt^2}$. We know the following dimensions: $v = [\text{L}]/[\text{T}]$, $x = [\text{L}]$, and $t = [\text{T}]$. Since the factor 4 is dimensionless, *z* has the dimensions of

$$\frac{v}{xt^2} = \frac{[\text{L}]/[\text{T}]}{[\text{L}][\text{T}]^2} = \boxed{\frac{1}{[\text{T}]^3}}$$

4. ***REASONING*** By multiplying the quantity $1.002\times10^{-3}\ \text{kg}/(\text{s}\cdot\text{m})$ by the appropriate conversions factors, we can convert the quantity to units of poise (P).

These conversion factors are obtainable from the following relationships between the various units:

$$1\ \text{kg} = 1.00 \times 10^3\ \text{g}$$
$$1\ \text{m} = 1.00 \times 10^2\ \text{cm}$$
$$1\ \text{P} = 1\ \text{g}/(\text{s}\cdot\text{cm})$$

SOLUTION The conversion from the unit $\text{kg}/(\text{s}\cdot\text{m})$ to the unit P proceeds as follows:

$$\left(1.78 \times 10^{-3}\ \frac{\cancel{\text{kg}}}{\cancel{\text{s}\cdot\text{m}}}\right)\left(\frac{1.00 \times 10^3\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}}\right)\left(\frac{1\ \cancel{\text{m}}}{1.00 \times 10^2\ \cancel{\text{cm}}}\right)\left[\frac{1\ \text{P}}{1\ \cancel{\text{g}}/(\cancel{\text{s}}\cdot\cancel{\text{cm}})}\right] = \boxed{1.002 \times 10^{-2}\ \text{P}}$$

5. ***REASONING*** Multiplying an equation by a factor of 1 does not alter the equation; this is the basis of our solution. We will use factors of 1 in the following forms:

$$\frac{1\ \text{gal}}{128\ \text{oz}} = 1, \text{ since } 1\ \text{gal} = 128\ \text{oz}$$

$$\frac{3.785\times10^{-3}\ \text{m}^3}{1\ \text{gal}} = 1, \text{ since } 3.785 \times 10^{-3}\ \text{m}^3 = 1\ \text{gal}$$

$$\frac{1\ \text{mL}}{10^{-6}\ \text{m}^3} = 1, \text{ since } 1\ \text{mL} = 10^{-6}\ \text{m}^3$$

SOLUTION The starting point for our solution is the fact that

$$\text{Volume} = 1\ \text{oz}$$

Multiplying this equation on the right by factors of 1 does not alter the equation, so it follows that

$$\text{Volume} = (1\ \text{oz})(1)(1)(1) = \left(1\ \cancel{\text{oz}}\right)\left(\frac{1\ \cancel{\text{gal}}}{128\ \cancel{\text{oz}}}\right)\left(\frac{3.785\times10^{-3}\ \cancel{\text{m}^3}}{1\ \cancel{\text{gal}}}\right)\left(\frac{1\ \text{mL}}{10^{-6}\ \cancel{\text{m}^3}}\right) = \boxed{29.6\ \text{mL}}$$

Note that all the units on the right, except one, are eliminated algebraically, leaving only the desired units of milliliters (mL).

6. ***REASONING*** To convert from liter to cubic centimeters, use the equivalence 1 L = 1000 mL and 1 mL = 1 cm^3 To find the thickness of the painted layer, we use

the fact that the paint's volume is the same, whether in the can or painted on the wall. The layer of paint on the wall can be thought of as a very thin "box" with a volume given by the product of the surface area (the "box top") and the thickness of the layer. Therefore, its thickness is the ratio of the volume to the painted surface area: Thickness = Volume/Area. That is, the larger the area it's spread over, the thinner the layer of paint.

SOLUTION

a. The conversion is

$$\left(2.75\ \cancel{L}\right)\left(\frac{1000\ \cancel{\text{mL}}}{\cancel{L}}\right)\left(\frac{1\ \text{cm}^3}{1\ \cancel{\text{mL}}}\right)=\boxed{2.75\times10^3\ \text{cm}^3}$$

b. The thickness is the volume found in (a) divided by the area (in cm^2), then converting to mm.

$$\text{Thickness}=\frac{\text{Volume}}{\text{Area}}=\frac{2.75\times10^3\ \text{cm}^3}{2.0\times10^5\ \text{cm}^2}=0.014\ \text{cm}=\boxed{0.14\ \text{mm}}$$

7. ***REASONING*** The dimension of the spring constant *k* can be determined by first solving the equation $T=2\pi\sqrt{m/k}$ for *k* in terms of the time *T* and the mass *m*. Then, the dimensions of *T* and *m* can be substituted into this expression to yield the dimension of *k*.

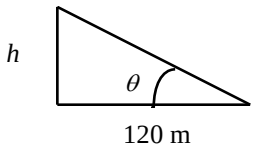
SOLUTION Algebraically solving the expression above for *k* gives $k=4\pi^2m/T^2$. The term $4\pi^2$ is a numerical factor that does not have a dimension, so it can be ignored in this analysis. Since the dimension for mass is [M] and that for time is [T], the dimension of *k* is

$$\text{Dimension of } k=\boxed{\frac{[\text{M}]}{[\text{T}]^2}}$$

8. ***REASONING AND SOLUTION*** The following figure (*not* drawn to scale) shows the geometry of the situation, when the observer is a distance *r* from the base of the obelisk.

The angle θ is related to *r* and *h* by $\tan\theta=h/r$. Solving for *h*, we find

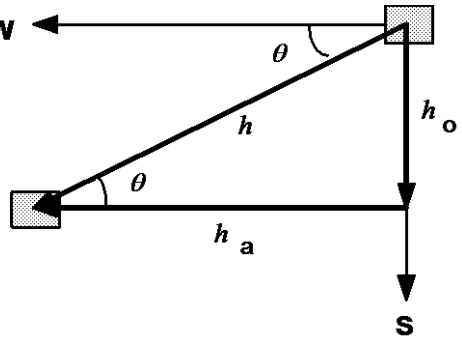
$$h=r\tan\theta=(120\ \text{m})\tan 12^\circ=\boxed{26\ \text{m}}$$



9. ***REASONING*** The shortest distance between the two towns is along the line that joins them. This distance, h, is the hypotenuse of a right triangle whose other sides are h_o = 18.0 km and h_a = 85.0 km, as shown in the figure below.

SOLUTION The angle θ is given by $\tan\theta = h_o / h_a$ so that

$$\theta = \tan^{-1}\left(\frac{18.0\text{ km}}{85.0\text{ km}}\right) = 12.0^\circ \text{ S of W}$$

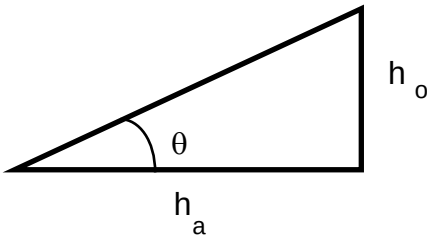


We can then use the Pythagorean theorem to find h.

$$h = \sqrt{h_o^2 + h_a^2} = \sqrt{(18.0\text{ km})^2 + (85.0\text{ km})^2} = \boxed{86.9\text{ km}}$$

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10. ***REASONING*** The drawing shows a schematic representation of the hill. We know that the hill rises 15.0 m vertically for every 100.0 m of distance in the horizontal direction, so that $h_o = 15.0$ m and $h_a = 100.0$ m. Moreover, according to Equation 1.3, the tangent function is $\tan\theta = h_o / h_a$. Thus, we can use the inverse tangent function to determine the angle θ.



