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## Problem Solutions, Chapter 1

1.1 Steady, fully developed, laminar, circular pipe flow:

(a) The radial equation of motion shows that $p$ does not depend on $r$. $u$ does not depend on $x$ because the flow is fully developed. Differentiation of the axial equation of motion then shows that $\partial p/\partial x$ does not depend on $x$.

(b) Since the pressure gradient does not depend on $r$, integrating this equation over $r$ and using $u = 0$ at the wall gives Eq. (1.1).

(c) straightforward

(d) The pumping power is the net axial pressure force on the fluid times its average velocity, $-\partial P/\partial x \times L \times A \times u_{ave}$ (J/s). The rate of mass flow is $\rho A u_{ave}$ (kg/s). Their ratio is $fLu_{ave}^2/2D$ (J/kg).

1.2 Generalize to include conduction heat transfer in the radial direction:

(a) The left side of the differentiated equation vanishes, so it reads

$$0 = \frac{\alpha}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\frac{\partial T}{\partial x}.$$

Thus $r\partial(\partial T/\partial x)/\partial r$ does not depend on $r$, so that $\partial(\partial T/\partial x)/\partial r = \text{const}/r$. The constant must vanish to prevent a singularity at $r = 0$, so $\partial T/\partial x$ does not depend on $r$. It suffices to show that Eq. 1.11 is a solution to this equation.

(b) This expression is Eq. (1.10) with the temperature gradient evaluated from Eq. (1.11).

1.3 A time average over an interval $T$ is defined as:

$$\overline{u}(\mathbf{x}, t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} u(\mathbf{x}, t + t')dt'.$$

Spatial differentiation commutes with this time averaging.

1.4 Term by term this produces (using the comma notation for differentiation) and defining $u_i u_i = 2e$

$$u_{i,t}u_i = e_{,t} \quad \text{(local time change)}$$

$$u_i u_j u_{i,j} = (u_j e)_{,j} \quad \text{(advection)}$$

$$-\frac{1}{\rho}p_{,i}u_i = -\frac{1}{\rho}(pu_i)_{,i} \quad \text{(pressure transport)}$$

$$\nu u_i u_{i,jj} = \nu e_{,jj} - \nu u_{i,j}u_{i,j} \quad \text{(diffusion, viscous dissipation)}$$

All but the first and last terms are divergences, and so by the Divergence

Theorem they integrate to zero over the flow volume. Thus $e$ decays in time due to viscous dissipation.

1.5 If we apply a body force per unit mass $f_i(\mathbf{x}, t)$ to the fluid, $u_i f_i$ appears on the right side of the $e$ equation. Now a volume-integrated equilibrium between the rate of kinetic energy production through $u_i f_i$ and its rate of loss by conversion to internal energy (viscous dissipation) is possible.

1.6 From Problem 1.4 $\epsilon = \nu u_{i,j} u_{i,j}$. It is found that $\epsilon \sim u^3/\ell$. But

$$\frac{\nu u^2}{\ell^2} \sim \frac{u^3}{\ell}\frac{\nu}{u\ell} = \frac{u^3}{\ell} R_t^{-1} \ll \epsilon.$$

1.7 Since $u_{i,j} = (u_{i,j} + u_{j,i})/2 + (u_{i,j} - u_{j,i})/2 = s_{ij} + r_{ij}$, the sum of strain-rate and rotation-rate tensors, and since $\epsilon = \nu u_{i,j} u_{i,j}$, we can write $\epsilon = \nu(s_{ij} + r_{ij})(s_{ij} + r_{ij})$. But $s_{ij} r_{ij} = 0$ so this simplifies to $\epsilon = \nu(s_{ij} s_{ij} + r_{ij} r_{ij}) = 2\nu s_{ij} s_{ij}$. From (1.24) $2\nu s_{ij} = \sigma_{ij}/\rho$, the kinematic viscous stress tensor. Thus $\epsilon$ is the scalar product, or contraction, of the viscous (kinematic) stress tensor and the strain-rate tensor.

1.8 The viscous term in (1.38) is $\nu \overline{u}_{i,jj}$. If we denote the velocity and length scales of the averaged velocity as $\overline{u}$ and $\overline{\ell}$, then this term is of the order of $\nu/\overline{u}\overline{\ell} = 1/\overline{R}_t$, the inverse Reynolds number of the averaged motion.

1.9 The computational domain is of order $\ell^3$. The mesh size $\Delta \sim \eta$. Thus one needs $\sim (\ell/\eta)^3$ grid points. From Eq. (1.37) $(\ell/\eta)^3 \sim R_t^{9/4}$.

1.10 Because its turbulence time scale $\ell/u$ is so large.

1.11 One takes the gradient of Eq. (1.31) to get Eq. (14.31).

1.12 Straightforward.

1.13 Linear equations (not the Navier-Stokes equation) have this property. We use it (without giving it a second thought) in adding the concentration fields downwind of multiple sources.

1.14 This is straightforward but "tricky". $\epsilon \sim u^3/\ell$ is the dissipation rate per unit mass, so the fluid density enters the total dissipation rate.

1.15 Straightforward.

1.16 The velocity derivative covariance must scale as $1/\nu$.

1.17 This property of nonlinearity is central to understanding of turbulence system: fluctuations tend to be correlated.

1.18 The identities make this straightforward.

1.19 Straightforward.

1.20 Here the volume of integration is bounded by the pipe wall and the upstream and downstream cross sections. By the Divergence Theorem a volume integral of a divergence of a vector quantity is the integral over the bounding surface of the dot product of the vector quantity and the outward normal of the bounding surface. Here the outward normals to these cross

sections are in opposite directions. As a result the integrated advection and molecular diffusion terms vanish. Because of the streamwise pressure gradient in the pipe the volume-integrated pressure-velocity term does not vanish; it balances the integrated viscous dissipation.

1.21 This is the case if the velocity appearing in the total time derivative depends on the second variable of differentiation.

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