SOLUTION With the aid of the inverse tangent function (see Equation 1.6) we find that

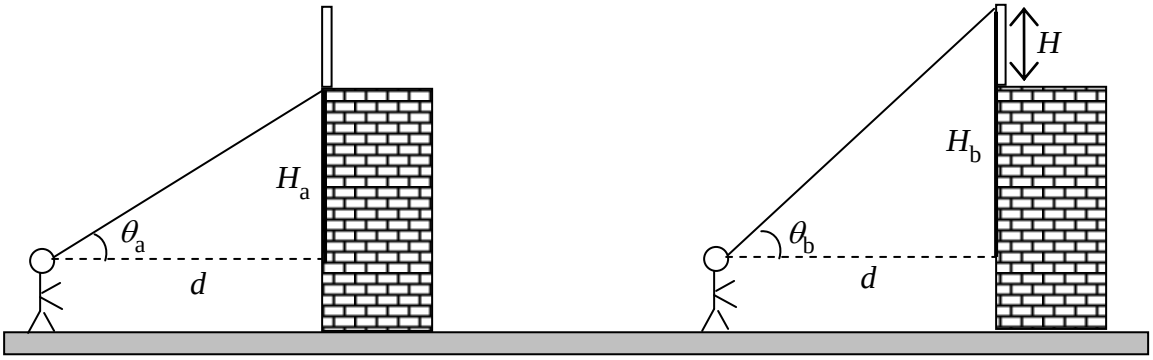
$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) = \tan^{-1}\left(\frac{15.0\text{ m}}{100.0\text{ m}}\right) = \boxed{8.53^\circ}$$

11. ***REASONING*** In both parts of the drawing the line of sight, the horizontal dashed line, and the vertical form a right triangle. The angles θ_a = 35.0° and θ_b = 38.0° at which the person's line of sight rises above the horizontal are known, as is the horizontal distance d = 85.0 m from the building. The unknown vertical sides of the right triangles correspond, respectively, to the heights H_a and H_b of the bottom and top of the antenna relative to the person's eyes. The antenna's height H is the *difference* between H_b and H_a: $H = H_b - H_a$. The horizontal side d of the triangle is adjacent to the angles θ_a and θ_b, while the vertical sides H_a and H_b are opposite these

angles. Thus, in either triangle, the angle θ is related to the horizontal and vertical sides by Equation 1.3 $\left(\tan\theta = \frac{h_o}{h_a} \right)$:

$$\tan\theta_a = \frac{H_a}{d} \tag{1}$$

$$\tan\theta_b = \frac{H_b}{d} \tag{2}$$



(*a*) (*b*)

SOLUTION Solving Equations (1) and (2) for the heights of the bottom and top of the antenna relative to the person's eyes, we find that

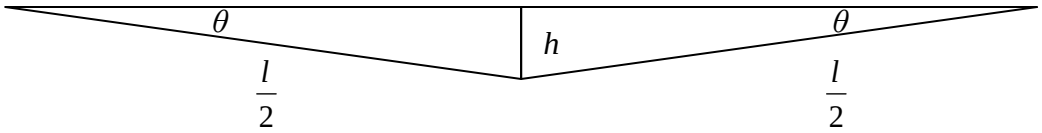
$$H_a = d\tan\theta_a \qquad \text{and} \qquad H_b = d\tan\theta_b$$

The height of the antenna is the difference between these two values:

$$H = H_b - H_a = d\tan\theta_b - d\tan\theta_a = d\left(\tan\theta_b - \tan\theta_a\right)$$

$$H = (85.0\text{ m})\left(\tan 38.0^\circ - \tan 35.0^\circ\right) = \boxed{6.9\text{ m}}$$

12. ***REASONING*** If we treat the clothesline and an imaginary line representing the clothesline in an unaltered position as two sides of a right triangle, we can solve for the missing side to find out how far below the horizontal the clothesline has sagged. We will use the fact that the shirt is in the exact center and the angle to use the sine function to solve the problem.



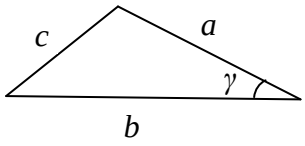
SOLUTION

The picture above shows the setup, where h is the sag, l represents the length of the clothesline (we're using $l/2$ since the shirt is in the middle) and θ is the angle given. Using the sine function to set $\sin\theta = h/(l/2)$ and solving for h we have:

$$h = \left(\frac{l}{2}\right)\sin\theta = \left(\frac{10\text{ m}}{2}\right)\sin\left(1^\circ\right) = 0.09\text{ m} = \boxed{9\text{ cm}}$$

13. ***REASONING*** As given in Appendix E, the law of cosines is

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$



where c is the side opposite angle γ, and a and b are the other two sides. Solving for γ, we have

$$\gamma = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

SOLUTION For c = 100 cm, a = 150 cm, and b = 190 cm

$$\gamma = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) = \cos^{-1}\left[\frac{(150\text{ cm})^2 + (190\text{ cm})^2 - (100\text{ cm})^2}{2(150\text{ cm})(190\text{ cm})}\right] = 32^\circ$$

Thus, the angle opposite the side of length 100 cm is $\boxed{31.5^\circ}$.

Similarly, for c = 150 cm, a = 100 cm, and b =190 cm, we find that the angle opposite the side of length 150 cm is $\boxed{51.6^0}$.

Finally, for c = 190 cm, a = 150 cm, and b = 100 cm, we find that the angle opposite the side of length 190 cm is $\boxed{96.8^\circ}$.

As a check on these calculations, we note that $31.5^\circ + 51.6^\circ + 96.8^\circ = 180^\circ$ (rounded off) which must be the case for the sum of the three angles in a triangle.

14. ***REASONING*** Since the initial force and the resultant force point along the east/west line, the second force must also point along the east/west line. The direction of the second force is not specified; it could point either due east or due west, so there are two answers. We use "N" to denote the units of the forces, which are specified in newtons.

SOLUTION If the second force points due east, both forces point in the same direction and the magnitude of the resultant force is the sum of the two magnitudes: $F_1 + F_2 = F_R$. Therefore,

$$F_2 = F_R - F_1 = 400 \text{ N} - 250 \text{ N} = \boxed{150 \text{ N}}$$

If the second force points due west, the two forces point in opposite directions, and the magnitude of the resultant force is the difference of the two magnitudes: $F_2 - F_1 = F_R$. Therefore,

$$F_2 = F_R + F_1 = 400 \text{ N} + 250 \text{ N} = \boxed{650 \text{ N}}$$

15. ***REASONING*** For convenience, we can assign due east to be the positive direction and due west to be the negative direction. Since all the vectors point along the same east-west line, the vectors can be added just like the usual algebraic addition of positive and negative scalars. We will carry out the addition for all of the possible choices for the two vectors and identify the resultants with the smallest and largest magnitudes.

SOLUTION There are six possible choices for the two vectors, leading to the following resultant vectors:

$\mathbf{F_1} + \mathbf{F_2} = 50.0 \text{ newtons} + 10.0 \text{ newtons} = +60.0 \text{ newtons} = 60.0 \text{ newtons, due east}$

$\mathbf{F_1} + \mathbf{F_3} = 50.0 \text{ newtons} - 40.0 \text{ newtons} = +10.0 \text{ newtons} = 10.0 \text{ newtons, due east}$

$\mathbf{F_1} + \mathbf{F_4} = 50.0 \text{ newtons} - 30.0 \text{ newtons} = +20.0 \text{ newtons} = 20.0 \text{ newtons, due east}$

$\mathbf{F_2} + \mathbf{F_3} = 10.0 \text{ newtons} - 40.0 \text{ newtons} = -30.0 \text{ newtons} = 30.0 \text{ newtons, due west}$

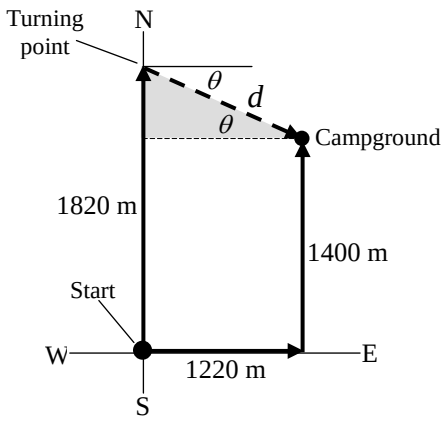
$\mathbf{F_2} + \mathbf{F_4} = 10.0 \text{ newtons} - 30.0 \text{ newtons} = -20.0 \text{ newtons} = 20.0 \text{ newtons, due west}$

$\mathbf{F_3} + \mathbf{F_4} = -40.0 \text{ newtons} - 30.0 \text{ newtons} = -70.0 \text{ newtons} = 70.0 \text{ newtons, due west}$

The resultant vector with the smallest magnitude is $\boxed{\mathbf{F_1} + \mathbf{F_3} = 10.0 \text{ newtons, due east}}$.

The resultant vector with the largest magnitude is $\boxed{\mathbf{F_3} + \mathbf{F_4} = 70.0 \text{ newtons, due west}}$.

16. ***REASONING*** At the turning point, the distance to the campground is labeled d in the drawing. Note that d is the length of the hypotenuse of a right triangle. Since we know the lengths of the other two sides of the triangle, the Pythagorean theorem can be used to find d. The direction that cyclist #2 must head during the last part of the trip is given by the angle θ. It can be determined by using the inverse tangent function.



SOLUTION

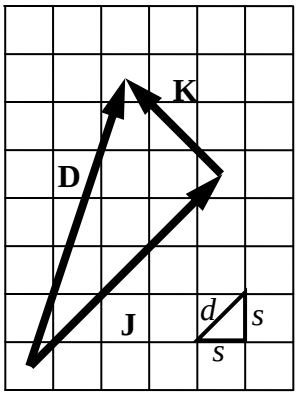
a. The two sides of the triangle have lengths of 1220 m and 420 m (1820 m – 1400 m = 420 m). The length d of the hypotenuse can be determined from the Pythagorean theorem, Equation (1.7), as

$$d = \sqrt{(1220\ \text{m})^2 + (420\ \text{m})^2} = \boxed{1290\ \text{m}}$$

b. Since the lengths of the sides opposite and adjacent to the angle θ are known, the inverse tangent function (Equation 1.6) can be used to find θ.

$$\theta = \tan^{-1}\left(\frac{420\ \text{m}}{1220\ \text{m}}\right) = \boxed{19^\circ \text{ south of east}}$$

17. ***REASONING*** The triple jump consists of a double jump in one direction, followed by a perpendicular single jump, which we can represent with displacement vectors **J** and **K** (see the drawing). These two perpendicular vectors form a right triangle with their resultant **D** = **J** + **K**, which is the displacement of the colored checker. In order to find the magnitude D of the displacement, we first need to find the magnitudes J and K of the double jump and the single jump. As the three sides of a right triangle, J, K, and D (the hypotenuse) are related to one another by the Pythagorean theorem (Equation 1.7) The double jump moves the colored checker a straight-line distance equal to the length of four square's diagonals d, and the single jump moves a length equal to two square's diagonals. Therefore,



$$J = 4d \qquad \text{and} \qquad K = 2d \tag{1}$$

Let the length of a square's side be s. Any two adjacent sides of a square form a right triangle with the square's diagonal (see the drawing). The Pythagorean theorem gives the diagonal length d in terms of the side length s:

$$d = \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2} \quad (2) \qquad (2)$$

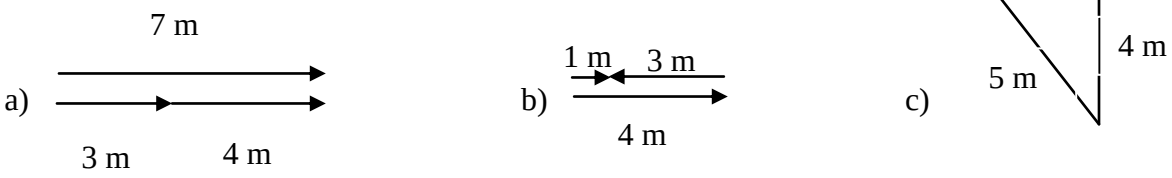
SOLUTION First, we apply the Pythagorean theorem to the right triangle formed by the three displacement vectors, using Equations (1) for *J* and *K*:

$$D = \sqrt{J^2 + K^2} = \sqrt{(4d)^2 + (2d)^2} = \sqrt{16d^2 + 4d^2} = \sqrt{20d^2} = d\sqrt{20} \qquad (3)$$

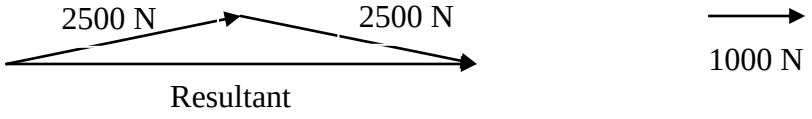
Substituting Equation (2) into Equation (3) gives

$$D = d\sqrt{20} = \left(s\sqrt{2}\right)\sqrt{20} = s\sqrt{40} = (5.5\text{ cm})\sqrt{40} = \boxed{35\text{ cm}}$$

18. ***REASONING AND SOLUTION*** In order to get the different resultants, we need to consider the different ways the two displacement vectors could be added together. If they are in the same direction the magnitudes will simply add together (7) and if they are in opposite directions then we would find the difference (1). Finally, if they are perpendicular to each other, then the resultant would be the hypotenuse of a 3-4-5 right triangle. Each case is drawn below:



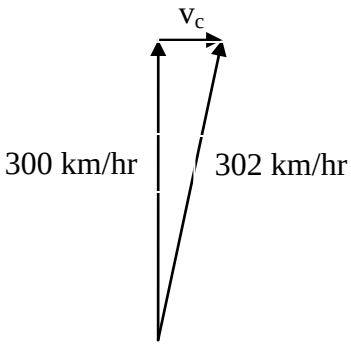
19. ***REASONING AND SOLUTION*** The single force needed to produce the same effect is equal to the resultant of the forces provided by the two ropes. The figure below shows the force vectors drawn to scale and arranged tail to head. The magnitude and direction of the resultant can be found by direct measurement using the scale factor shown in the figure.



a. From the figure, the magnitude of the resultant is $\boxed{4900\text{ N}}$

b. The single rope should be directed $\boxed{\text{along the dashed line}}$ in the text drawing.

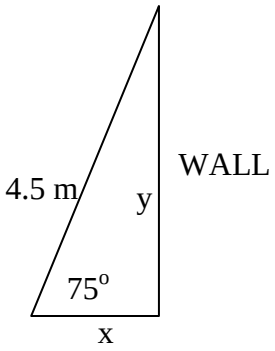
20. ***REASONING*** The result of the two velocities is given. The two components (the airplane's ground speed and the crosswind) are at right angles to each other and result in the overall speed of the airplane. The vector addition is shown at the right. By using the Pythagorean Theorem, we can solve for the cross-wind magnitude, v_c.



SOLUTION

$$v_c = \sqrt{(302 \text{ km/hr})^2 - (300 \text{ km/hr})^2} = \boxed{34.7 \text{ km/hr}}$$

21. ***REASONING*** The ladder leaning against the wall forms a right triangle with itself as the hypotenuse. As you climb along the ladder, you will travel the distances shown in the diagram to the right. We can use the sine and cosine functions to find the two unknowns.



SOLUTION

(a) $\cos\left(75^o\right) = \dfrac{x}{4.5 \text{ m}}$ $\qquad x = (4.5 \text{ m})\cos\left(75^o\right) = \boxed{1.1 \text{ m}}$

(b) $\sin\left(75^o\right) = \dfrac{y}{4.5 \text{ m}}$ $\qquad y = (4.5 \text{ m})\sin\left(75^o\right) = \boxed{4.3 \text{ m}}$

22. ***REASONING AND SOLUTION*** In order to determine which vector has the largest *x* and *y* components, we calculate the magnitude of the *x* and *y* components explicitly and compare them. In the calculations, the symbol u denotes the units of the vectors.

A_x = (200.0 u) cos 90.0° = 0.00 u $\qquad$ A_y = (200.0 u) sin 90.0° = 200.0 u

B_x = (250.0 u) cos 60.0° = 125.0 u $\qquad$ B_y = (250.0 u) sin 60.0° = 216.5 u

C_x = (150.0 u) cos 0.00° = 150.0 u $\qquad$ C_y = (150.0 u) sin 0.00° = 0.00 u

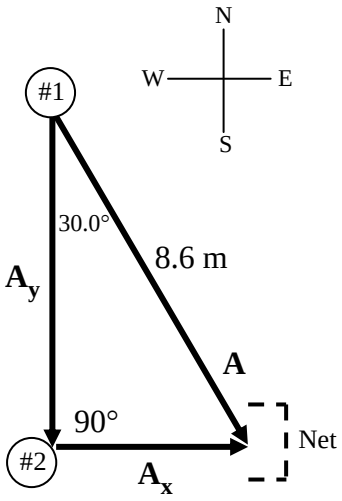
a. **C** has the largest *x* component.

b. **B** has the largest *y* component.

23. ***REASONING*** The triangle in the drawing is a right triangle. We know one of its angles is 30.0°, and the length of the hypotenuse is 8.6 m. Therefore, the sine and cosine functions can be used to find the magnitudes of $\mathbf{A_x}$ and $\mathbf{A_y}$. The directions of these vectors can be found by examining the diagram.

SOLUTION

a. The magnitude A_x of the displacement vector $\mathbf{A_x}$ is related to the length of the hypotenuse and the 30.0° angle by the sine function (Equation 1.1). The drawing shows that the direction of $\mathbf{A_x}$ is due east.

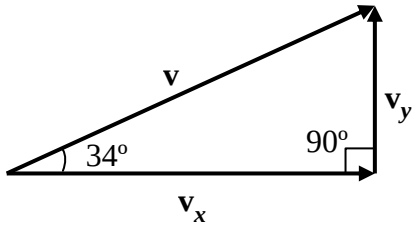


$$A_x = A\sin 30.0° = (8.6\text{ m})\sin 30.0° = \boxed{4.3\text{ m, due east}}$$

b. In a similar manner, the magnitude A_y of $\mathbf{A_y}$ can be found by using the cosine function (Equation 1.2). Its direction is due south.

$$A_y = A\cos 30.0° = (8.6\text{ m})\cos 30.0° = \boxed{7.4\text{ m, due south}}$$

24. ***REASONING*** The drawing assumes that the horizontal direction along the ground is the x direction and shows the plane's velocity vector $\mathbf{v}$, along with its horizontal component $\mathbf{v}_x$ and vertical component $\mathbf{v}_y$. These components, together with the velocity vector, form a right triangle, as indicated. Based on this right triangle, we can use the cosine function to determine the horizontal velocity component.



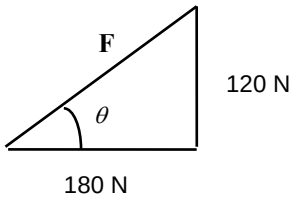
SOLUTION According to Equation 1.2, the cosine of an angle is the side of the right triangle adjacent to the angle divided by the hypotenuse. Thus, for the 34° angle in the drawing we have

$$\cos 34° = \frac{v_x}{v} \qquad \text{or} \qquad v_x = v\cos 34° = (150\text{ m/s})\cos 34° = \boxed{120\text{ m/s}}$$

25. ***REASONING AND SOLUTION***

a. From the Pythagorean theorem, we have

$$F = \sqrt{(120\text{ N})^2 + (180\text{ N})^2} = \boxed{220\text{ N}}$$



b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{120\text{ N}}{180\text{ N}}\right) = \boxed{34^\circ}$$

26. ***REASONING*** Because both boats travel at 125 km per hour, each one ends up (0.750 h)(125 km/h) = 93.8 km from the dock after a half-hour. They travel along straight paths, so this is the magnitude of both displacement vectors: $B = G = 93.8$ km. Since the displacement vector **G** makes an angle of 37° south of due west, its direction can also be expressed as 90° − 37° = 53° west of south. With these angles and the magnitudes of both vectors in hand, we can consider the westward and southward components of each vector.

SOLUTION

a. First we calculate the magnitudes of the westward component of the displacement of each boat and then subtract them to find the difference:

$$\text{Magnitude of } B_{\text{west}} = B\cos 25.0^\circ = (93.8\text{ km})\cos 25.0^\circ = 85.0\text{ km}$$

$$\text{Magnitude of } G_{\text{west}} = G\sin 53.0^\circ = (93.8\text{ km})\sin 53.0^\circ = 74.9\text{ km}$$

The blue boat travels farther by the following amount:

$$85.0\text{ km} - 74.9\text{ km} = \boxed{10.1\text{ km}}$$

b. Similarly, we find for the magnitudes of the southward components that

$$\text{Magnitude of } B_{\text{south}} = B\sin 25.0^\circ = (93.8\text{ km})\sin 25.0^\circ = 39.6\text{ km}$$

$$\text{Magnitude of } G_{\text{south}} = G\cos 53.0^\circ = (93.8\text{ km})\cos 53.0^\circ = 56.5\text{ km}$$

The green boat travels farther by the following amount:

$$56.5\text{ km} - 39.6\text{ km} = \boxed{16.9\text{ km}}$$

27. ***REASONING*** The force **F** and its two components form a right triangle. The hypotenuse is 82.3 newtons, and the side parallel to the $+x$ axis is F_x = 74.6 newtons. Therefore, we can use the trigonometric cosine and sine functions to determine the angle of **F** relative to the $+x$ axis and the component F_y of **F** along the $+y$ axis.

SOLUTION

a. The direction of **F** relative to the $+x$ axis is specified by the angle θ as

$$\theta = \cos^{-1}\left(\frac{74.6 \text{ newtons}}{82.3 \text{ newtons}}\right) = \boxed{25.0^\circ} \qquad (1.5)$$

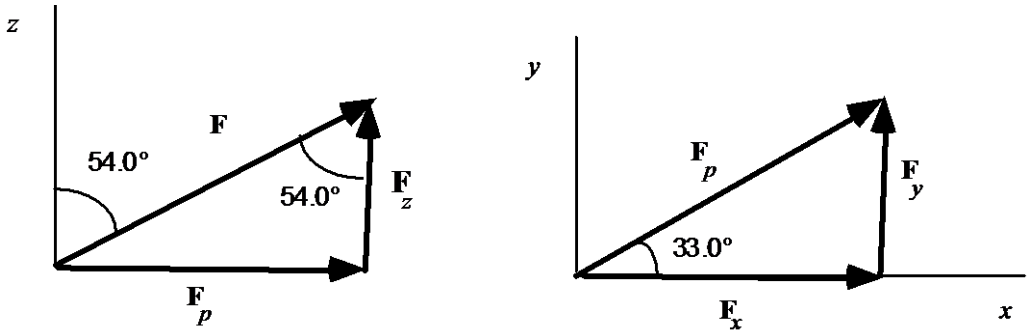
b. The component of **F** along the $+y$ axis is

$$F_y = F \sin 25.0^\circ = (82.3 \text{ newtons}) \sin 25.0^\circ = \boxed{34.8 \text{ newtons}} \qquad (1.4)$$

28. ***REASONING AND SOLUTION*** The force **F** can be first resolved into two components; the z component F_z and the projection onto the x-y plane, F_p as shown in the figure below on the left. According to that figure,

$$F_p = F \sin 54.0^\circ = (475 \text{ N}) \sin 54.0^\circ = 384 \text{ N}$$

The projection onto the x-y plane, F_p, can then be resolved into x and y components.



a. From the figure on the right,

$$F_x = F_p \cos 33.0^\circ = (384 \text{ N}) \cos 33.0^\circ = \boxed{322 \text{ N}}$$

b. Also from the figure on the right,

$$F_y = F_p \sin 33.0° = (384 \text{ N}) \sin 33.0° = \boxed{209 \text{ N}}$$

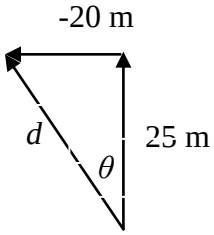
c. From the figure on the left,

$$F_z = F \cos 54.0° = (475 \text{ N}) \cos 54.0° = \boxed{279 \text{ N}}$$

29. ***REASONING AND SOLUTION*** Since the treasure hunter is always moving in the cardinal directions, we can treat each of these like vectors along the x and y axes and simple add the movements along each axis together. Finally, we can use the two resulting vectors as components of the solution we need.

Adding up all the of movement in the E/W direction and the N/S direction (treating E and N as positive):

$$x_{result} = -25 \text{ m} + 5 \text{ m} = -20 \text{ m}$$
$$y_{result} = 15 \text{ m} + 10 \text{ m} = 25 \text{ m}$$



Now we can add these two components to get the result and find the angle

$$d = \sqrt{(25 \text{ m})^2 + (-20 \text{ m})^2} = \boxed{32 \text{ m}}$$

at an angle of

$$\theta = \tan^{-1}\left(\frac{20 \text{ m}}{25 \text{ m}}\right) = \boxed{39° \text{ W of N}}$$

30. ***REASONING*** To apply the component method for vector addition, we must first determine the *x* and *y* components of each vector. The algebraic sum of the three *x* components gives the *x* component of the resultant. The algebraic sum of the three *y* components gives the *y* component of the resultant. Knowing the *x* and *y* components of the resultant will allow us to use the Pythagorean theorem to determine the magnitude of the resultant. Finally, the directional angle of the resultant will be obtained using the trigonometric sine function.

SOLUTION Referring to the drawing in the text, we see that the *x* and *y* components of the vectors are

$A_x = -A\cos 20.0° = -(5.00\text{ m})\cos 20.0° = -4.70\text{ m}$ | $A_y = A\sin 20.0° = (5.00\text{ m})\sin 20.0° = 1.71\text{ m}$

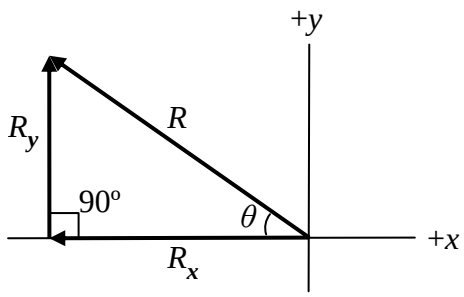
$B_x = B\cos 60.0° = (6.00\text{ m})\cos 60.0° = 3.00\text{ m}$ | $B_y = B\sin 60.0° = (6.00\text{ m})\sin 60.0° = 5.20\text{ m}$

$C_x = 0.00\text{ m}$ | $C_y = -4.00\text{ m}$

$R_x = -4.70\text{ m} + 3.00\text{ m} + 0.00\text{ m} = -1.70\text{ m}$ | $R_y = 1.71\text{ m} + 5.20\text{ m} - 4.00\text{ m} = 2.91\text{ m}$

Note that the value for A_x is negative because this component points in the $-x$ direction and that the value for C_x is zero because the vector **C** points along the $-y$ axis. Note also that the value for C_y is negative because the vector **C** points along the $-y$ axis.

The x component R_x of the resultant vector, being negative, points in the $-x$ direction. The y component R_y of the resultant vector, being positive, points in the $+y$ direction. The drawing shows these two components and the resultant vector. Since the components are perpendicular, the magnitude R of the resultant can be obtained using the Pythagorean theorem.



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-1.70\text{ m})^2 + (2.91\text{ m})^2} = \boxed{3.37\text{ m}}$$

Referring to the drawing, we can see that $\sin\theta = R_x / R$, so that the directional angle θ is

$$\theta = \sin^{-1}\left(\frac{R_y}{R}\right) = \sin^{-1}\left(\frac{2.91\text{ m}}{3.37\text{ m}}\right) = 59.7°$$

Thus, the resultant vector points in a direction of $\boxed{30.3°\text{ above the negative } x \text{ axis}}$.

31. ***REASONING*** Using the component method for vector addition, we will find the x component of the resultant force vector by adding the x components of the individual vectors. Then we will find the y component of the resultant vector by adding the y components of the individual vectors. Once the x and y components of the resultant are known, we will use the Pythagorean theorem to find the magnitude of the resultant and trigonometry to find its direction. We will take east as the $+x$ direction and north as the $+y$ direction.

SOLUTION The *x* component of the resultant force **F** is

$$F_x = \underbrace{(2240\ \text{N})\cos 34.0^\circ}_{F_{Ax}} + \underbrace{(3160\ \text{N})\cos 90.0^\circ}_{F_{Bx}} = (2240\ \text{N})\cos 34.0^\circ$$

The *y* component of the resultant force **F** is

$$F_y = \underbrace{-(2240\ \text{N})\sin 34.0^\circ}_{F_{Ay}} + \underbrace{(-3160\ \text{N})}_{F_{By}}$$

Using the Pythagorean theorem, we find that the magnitude of the resultant force is

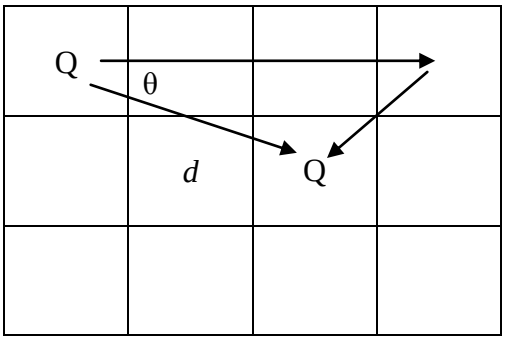
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2240\ \text{N})\cos 34.0^{\circ\ 2} + \left[-(2240\ \text{N})\sin 34.0^\circ - 3160\ \text{N}\right]^2} = \boxed{4790\ \text{N}}$$

Using trigonometry, we find that the direction of the resultant force is

$$\theta = \tan^{-1}\left[\frac{(2240\ \text{N})\sin 34.0^\circ + 3160\ \text{N}}{(2240\ \text{N})\cos 34.0^\circ}\right] \quad \boxed{67.2^\circ \text{ south of east}}$$

32. ***REASONING***

We can diagram the situation as shown to the right. Each square can be considered a "unit" (of length 50 mm) and we can simply add the x and y components of each movement and use them to calculate the resultant displacement using the Pythagorean Theorem and the tangent function.



SOLUTION

a)

Movement 1 components: x = + 3 units, y = 0 units

Movement 2 components: x = -1 units, -1 units

$$d_x = \left[(+3\ \text{units}) + (-1\ \text{units})\right](50\ \text{mm}) = 100\ \text{mm}$$

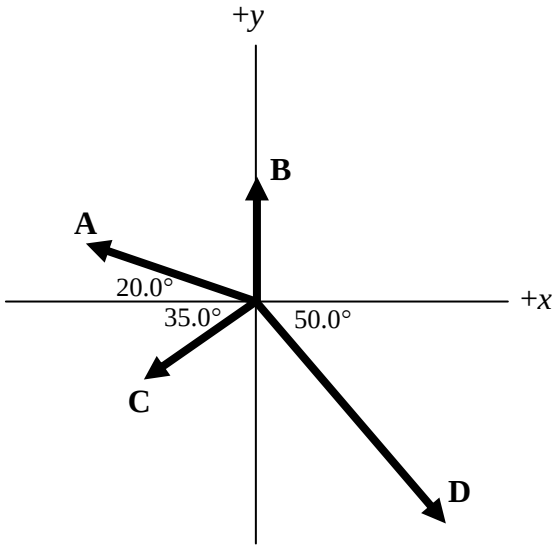
$$d_y = \left[(0\ \text{units}) + (-1\ \text{units})\right](50\ \text{mm}) = -50\ \text{mm}$$

$$d = \sqrt{(100\ \text{mm})^2 + (-50\ \text{mm})^2} = \boxed{110\ \text{mm}}$$

b)

$$\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-50\text{ mm}}{100\text{ mm}}\right) = \boxed{-27^\circ}$$

33. ***REASONING*** We will use the scalar x and y components of the resultant vector to obtain its magnitude and direction. To obtain the x component of the resultant we will add together the x components of each of the vectors. To obtain the y component of the resultant we will add together the y components of each of the vectors.



SOLUTION

The x and y components of the resultant vector **R** are R_x and R_y, respectively. In terms of these components, the magnitude R and the directional angle θ (with respect to the x axis) for the resultant are

$$R = \sqrt{R_x^2 + R_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \qquad (1)$$

The following table summarizes the components of the individual vectors shown in the drawing:

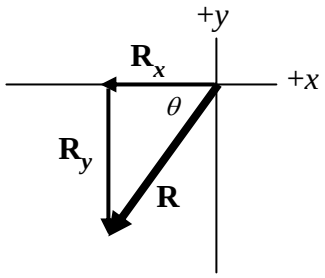
Vector	***x* component**	***y* component**
A	$A_x = -(20.0\text{ m})\cos 20.0^\circ = -18.8\text{ m}$	$A_y = (20.0\text{ m})\sin 20.0^\circ = 6.84\text{ m}$
B	$B_x = 0\text{ m}$	$B_y = 15.0\text{ m}$
C	$C_x = -(12.0\text{ m})\cos 35.0^\circ = -9.83\text{ m}$	$C_y = -(12.0\text{ m})\sin 35.0^\circ = -6.88\text{ m}$
D	$D_x = (36.0\text{ m})\cos 50.0^\circ = 23.1\text{ m}$	$D_y = -(36.0\text{ m})\sin 50.0^\circ = -27.6\text{ m}$
R	$R_x = -18.8\text{ m} + 0\text{ m} - 9.83\text{ m} + 23.1\text{ m}$ $= -5.5\text{ m}$	$R_y = 6.84\text{ m} + 11.0\text{ m} - 6.88\text{ m} - 27.6\text{ m}$ $= -12.6\text{ m}$

Note that the component B_x is zero, because **B** points along the y axis. Note also that the components C_x and C_y are both negative, since **C** points between the $-x$ and $-y$ axes. Finally, note that the component D_y is negative since **D** points below the $+x$ axis.

Using equation (1), we find that

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-5.5\text{ m})^2 + (-12.6\text{ m})^2} = \boxed{13.8\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-12.6\text{ m}}{-5.5\text{ m}}\right) = \boxed{66°}$$



Since both R_x and R_y are negative, the resultant points between the $-x$ and $-y$ axes.

34. ***REASONING*** If we let the directions due east and due north be the positive directions, then the desired displacement **A** has components

$$A_E = (4.8\text{ km})\cos 42° = 3.57\text{ km}$$

$$A_N = (4.8\text{ km})\sin 42° = 3.21\text{ km}$$

while the actual displacement **B** has components

$$B_E = (3.2\text{ km})\cos 22° = 2.97\text{ km}$$

$$B_N = (3.2\text{ km})\sin 22° = 1.20\text{ km}$$

Therefore, to reach the research station, the research team must go

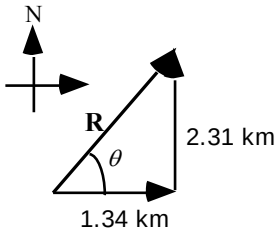
$$3.57\text{ km} - 2.97\text{ km} = 0.60\text{ km, eastward}$$

and

$$3.21\text{ km} - 1.20\text{ km} = 2.01\text{ km, northward}$$

SOLUTION

a. From the Pythagorean theorem, we find that the magnitude of the displacement vector required to bring the team to the research station is



$$R = \sqrt{(0.60\text{ km})^2 + (2.01\text{ km})^2} = \boxed{2.10\text{ km}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{2.01\text{ km}}{0.60\text{ km}}\right) = \boxed{73.6^\circ}$$

35. ***REASONING*** Since the finish line is coincident with the starting line, the net displacement of the sailboat is zero. Hence the sum of the components of the displacement vectors of the individual legs must be zero. In the drawing in the text, the directions to the right and upward are taken as positive.

SOLUTION In the horizontal direction $R_h = A_h + B_h + C_h + D_h = 0$

$$R_h = (3.20\text{ km})\cos 40.0^\circ - (5.10\text{ km})\cos 35.0^\circ - (4.80\text{ km})\cos 23.0^\circ + D\cos\theta = 0$$

$$D\cos\theta = 6.14\text{ km} \qquad (1)$$

In the vertical direction $R_v = A_v + B_v + C_v + D_v = 0$.

$$R_v = (3.20\text{ km})\sin 40.0^\circ + (5.10\text{ km})\sin 35.0^\circ - (4.80\text{ km})\sin 23.0^\circ - D\sin\theta = 0.$$

$$D\sin\theta = 3.11\text{ km} \qquad (2)$$

Dividing (2) by (1) gives

$$\tan\theta = (3.11\text{ km})/(6.14\text{ km}) \quad \text{or} \quad \theta = \boxed{26.9^\circ}$$

Solving (1) gives

$$D = (6.14\text{ km})/\cos 26.9^\circ = \boxed{6.88\text{ km}}$$

36. ***REASONING*** The following table shows the components of the individual displacements and the components of the resultant. The directions due east and due north are taken as the positive directions.

Displacement	*East/West Component*	*North/South Component*
(1)	–22.0 cm	0
(2)	–(15.0 cm) cos 40.0° = –11.49 cm	–(15.0 cm) sin 40.0° = –9.64 cm
(3)	(18.5 cm) cos 55.0° = 10.61 cm	–(18.5 cm) sin 55.0° = –15.15 cm
(4)	(22.0 cm) cos 30.0° = 19.05 cm	(22.0 cm) sin 30.0° = 11 cm
Resultant	–3.83 cm	–13.79 cm

SOLUTION

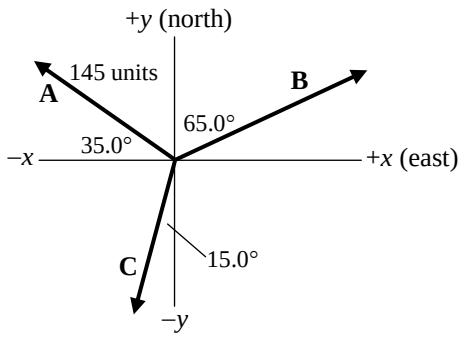
a. From the Pythagorean theorem, we find that the magnitude of the resultant displacement vector is

$$R = \sqrt{(\text{-}3.83\text{ cm})^2 + (\text{-}13.79\text{ cm})^2} = \boxed{14.3\text{ cm}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{13.79\text{ cm}}{3.83\text{ cm}}\right) = \boxed{74.5°,\text{ south of west}}$$

37. ***REASONING*** The drawing shows the vectors **A**, **B**, and **C**. Since these vectors add to give a resultant that is zero, we can write that **A + B + C =** 0. This addition will be carried out by the component method. This means that the *x*-component of this equation must be zero ($A_x + B_x + C_x = 0$) and the *y*-component must be zero ($A_y + B_y + C_y = 0$). These two equations will allow us to find the magnitudes of **B** and **C**.



SOLUTION The *x*- and *y*-components of **A**, **B**, and **C** are given in the table below. The plus and minus signs indicate whether the components point along the positive or negative axes.

Vector	*x Component*	*y Component*
A	–(145 units) cos 35.0° = –119 units	+(145 units) sin 35.0° = +83.2 units
B	+B sin 65.0° = +B (0.906)	+B cos 65.0° = +B (0.423)
C	–C sin 15.0° = –C (0.259)	–C cos 15.0° = –C (0.966)
A + B + C	–119 units + B (0.906) – C (0.259)	+83.2 units + B (0.423) – C (0.966)

Setting the separate *x*- and *y*- components of **A + B + C** equal to zero gives

x-component (–119 units) + B (0.906) – C (0.259) = 0

y-component (+83.2 units) + B (0.423) – C (0.966) = 0

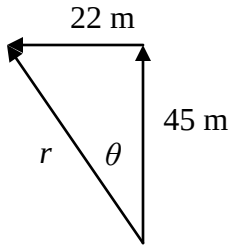
Solving these two equations simultaneously, we find that

a. B = 178 units b. C = 164 units

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38. ***REASONING*** The shortest distance between the tree and the termite mound is equal to the magnitude of the chimpanzee's displacement **r**, shown below.

SOLUTION



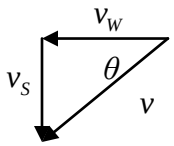
a. From the Pythagorean theorem, we have

$$r = \sqrt{(45\text{ m})^2 + (22\text{ m})^2} = \boxed{50\text{ m}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{22\text{ m}}{45\text{ m}}\right) = \boxed{26.1^\circ}$$

39. ***REASONING*** The ostrich's velocity vector **v** and the desired components are shown in the figure at the right. The components of the velocity in the directions due west and due south are v_W and v_S , respectively. The sine and cosine functions can be used to find the components.



SOLUTION

a. According to the definition of the sine function, we have for the vectors in the figure

$$\sin\theta = \frac{v_S}{v} \quad \text{or} \quad v_S = v\sin\theta = (15.0\text{ m/s})\sin(22.0^\circ) = \boxed{5.62\text{ m/s}}$$

b. Similarly,

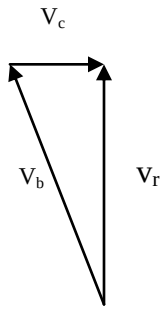
$$\cos\theta = \frac{v_W}{v} \quad \text{or} \quad v_W = v\cos\theta = (15.0\text{ m/s})\cos(22.0^\circ) = \boxed{13.9\text{ m/s}}$$

40. ***REASONING AND SOLUTION*** The east and north components are, respectively

a. $A_e = A \cos\theta = (250 \text{ km}) \cos 70.0° = \boxed{85.5 \text{ km}}$

b. $A_n = A \sin\theta = (250 \text{ km}) \sin 70.0° = \boxed{235 \text{ km}}$

41. ***REASONING*** The boat will need to be directed upstream so that the upstream component of the velocity is exactly canceled by the current, resulting in the boat traveling straight across the river. This situation, using vector addition, is shown in the figure to the right.



Since V_b and V_c (the velocity of the boat and the velocity of the current, respectively) are known, we can use the sine function to solve for the magnitude and direction.

SOLUTION

$$\theta = \sin\left(\frac{2 \text{ km/hr}}{10 \text{ km/hr}}\right) = \boxed{11.5^\circ \text{ W of N}}$$

42. ***REASONING*** We can find the total displacement of the ball by adding the components of the two separate displacements, one as the ball travels down the ramp and the other as it travels across the floor. We can then use these components to find the final displacement (both the magnitude and direction)

SOLUTION

Displacement	***Horizontal Component***	***Vertical Component***
Down Ramp	$(1 \text{ m})/\tan(40^o) = 1.2 \text{ m}$	1.0 m
Across Floor	7 m	0 m
Resultant	7.2 m	1.0 m

From the Pythagorean theorem, we find that the magnitude of the resultant displacement vector is

$$R = \sqrt{(7.2\text{ m})^2 + (1.0\text{ m})^2} = \boxed{7.3\text{ m}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{1.0\text{ m}}{7.3\text{ cm}}\right) = \boxed{-7.8^\circ \text{ with respect to the horizontal}}$$

43. ***REASONING*** We are given that the vector sum of the three forces is zero, so $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = 0$ N. Since $\mathbf{F_1}$ and $\mathbf{F_2}$ are known, $\mathbf{F_3}$ can be found from the relation $\mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2})$. We will use the *x*- and *y*-components of this equation to find the magnitude and direction of $\mathbf{F_3}$.

SOLUTION The *x*- and *y*-components of the equation $\mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2})$ are:

x-component $F_{3x} = -\left(F_{1x} + F_{2x}\right)$ (1)

y-component $F_{3y} = -\left(F_{1y} + F_{2y}\right)$ (2)

The table below gives the *x*- and *y*-components of $\mathbf{F_1}$ and $\mathbf{F_2}$:

Vector	***x* component**	***y* component**
$\mathbf{F_1}$	$F_{1x} = -(21.0\text{ N})\sin 30.0^\circ = -10.5\text{ N}$	$F_{1y} = -(21.0\text{ N})\cos 30.0^\circ = +18.2\text{ N}$
$\mathbf{F_2}$	$F_{2x} = +15.0\text{ N}$	$F_{2y} = 0\text{ N}$

Substituting the values for F_{1x} and F_{2x} into Equation (1) gives

$$F_{3x} = -\left(F_{1x} + F_{2x}\right) = -\left(-10.5\text{ N} + 15.2\text{ N}\right) = -4.5\text{ N}$$

Substituting F_{1y} and F_{2y} into Equation (2) gives

$$F_{3y} = -\left(F_{1y} + F_{2y}\right) = -\left(+18.2\text{ N} + 0\text{ N}\right) = -18.2\text{ N}$$

The magnitude of $\mathbf{F_3}$ can now be obtained by employing the Pythagorean theorem:

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \sqrt{(-4.5\text{ N})^2 + (-18.2\text{ N})^2} = \boxed{18.7\text{ N}}$$

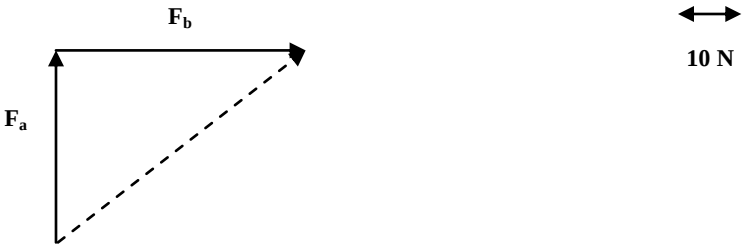
The angle θ that $\mathbf{F_3}$ makes with respect to the $-x$ axis can be determined from the inverse tangent function (Equation 1.6),

$$\theta = \tan^{-1}\left(\frac{F_{3y}}{F_{3x}}\right) = \tan^{-1}\left(\frac{-18.2\text{ N}}{-4.5\text{ N}}\right) = \boxed{76°}$$

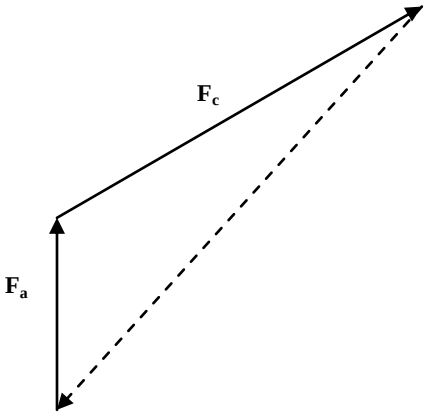
44. ***REASONING*** The vectors can be added "tip-to-tail" using a proper scale factor and the directions measured with a protractor. Each situation is diagramed below (resultant given as a dotted line) with then given scale factor. Note that in order to subtract a vector we reverse the direction while keeping the magnitude the same.

SOLUTION

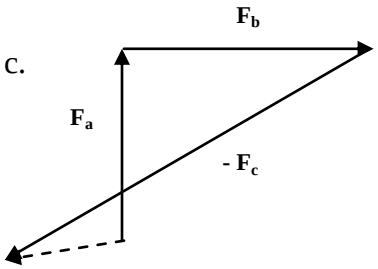
a.



b.



c.



45. ***REASONING AND SOLUTION*** If **D** is the unknown vector, then **A** + **B** + **C** + **D** = 0 requires that $D_E = -(A_E + B_E + C_E)$ or

$$D_E = (113 \text{ u}) \cos 60.0° - (222 \text{ u}) \cos 35.0° - (177 \text{ u}) \cos 23.0° = \boxed{-288 \text{ units}}$$

The minus sign indicates that D_E has a direction of due west.

Also, $D_N = -(A_N + B_N + C_N)$ or

$$D_N = (113 \text{ u}) \sin 60.0° + (222 \text{ u}) \sin 35.0° - (177 \text{ u}) \sin 23.0° = \boxed{156 \text{ units}}$$